CS412/413

Introduction to Compilers
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Lecture 13 : Static Semantics
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Static Semantics

- Can describe the types used in a program
- How to describe type checking?
- Formal description: static semantics for the programming language
- Is to type-checking:
  - As grammar is to syntax analysis
  - As regular expression is to lexical analysis
- Static semantics defines types for legal ASTs in the language

Type Judgments

- Static semantics = formal notation which describes type judgments:

  E : T

  means "E is a well-typed expression of type T"
- Type judgment examples:

  2 : int 2 * (3 + 4) : int
  true : bool "Hello" : string

Type Judgments for Statements

- Statements may be expressions (i.e. represent values)
- Use type judgments for statements:

  if (b) then 2 else 3 : int
  x = false : bool
  b = true, y = 2 : int

- For statements which are not expressions: use a special unit type (empty type):

  S : unit

  means "S is a well-typed statement with no result type"

Deriving a Judgment

- Consider the judgment:

  if (b) then 2 else 3 : int

- What do we need to decide that this is a well-typed expression of type int?
- b must be a bool (b: bool)
- 2 must be an int (2: int)
- 3 must be an int (3: int)

Type Judgments

- Type judgment notation: $A \vdash E : T$

  means "In the context A the expression E is a well-typed expression with the type T "
- Type context is a set of type bindings id : T

  (i.e. type context = symbol table)

  b: bool, x: int $\vdash$ b : bool
  b: bool, x: int $\vdash$ if (b) then 2 else x : int
  $\vdash$ 2 + 2 : int

Deriving a Judgement

- To show:

  $b\text{: bool, x: int} \vdash \text{if (b) then 2 else x : int}$

- Need to show:

  $b\text{: bool, x: int} \vdash \text{b : bool}$

  $b\text{: bool, x: int} \vdash \text{2 : int}$

  $b\text{: bool, x: int} \vdash \text{x : int}$

General Rule

- For any environment A, expression E, statements S_1 and S_2, the judgment

  $A \vdash \text{if (E) then } S_1 \text{ else } S_2 : T$

  is true if:

  $A \vdash E : \text{bool}$

  $A \vdash S_1 : T$

  $A \vdash S_2 : T$

Inference Rules

$$\overbrace{\dfrac{A \vdash E\text{: bool} \quad A \vdash S_1\text{: T} \quad A \vdash S_2\text{: T}}{\underbrace{A \vdash \text{if (E) then } S_1 \text{ else } S_2 : T}_{\text{Conclusion}}}}^{\text{Premises}} \text{(if-rule)}$$

- Holds for any choice of E, S_1, S_2, T

Why Inference Rules?

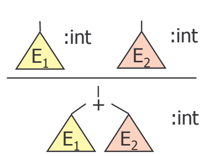
- Inference rules: compact, precise language for specifying static semantics (can specify languages in ~20 pages vs. 100's of pages of Java Language Specification)
- Inference rules correspond directly to recursive AST traversal that implements them
- Type checking is attempt to prove type judgments $A \vdash E : T$ true by walking backward through rules

Meaning of Inference Rule

- Inference rule says:
  given that antecedent judgments are true
  - with some substitution for A, E_1, E_2

  then, consequent judgment is true
  - with a consistent substitution

$$\dfrac{A \vdash E_1 : \text{int} \qquad A \vdash E_2 : \text{int}}{A \vdash E_1 + E_2 : \text{int}} \text{(+)}$$



Proof Tree

- Expression is well-typed if there exists a type derivation for a type judgment
- Type derivation is a proof tree
- Example: if A1 = b: bool, x: int, then:

$$\dfrac{\dfrac{A1 \vdash \text{b: bool}}{A1 \vdash \text{!b: bool}} \quad \dfrac{A1 \vdash \text{2: int} \quad A1 \vdash \text{3: int}}{A1 \vdash \text{2+3 : int}} \quad A1 \vdash \text{x : int}}{\text{b: bool, x: int} \vdash \text{if (!b) then 2+3 else x : int}}$$

More about Inference Rules

- No premises = axiom

$$\frac{}{A \vdash \text{true} : \text{bool}}$$

- A goal judgment may be proved in more than one way

$$\frac{A \vdash E_1 : \text{float} \quad A \vdash E_2 : \text{float}}{A \vdash E_1 + E_2 : \text{float}} \qquad \frac{A \vdash E_1 : \text{float} \quad A \vdash E_2 : \text{int}}{A \vdash E_1 + E_2 : \text{float}}$$

- No need to search for rules to apply -- they correspond to nodes in the AST

While Statements

- Rule for while statements:

$$\frac{A \vdash E : \text{bool} \quad A \vdash S : T}{A \vdash \text{while (E) S} : \text{unit}} \text{(while)}$$

- Why unit type?

