CS 412
Introduction to Compilers

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Lecture 36: Parametric Polymorphism
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Polymorphism

- *poly*=many, *morph*=shape
- Code is *polymorphic* if it can be used with values from more than one type
- Have already seen *subtype polymorphism* in Iota+

```
interface set { … }
class hashSet implements set { … }
class arraySet implements set { … }
intersect(s1, s2: set): set
```

- intersect works with s1, s2 from *any* class that implements the set interface

Outline

- Two other forms of polymorphism
  - ad hoc polymorphism (overloading)
  - parametric polymorphism
    - template mechanisms (C++)
    - fully parametric code (ML)
    - bounded parameters (CLU, PolyJ)
- Language design issues
- Type checking
- Code generation
- Appel Chap. 16

Overloading

- Overloading: same name can be reused with different types, as in Java (also: *ad-hoc polymorphism*)
- Ambiguity resolved by static argument types

```
print(int x)
print(string s)     ← print(3)
print(float f)
```

- *Looks* like one polymorphic function print
- Reality: three different functions bound to same name—not true polymorphism

⇒ Three separate entries in symbol table

- Overloading relies on knowing argument types; conflicts with templates, type inference

Parametric polymorphism

- Subtype, ad-hoc polymorphism don't allow abstraction over types
- Example: generic array sort routine

  sort(a: array[T]) (for all T)
- Type T is a *parameter* to function

  sort(T: type, a: array[T])
- *Types* used like values (can pass as arguments). Useful to separate different kinds of arguments:

  sort[T: type](a: array[T])

Generic array sort

```
sort[T: type](a: array[T]) = (
   i,j:int = 1;
   while (j < n) (
        e: T = a[j];
        i = j-1;
        while (i >= 0 && a[i] > e)
              ( a[i+1] = a[i]; i-- )
        a[i+1] = e
   )
)
```

When to type-check? (pointing to `a[i] > e`)

Templates (C++)

- Idea: instantiate `sort(T,a)` for each distinct `T` used in program; type-check instantiated code

```
sort[int]( a: array[int] ) = (
  i,j:int = 1;
  while (j < n) (
    e: int = a[j];
    i = j-1;
    while (i >= 0 && a[i] > e) ( a[i+1] = a[i]; i-- )
    a[i+1] = e))
```

int > int : ok

- But: get type-checking errors in library code
- Libraries must be shipped in source form
- ">" code differs: must recompile code—bloat!

Parametricity

- Can write code in way that doesn't depend on T using 1st class function:

```
sort[T: type](a: array[T], gt: function(T,T): bool) = (
  i,j:int = 1;
  while (j < n) (
    e: T = a[j];
    i = j-1;
    while (i >= 0 && gt(a[i], e)) ( a[i+1] = a[i]; i-- )
    a[i+1] = e))
```

~~a[i] > e~~

- Now: code doesn't depend on what T is: fully *parametric* w/ respect to T
- Can type-check and generate code *once* for all instantiations (ML)

Type checking

- Set of legal type identifiers differs inside a parameterized function

```
sort[T: type]
  (a: array[T], gt: function(T,T): bool) = ( ...e: T = a[j]...)
```

`T` in scope as type here

$$\frac{\begin{array}{c} A,\ \alpha_1\text{:type},\ldots,\alpha_m\text{:type},\ x_1{:}T_1,\ldots,\ x_n{:}T_n \vdash e : T_R \\ A,\ \alpha_1\text{:type},\ldots,\alpha_m\text{:type} \vdash T_i :: \textbf{type} \quad \forall i \in 1..n \\ A,\ \alpha_1\text{:type},\ldots,\alpha_m\text{:type} \vdash T_R :: \textbf{type} \end{array}}{A \vdash f[\alpha_1,\ldots,\alpha_m](x_1{:}T_1,\ldots,\ x_n{:}T_n)\text{: } T_R = e\ \textbf{defn}}$$

Type variables

- Identifier considered a legal type = *kind judgement*

$$\frac{\alpha\text{: type} \in A}{A \vdash \alpha :: \textbf{type}} \qquad \frac{A \vdash T :: \textbf{type}}{A \vdash \text{array}[T] :: \textbf{type}} \qquad \text{\&c.}$$

- No other typing rules mention types α explicitly
  - only rules mentioning no specific types can be used
  - ensures static checking works for all actual types used as parameters to code

$$\frac{id : T \in A \qquad A \vdash E : T}{A \vdash id = E : T}\ (\text{assign})$$

Code generation

- Can generate same code for all instantiations of a parameterized abstraction *if* all types have the same size

```
sort[T: type](a: array[T], gt: function(T,T): bool) = (
  i,j:int = 1;
  while (j < n) (
    e: T = a[j];
    i = j-1;
    while (i >= 0 && gt(a[i], e)) ( a[i+1] = a[i]; i-- )
    a[i+1] = e))
```

How much stack space?

