CS412/413

Introduction to Compilers
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Lecture 25: Control Flow Analysis
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Problem 4: Constant Folding

- Compute constant variables at each program point
- Constant variable = variable having a constant value on all program executions

- Dataflow information: sets of constant values
- Example: {x=2, y=3} at program point p
- Is a forward analysis

- Let V = set of all variables in the program, nvar = |V|
- Let N = set of integer constants
- Use a lattice over the set V x N
- Construct the lattice starting from a lattice for N

- Problem: $(N, \leq)$ is not a complete lattice!
  … why?

Constant Folding Lattice

- Second try: lattice $(N \cup \{\top, \bot\}, \leq)$
  - Where $\bot \leq n$, for all $n \in N$
  - And $n \leq \top$, for all $n \in N$
  - Is complete!

- Meaning:
  - $v = \top$: don't know if v is constant
  - $v = \bot$: v is not constant

Constant Folding Lattice

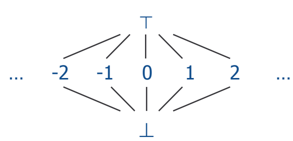
- Second try: lattice $(N \cup \{\top, \bot\}, \leq)$
  - Where $\bot \leq n$, for all $n \in N$
  - And $n \leq \top$, for all $n \in N$
  - Is complete!

- Problem:
  - Is incorrect for constant folding
  - Meet of two constants $c \neq d$ is min(c,d)
  - Meet of different constants should be $\bot$

- Another problem: has infinite height …

Constant Folding Lattice

- Solution: flat lattice $L = (N \cup \{\top, \bot\}, \sqsubseteq)$
  - Where $\bot \sqsubseteq n$, for all $n \in N$
  - And $n \sqsubseteq \top$, for all $n \in N$
  - And distinct integer constants are not comparable



- Note: meet of any two distinct numbers is $\bot$!

Constant Folding Lattice

- Denote $N^* = N \cup \{\top, \bot\}$
- Use flat lattice $L = (N^*, \sqsubseteq)$

- Constant folding lattice: $L' = (V \to N^*, \sqsubseteq_C)$
- Where partial order on $V \to N^*$ is defined as:

  $X \sqsubseteq_C Y$ iff for each variable v: $X(v) \sqsubseteq Y(v)$

- Can represent a function in $V \to N^*$ as a set of assignments: { {v1=c1}, {v2=c2}, …, {vn=cn} }

CF: Transfer Functions

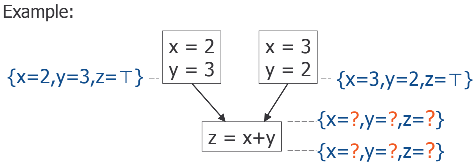
- Transfer function for instruction I:
  $F_I(X) = (X - kill[I]) \cup gen[I]$
  where:
  kill[I] = constants "killed" by I
  gen[I] = constants "generated" by I
- $X[v] = c \in N^*$ if $\{v=c\} \in X$
- If I is v = c (constant): gen[I] = {v=c} kill[I] = {v} x N*
- If I is v = u+w: gen[I] = {v=e} kill[I] = {v} x N*
  where e = X[u] + X[w], if X[u] and X[w] are not $\top, \bot$
  e = $\bot$, if X[u] = $\bot$ or X[w] = $\bot$
  e = $\top$, if X[u] = $\top$ and X[w] = $\top$

CF: Transfer Functions

- Transfer function for instruction I:
  $F_I(X) = (X - kill[I]) \cup gen[I]$
- Here gen[I] is not constant, it depends on X
- However transfer functions are monotonic (easy to prove)
- … but are transfer functions distributive?

CF: Distributivity

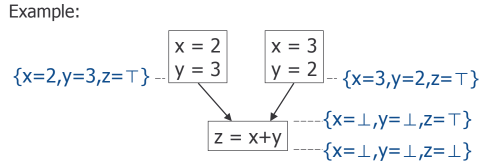
- Example:



- At join point, apply meet operator
- Then use transfer function for z=x+y

CF: Distributivity

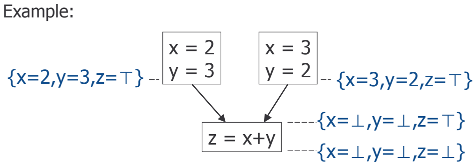
- Example:



- Dataflow result (MFP) at the end: $\{x=\bot,y=\bot,z=\bot\}$
- MOP solution at the end: $\{x=\bot,y=\bot,z=5\}$!