If Statements

- If statement as an expression (e.g., in ML): its value is the value of the branch that is executed

$$\frac{A \vdash E : \text{bool} \quad A \vdash S_1 : T \quad A \vdash S_2 : T}{A \vdash \text{if (E) then } S_1 \text{ else } S_2 : T} \text{(if-then-else)}$$

- If no else clause:

$$\frac{A \vdash E : \text{bool} \quad A \vdash S : T}{A \vdash \text{if (E) S} : ?} \text{(if-then)}$$

Assignment Statements

$$\frac{\text{id} : T \in A \quad A \vdash E : T}{A \vdash \text{id} = E : T} \text{(variable-assign)}$$

$$\frac{A \vdash E_3 : T \quad A \vdash E_2 : \text{int} \quad A \vdash E_1 : \text{array}[T]}{A \vdash E_1[E_2] = E_3 : T} \text{(array-assign)}$$

Sequence Statements

- Rule: A sequence of statements is well-typed if the first statement is well-typed, and the remaining are well-typed too:

$$\frac{A \vdash S_1 : T_1 \quad A \vdash (S_2 ; \ldots ; S_n) : T_n}{A \vdash (S_1 ; S_2 ; \ldots ; S_n) : T_n} \text{(sequence)}$$

- What about variable declarations ?

Declarations

= unit
if no E

$$\frac{A \vdash \text{id} : T \,[\, = E \,]\, : T_1 \quad A, \text{id} : T \vdash (S_2 ; \ldots ; S_n) : T_n}{A \vdash (\text{id} : T \,[\, = E \,];\, S_2 ; \ldots ; S_n) : T_n} \text{(declaration)}$$

- Declarations add entries to the environment (in the symbol table)

Function Calls

- If expression E is a function value, it has a type $T_1 \times T_2 \times \ldots \times T_n \rightarrow T_r$
- T_i are argument types; T_r is return type
- How to type-check function call $E(E_1,\ldots,E_n)$?

$$\frac{A \vdash E : T_1 \times T_2 \times \ldots \times T_n \rightarrow T_r \qquad A \vdash E_i : T_i \ (i \in 1..n)}{A \vdash E(E_1,\ldots,E_n) : T_r} \ \text{(function-call)}$$

Function Declarations

- Consider a function declaration of the form

  T_r fun (T_1 a_1,…, T_n a_n) { return E; }

- Type of function body S must match declared return type of function, i.e. $E : T_r$
- … but in what type context?

Add Arguments to Environment!

- Let A be the context surrounding the function declaration. Function declaration:

  T_r fun (T_1 a_1,…, T_n a_n) { return E; }

  is well-formed if

  $$A, a_1 : T_1, \ldots, a_n : T_n \vdash E : T_r$$

- …what about recursion?

  Need: fun: $T_1 \times T_2 \times \ldots \times T_n \rightarrow T_r \in A$

Recursive Function Example

- Factorial:

```
int fact(int x) {
    if (x==0) return 1;
    else return x * fact(x - 1);
}
```

- Prove: $A \vdash x * fact(x-1) : int$

  Where: $A = \{ fact: int \rightarrow int, x : int \}$

Mutual Recursion

- Example:

```
int f(int x) { return g(x) + 1; }
int g(int x) {return f(x) – 1; }
```

- Need environment containing at least

  f: int → int, g: int → int

  when checking both f and g

- Two-pass approach:
  - Scan top level of AST picking up all function signatures and creating an environment binding all global identifiers
  - Type-check each function individually using this global environment

How to Check Return?

$$\frac{A \vdash E : T}{A \vdash \text{return } E : \text{unit}} \ \text{(return1)}$$

- A return statement produces no value for its containing context to use
- Does not return control to containing context

- Suppose we use type unit…
- …then how to make sure the return type of the current function is T ?

Put Return in the Symbol Table

- Add a special entry { return_fun : T } when we start checking the function "fun", look up this entry when we hit a return statement.
- To check T_r fun (T_1 a_1,…, T_n a_n) { return S; } in environment A, need to check:

$$A, a_1 : T_1, \ldots, a_n : T_n, \text{return\_fun} : T_r \vdash S : T_r$$

$$\frac{A \vdash E : T \qquad \text{return\_fun} : T \in A}{A \vdash \text{return } E : \text{unit}} \text{(return)}$$

Static Semantics Summary

- Static semantics = formal specification of type-checking rules
- Concise form of static semantics: typing rules expressed as inference rules
- Expression and statements are well-formed (or well-typed) if a typing derivation (proof tree) can be constructed using the inference rules