Analysis of Boolean Functions

CS 6817: Special Topics in Complexity Theory, Fall 2020

1 Essential information

- Instructor: Eshan Chattopadhyay
  - Email: eshan@cs.cornell.edu
  - Website: `https://www.cs.cornell.edu/~eshan/`
- Course webpage: `https://courses.cs.cornell.edu/cs6817/2020fa/`
- Lecture
  - Timing: TR 9:55-11:10am
  - Location: Schwartz Ctr Performing Arts, Room 111
  - Synchronous attendance is **not** mandatory.

2 Course description and resources

A central focus of this course will be to uncover properties of Boolean functions (i.e., functions of the form $f : \{0,1\}^n \to \{0,1\}$) via analytic methods (such as studying their Fourier spectrum). Such methods have seen remarkable applications in various fields of theoretical computer science such as property testing, hardness of approximation, learning theory, pseudorandomness, etc, and is by now an essential ingredient in a theorist's toolkit. These methods have also found nice applications beyond computer science, in areas such as combinatorics, random graphs and metric spaces. We will develop this theory from the basics and cover a variety of applications in the course.

Textbook: *Analysis of Boolean Functions* by Ryan O'Donnell. You can buy the book or find a draft of the book here: `http://www.contrib.andrew.cmu.edu/~ryanod/?page_id=2334`. The course website will be populated with other resources as the semester progresses.

3 Prerequisites

CS 4820 or permission from the instructor. In general, some mathematical maturity is expected. Familiarity with basic notions of algebra (such as finite fields, basics of vector spaces, polynomials), linear algebra, discrete probability, and basics of computational complexity theory will come in handy.

4 Tentative topics

A brief overview of topics that will be covered:

- Basics
  - Fourier expansion of Boolean functions
  - Linearity testing
  - Influence of variables, Noise stability applications to social choice theory (Arrow's theorem), Isoperimetric inequalities
- Fourier concentration
  - Learning algorithms based on Fourier concentration (Goldreich-Levin Algorithm)
  - Random restrictions, Switching Lemmas, Fourier concentration of small-depth circuits.
  - Applications to Pseudorandomness
- Noise stability
  - Fourier properties of linear threshold functions, Central limit theorems
  - Noise stability of linear threshold functions.
- Hypercontractivity
  - Basic hypercontractivity theorem (The Bonami Lemma)
  - Applications: The Kahn-Kalai-Linial Theorem, Freidgut's Junta theorem, Small-set expansion of the hypercube

5 Evaluation

- Homework: 50% (tentatively 4 problem sets spread evenly accross the semester),
- Project and final presentation: 50%. The expectation is that students work in groups (of 2-3 members) to write a high quality survey based on reading a few related research papers (that is relevant to this course) and give a final presentation. Alternatively, a group may also choose to work on an open problem, and report their findings in a well drafted write-up and a final presentation. We encourage students to start thinking about potential project topics quite early into the semester.