CF: Distributivity

- Example:



- Reason for MOP $\neq$ MFP:
  transfer function F of z=x+y is not distributive!
  $F(X1 \sqcap X2) \neq F(X1) \sqcap F(X2)$
  where $X1 = \{x=2,y=3,z=\top\}$ and $X2 = \{x=3,y=2,z=\top\}$

Classification of Analyses

- Forward analyses: information flows from
  - CFG entry block to CFG exit block
  - Input of each block to its output
  - Output of each block to input of its successor blocks
  - Examples: available expressions, reaching definitions, constant folding
- Backward analyses: information flows from
  - CFG exit block to entry block
  - Output of each block to its input
  - Input of each block to output of its predecessor blocks
  - Example: live variable analysis

Another Classification

- "may" analyses:
  - information describes a property that MAY hold in SOME executions of the program
  - Usually: $\sqcap = \cup$, $\top = \emptyset$
  - Hence, initialize info to empty sets
  - Examples: live variable analysis, reaching definitions
- "must" analyses:
  - information describes a property that MUST hold in ALL executions of the program
  - Usually: $\sqcap = \cap$, $\top = S$
  - Hence, initialize info to the whole set
  - Examples: available expressions

Next

- Control flow analysis
  - Detect loops in control flow graphs
  - Dominators
- Loop optimizations
  - Code motion
  - Strength reduction for induction variables
  - Induction variable elimination

Program Loops

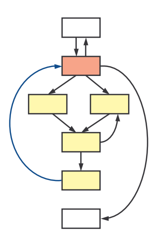
- Loop = a computation repeatedly executed until a terminating condition is reached
- High-level loop constructs:
  - While loop: while(E) S
  - Do-while loop: do S while(E)
  - For loop: for(i=1, i<=u, i+=c) S
- Why are loops important:
  - Most of the execution time is spent in loops
  - Typically: 90/10 rule, 10% code is a loop
- Therefore, loops are important targets of optimizations

Detecting Loops

- Need to identify loops in the program
  - Easy to detect loops in high-level constructs
  - Difficult to detect loops in low-level code or in general control-flow graphs
- Examples where loop detection is difficult:
  - Languages with unstructured "goto" constructs: structure of high-level loop constructs may be destroyed
  - Optimizing Java bytecodes (without high-level source program): only low-level code is available

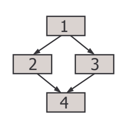
Control-Flow Analysis

- Goal: identify loops in the control flow graph
- A loop in the CFG:
  - Is a set of CFG nodes (basic blocks)
  - Has a loop header such that control to all nodes in the loop always goes through the header
  - Has a back edge from one of its nodes to the header



Dominators

- Use concept of dominators to identify loops:
  "CFG node d dominates CFG node n if all the paths from entry node to n go through d"



1 dominates 2, 3, 4
2 doesn't dominate 4
3 doesn't dominate 4

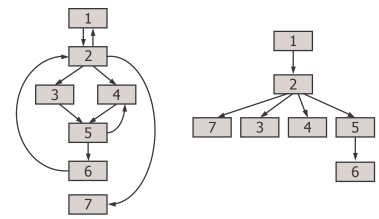
- Intuition:
  - Header of a loop dominates all nodes in loop body
  - Back edges = edges whose heads dominate their tails
  - Loop identification = back edge identification

Immediate Dominators

- Properties:
  1. CFG entry node n_0 in dominates all CFG nodes
  2. If d1 and d2 dominate n, then either
     - d1 dominates d2, or
     - d2 dominates d1
- Immediate dominator idom(n) of node n:
  - idom(n) $\neq$ n
  - idom(n) dominates n
  - If m dominates n, then m dominates idom(n)
- Immediate dominator idom(n) exists and is unique because of properties 1 and 2

Dominator Tree

- Build a dominator tree as follows:
  - Root is CFG entry node n_0
  - m is child of node n iff n=idom(m)
- Example:



Computing Dominators

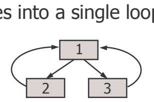
- Formulate problem as a system of constraints:
  - dom(n) is set of nodes who dominate n
  - $dom(n_0) = \{n_0\}$
  - $dom(n) = \cap \{ dom(m) \mid m \in pred(n) \}$
- Can also formulate problem in the dataflow framework
  - What is the dataflow information?
  - What is the lattice?
  - What are the transfer functions?
  - Use dataflow analysis to compute dominators

Natural Loops

- Back edge: edge n$\rightarrow$h such that h dominates n
- Natural loop of a back edge n$\rightarrow$h:
  - h is loop header
  - Loop nodes is set of all nodes that can reach n without going through h
- Algorithm to identify natural loops in CFG:
  - Compute dominator relation
  - Identify back edges
  - Compute the loop for each back edge

Disjoint and Nested Loops

- Property: for any two natural loops in the flow graph, one of the following is true:
  1. They are disjoint
  2. They are nested
  3. They have the same header
- Eliminate alternative 3: if two loops have the same header and none is nested in the other, combine all nodes into a single loop



Two loops: {1,2} and {1,3}
Combine into one loop: {1,2,3}

Loop Preheader

- Several optimizations add code before header
- Insert a new basic block (called preheader) in the CFG to hold this code