Foundations of Artificial Intelligence

Constraint Satisfaction

CS472 – Fall 2007
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Moving to a different formalism…

```
  SEND
+ MORE
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 MONEY
```

Consider state space for cryptarithmetic (e.g. DFS).

Is this (DFS) how humans tackle the problem?

Human problem solving appears more **sophisticated**!
For example, we derive new constraints on the fly.
→ **little** or **no** search!

Constraint Satisfaction Problems (CSP)

A powerful representation for (discrete) search problems

A **Constraint Satisfaction Problem (CSP)** is defined by:

- **X** is a set of n variables $X_1, X_2, \ldots, X_n$ each defined by a finite domain $D_1, D_2, \ldots D_n$ of possible values.
- **C** is a set of constraints $C_1, C_2, \ldots, C_m$. Each C_i involves a subset of the variables; specifies the allowable combinations of values for that subset.

A solution is an assignment of values to the variables that satisfies all constraints.

Cryptarithmetic as a CSP

Problem: TWO + TWO = FOUR
Variables:

$\boldsymbol{T} \in \{0,\ldots,9\}; \boldsymbol{W} \in \{0,\ldots,9\}; 0 \in \{0,\ldots,9\};$

$\boldsymbol{F} \in \{0,\ldots,9\}; \boldsymbol{U} \in \{0,\ldots,9\}; \boldsymbol{R} \in \{0,\ldots,9\};$

$\boldsymbol{X}_1 \in \{0,\ldots,9\}; \boldsymbol{X}_2 \in \{0,\ldots,9\}; \boldsymbol{X}_3 \in \{0,\ldots,9\};$

Constraints:

$0 + 0 = \boldsymbol{R} + 10 * \boldsymbol{X}_1$

$\boldsymbol{X}_1 + \boldsymbol{W} + \boldsymbol{W} = \boldsymbol{U} + 10 * \boldsymbol{X}_2$

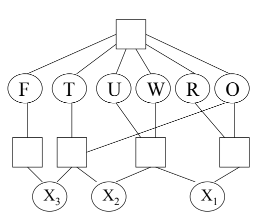
$\boldsymbol{X}_2 + \boldsymbol{T} + \boldsymbol{T} = \boldsymbol{O} + 10 * \boldsymbol{X}_3$

$\boldsymbol{X}_3 = \boldsymbol{F}$

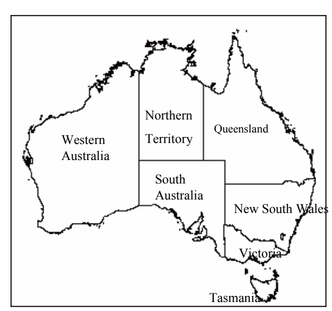
each letter has a different digit ($F \neq T, F \neq U$, etc.);

Cryptarithmetic Constraint Graph

```
  TWO
+TWO
 FOUR
```



Map Coloring Problem



Types of Constraints

Unary Constraints:
Restriction on single variable

Binary Constraints:
Restriction on pairs of variables

Higher-Order Constraints:
Restriction on more than two variables

Constraint Satisfaction Problems (CSP)

For a given CSP the problem is one of the following:

1. find all solutions

2. find one solution
   · just a feasible solution, or
   · A "reasonably good" feasible solution, or
   · the optimal solution given an objective function

3. determine if a solution exists

How to View a CSP as a Search Problem?

Initial State - state in which all the variables are unassigned.

Successor function - assign a value to a variable from a set of possible values.

Goal test - check if all the variables are assigned and all the constraints are satisfied.

Path cost - assumes constant cost for each step

Branching Factor

Approach 1- any unassigned variable at a given state can be assigned a value by an operator: branching factor as high as sum of size of all domains.

Approach 2 - since order of variable assignment not relevant, consider as the successors of a node just the different values of a *single* unassigned variable: max branching factor = max size of domain.

Maximum Depth of Search Tree
n the number of variables → all solutions at depth n.

Prefer DFS or BFS?

CSP – Goal Decomposed into Constraints

Backtracking Search: a DFS that

- chooses values for variables one at a time
- checks for *consistency* with the constraints.

Decisions during search:

- Which variable to choose next for assignment.
- Which value to choose next for the variable.

Forward Checking

- **Idea:** Reduce domain of unassigned variables based on assigned variables.

- Each time variable is instantiated, delete from domains of the uninstantiated variables all of those values that conflict with current variable assignment.

- Identify dead ends without having to try them via backtracking.

General Purpose Heuristics

Variable and value ordering:

Minimum remaining values (MRV): choose the variable with the *fewest* possible values.

Degree heuristic: assign a value to the variable that is involved in the largest number of constraints on other unassigned variables.

Least-constraining value heuristic: choose a value that rules out the smallest number of values in variables connected to the current variable by constraints.

Comparison of CSP Algorithms

Problem	BT	BT+MRV	BT+FC	BT+FC+MRV
USA	(>1,000K)	(>1,000K)	2K	60
N-queens	(>40,000K)	13,500K	(>40,000K)	817K

Constraint Propagation (Arc Consistency)

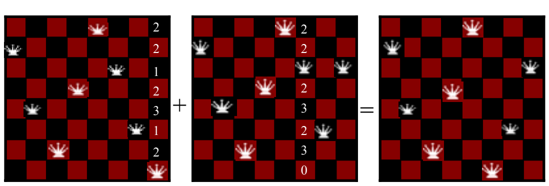
Arc Consistency - state is arc-consistent, if every variable has some value that is consistent with each of its constraints (consider pairs of variables)

- Init: Q is queue with all (directed) arcs (X_i,X_j) in CSP
- WHILE Q is not empty
  - (X_i,X_j) = remove_first(Q)
  - FOREACH $x \in dom(X_i)$
    - *IF no $y \in dom(Xj)$ satisfies constraint (X_i,X_j)
    - ·THEN remove x from $dom(X_i)$
  - IF $dom(X_i)$ changed
    - *THEN add all arcs $(X_k, X_i) \notin Q$ to Q

Constraint Propagation (K-Consistency)

- **K-Consistency** generalizes arc-consistency (2-consistency).
- Consistency of groups of K variables.

Local Search for CSPs



Remarks

- Infinite discrete domains and continuous domains
- Exploiting special problem structure
- Dramatic recent progress in Constraint Satisfaction. Methods can now handle problems with **10,000** to **100,000** variables, and up to **1,000,000** constraints.