CS412/413

Introduction to Compilers
Radu Rugina

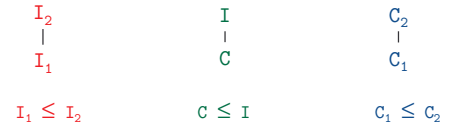
Lecture 15: Subtyping
24 Feb 06

Review

- Objects: fields, methods, public/private qualifiers
- Object types: field types + method signatures
  - Interfaces = pure types
  - Objects = types and implementation
- Object inheritance
  - Induces a subtyping relationship $S \leq T$
  - Similar for interfaces
  - Subtyping allows multiple implementations
  - Java: extends, implements

Subtypes in Java

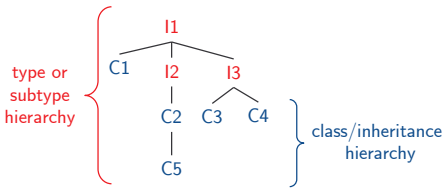
```
interface I1        class C              class C1
   extends I2 { ...    implements I {       extends C2
  }                     ... }
```



$I_1 \leq I_2$ $\quad C \leq I$ $\quad C_1 \leq C_2$

Subtype Hierarchy

- Introduction of subtype relation creates a hierarchy of types: subtype hierarchy



Type-checking

- Problem: what are the valid types for an object?
- Subsumption rule connects subtyping relation and ordinary typing judgements

$$\frac{A \vdash E : S \qquad S \leq T}{A \vdash E : T}$$

$S \leq T \rightarrow$ values(S) $\subseteq$ values(T)

- "If expression E has type S, it also has type T for every T such that $S \leq T$"

Type-checking

- Rules for checking code must allow a subtype where a supertype was expected
- Old rule for assignment:

$$\frac{id : T \in A \qquad A \vdash E : T}{A \vdash id = E}$$

What needs to change here?

Type-checking Overview

- Rules for checking code must allow a subtype where a supertype was expected
- New rule for assignment:

$$\frac{A \vdash E : T' \quad T' \le T \quad id : T \in A}{A \vdash id{=}E} = \frac{A \vdash E : T' \quad T' \le T}{A \vdash E : T} + \frac{id : T \in A \quad A \vdash E : T}{A \vdash id = E}$$

Type-checking Code

```
class Assignment extends ASTNode {
   Symbol id; ExprNode e;
   Type typeCheck(Symtab s) {
        Type Tp = e.typeCheck(s);
        Type T = s.lookup(id);
        if (Tp.subtypeOf(T)) return T;
        else throw new TypecheckError(E);
   }
}
```

$$\frac{A \vdash E : T' \quad T' \le T \quad id : T \in A}{A \vdash id{=}E}$$

Issues

- When are two object/record types identical?
  - Do `struct foo { int x,y; }` and `struct bar { int x,y; }` have the same type?
- We know inheritance (i.e. adding methods and fields) induces subtyping relation
- Issues in the presence of subtyping:
  1. Types of records with object fields
     `class C1 { Point p; }  class C2 { ColoredPoint p; }`
  2. Is it safe to allow fields to be written?
  3. Types of functions (methods)
     `Point foo(Point p)  ColoredPoint bar(ColoredPoint p)`

Type Equivalence

- Types derived with constructors have names
- When are record types equivalent?
- When they have the same fields (i.e. same structure)?
  `struct point { int x,y; } =? struct edge { int n1, n2; }`
- … or only when they have the same names?
  - Types with the same structure are different if they have different names

Type Equivalence

```
class C1 {
     int x, y;
}
class C2 {
     int x, y;
}
C1 a = new C2();
```

Java: name

```
TYPE t1 = OBJECT
     x,y: INTEGER
END
TYPE t2 = OBJECT
     x,y: INTEGER
END;
VAR a: t1 := NEW(t2);
```

Modula-3: structure

Is this code legal?

Type Equivalence

- Name equivalence: types are equal if they are defined by the same type constructor expression and bound to the same name
  - C/C++ example:
    `struct foo { int x; };`
    `struct bar { int x; }`
    `struct foo ≠ struct bar`
- Structural equivalence: two types are equal if their constructor expressions are equivalent
  - C/C++ example:
    `typedef struct foo t1[ ];`
    `typedef struct foo t2[ ];`
    `t1 = t2`

Declared vs. Implicit Subtyping

Java

```
class C1 {
     int x, y;
}
class C2 extends C1 {
     int z;
}
C1 a = new C2();
```

Modula-3

```
TYPE t1 = OBJECT
     x,y: INTEGER
END
TYPE t2 = OBJECT
     x,y,z: INTEGER
END;
VAR a: t1 := NEW(t2);
```


Named vs. Structural Subtyping

- Name equivalence of types (e.g. Java): direct subtypes explicitly declared; subtype relationships inferred by transitivity
- Structural equivalence of types (e.g., Modula-3): subtypes inferred based on structure of types; extends declaration is optional
- Java: still need to check explicit interface declarations similarly to structural subtyping

Width Subtyping for Records

- How to formally express subtyping in the presence of structural equivalence?
- Example:

  `{int x; int y; int color; }` ≤ `{ int x; int y; }`

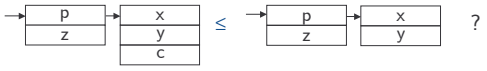
- General rule:

$$\frac{n \le m}{A \vdash \{a_1: T_1, \ldots, a_m: T_m\} \le \{a_1: T_1, \ldots, a_n: T_n\}}$$

Object Fields

- Assume fields can be references to objects
- Subtype relations for individual fields
- How does it translate to subtyping for the whole record?
- If `ColoredPoint` ≤ `Point`, should we allow

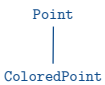
  `{ ColoredPoint p; int z; }` ≤ `{ Point p; int z; }` ?



Field Invariance

- Try `{ p: ColoredPoint; int z; }` ≤ `{ p: Point; int z; }`

```
class C1 { Point p; int z; }
class C2 { ColoredPoint p; int z; }
C2 o2 = new C2();
C1 o1 = o2;
o1.p = new Point( );
o2.p.c = 10;
```



- Mutable (assignable) fields must be type invariant!

Immutable Record Subtyping

- Rule: corresponding immutable fields may be subtypes (also known as "depth subtyping")

$$\frac{A \vdash T_i \le T_i' \ (i \in 1..n)}{A \vdash \{a_1: T_1 \ldots a_n: T_n\} \le \{a_1: T_1' \ldots a_n: T_n'\}}$$

Signature Conformance

- Subclass method signatures must conform to those of superclass
  - Argument types
  - Return type
  - Exceptions
  - How much conformance is really needed?
- Java rule: arguments and returns must have identical types, may remove exceptions

Example 1

- Consider the program:

```
interface List { List rest(int); }
class SimpleList implements List
    { SimpleList rest(int); }
```

- Is the following subtyping relation safe?

```
{ rest: int→SimpleList } ≤ { rest: int→List }
```

```
int→SimpleList ≤ int→List ?
```


Example 2

- Consider the program:

```
class Shape { int setLLCorner(Point p); }
class ColoredRectangle extends Shape
    { int setLLCorner(ColoredPoint p); }
```

- Legal in language Eiffel
- Is this safe?

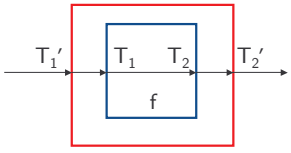
```
ColoredPoint → int ≤ Point → int  ?
```


Function Subtyping

- From definition of subtyping: F: $T_1 \to T_2 \le$ F': $T_1' \to T_2'$ if a value of type $T_1 \to T_2$ can be used wherever $T_1' \to T_2'$ is expected
- Requirement 1: whenever result of F' is used, result of F can also be used
  - Implies $T_2 \le T_2'$
- Requirement 2: any argument to F' must be a valid argument for F
  - Implies $T_1' \le T_1$

General Rule

- Function subtyping: $T_1 \to T_2 \le T_1' \to T_2'$
- Consider function f of type $T_1 \to T_2$:



Contravariance/Covariance

- Function argument types may be contravariant
- Function result types may be covariant

$$\frac{T_1' \le T_1 \qquad T_2 \le T_2'}{T_1 \to T_2 \le T_1' \to T_2'}$$

- Java is conservative!

```
{ rest: int→SimpleList } ≤ { rest: int→List }
```

Java Arrays

- Java has an array type constructor; for any type T $array(T)$ is an array of T's
- Java has the following rule for array subtyping:

$$\frac{T_1 \leq T_2}{array(T_1) \leq array(T_2)}$$

- Is this rule safe?

Java Array Subtype Problems

- Example:

```
Elephant <: Animal, Whale <: Animal
Elephant [ ] y = new Elephant[10];
Animal [ ] x = y;
x[0] = new Whale();
y[0].trunk = new Trunk(); // oops!
```

- Covariant modification: unsound
- Java does run-time check!
  - Example above throws `java.lang.ArrayStoreException`

Unification

- Some rules more problematic
- Rule:

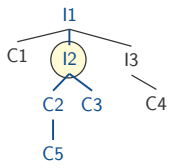
$$\frac{A \vdash E_1 : bool \quad A \vdash E_2 : T_2 \quad A \vdash E_3 : T_3}{A \vdash (E_1 \; ? \; E_2 : E_3) : ?}$$

- Problem: How should we combine T_2 and T_3?

Unification

- Idea: unified type is least common ancestor in type hierarchy (least upper bound)
- Partial order of types must be a lattice

b : new C5() : new C3() : I2



LUB(C3, C5) = I2

Logic: I2 must be same as or a subtype of any type (e.g. I1) that could be the type of both a value of type C3 and a value of type C5

What if no LUB?