- Option 1: different code for `sort[int/long]`
- Option 2: auto *box/unbox* to ensure 1-word size

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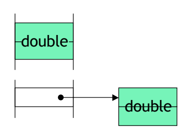
Boxing/unboxing

- Idea: some types T have two different run-time representations
  1. inline representation
  2. reference to heap



- To support primitive type `T <: object`, primitive values boxed when cast to `object`
- To support parametric polymorphism: large types (long, double, closures) boxed when used as parameters

First-class vs. second-class polymorphic values

sort[T: type] (a: array[T], gt: function(T,T): bool)

- sort[int] has type

  $\text{array[int]} \times (\text{int}\times\text{int}\rightarrow\text{bool}) \rightarrow \text{unit}$
- sort is a *polymorphic value,* type

  $\forall\text{T. array[int]} \times (\text{T}\times\text{T}\rightarrow\text{bool}) \rightarrow \text{unit}$
- Most languages (incl. ML): polymorphic values are in environment but can only be instantiated
- First class polymorphic values: powerful, but can't figure out all instantiations in advance! (must box)

Parameterized types

- sort[T: type] : parameterized function that works for any type *T*
- array[T]: parameterized *type* that can be constructed for any type *T*

  array: type $\rightarrow$ type
- Can a language allow user-defined parameterized types?
- Useful for data structures: Set[T], Map[K,V], Stack[T], Vector[T], Hashtable[K, V]

Example

- PolyJ: Java extended w/ parameterized types
- Java 1.2+:

```
interface Map {
      boolean containsKey(Object k);
      Object get(Object k);
}
```

- PolyJ:

```
interface Map[Key, Value] {
      boolean containsKey(Key k);
      Value get(Key k);
}
class HashMap[Key, Value] implements Map[Key,Value] { … }
Map[String,int] m = new HashMap[String, int] …
```


Parameterized types vs. Instantiations

```
class Vector[T] {
  void add(T e);
  T get(int i);
  T set(int i, T e);
}
```

Vector: type→type

```
Vector[int] = class {
        void add(int e);
        int get(int i);
        int set(int i, int e);
}
```

- Parameterized types (Vector) are not types; can't have a value of type Vector (in PolyJ)

  Vector x; … Vector[int] y = x[int]; // ?

- Instantiations (Vector[int]) *are* types

Using parameterized types

```
class Vector {
   void add(Object e);
   Object get(int i);
   Object set(int i, Object e);
}

Vector v;
Animal a;
v.add(a);       //unchecked
a = (Animal)v.get(i);
```

```
class Vector[T] {
   void add(T e);
   T get(int i);
   T set(int i, T e);
}

Vector[Animal] v;
Animal a;
v.add(a);
a = v.get(i);
```


Subtyping relations

- What subtyping relations can hold for instantiation types? –depends on parameterized type

```
class Vector[T] {
     void add(T e);
     T get(int i);
}
Elephant <: Animal

Vector[Elephant] <: Vector[Animal]?
Vector[Animal] <: Vector[Elephant]?
```

Applying subtyping rules

- Vector[Elephant] <: Vector[Animal]?

```
{                                   {
  void add(Elephant e);               void add(Animal e);
  Elephant get(int i);      <:        Animal get(int i);
}                                   }
```

Nope: add

- Vector[Animal] <: Vector[Elephant]?

Nope: get

- Rule:
  – Subtyping on instantiation is covariant if type parameters appear only as return values
  – is contravariant if appear only as arguments

Constrained parametric polymorphism

```
class set[T] {
  T[] elements;
  boolean contains(e: T) {
    for (int i=0; i < elements.length(); i++) {
      if e.equals(elements[i])
        return true;
  }
}
```

oops!

- Set[T] doesn't make *sense* unless T has a notion of equality
- SortedSet[T] : T must have a total ordering relation

Signature constraints

```
SortedSet[T] where T { int compare(T); } {
  T[] elements;
  boolean contains(e: T) {
    for (int i=0; i < elements.length(); i++) {
      if (0 == e.compare(elements[i]))
        return true;
  }
}
```

- Constraint: contract between instantiator and parameterized code
- Can only be instantiated on types *w/* compare
- SortedSet code only uses operations guaranteed to exist on any actual type used in instantiation

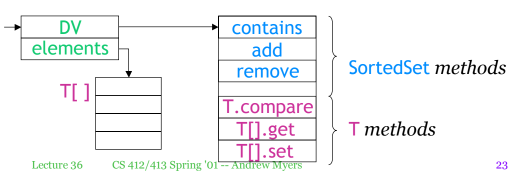
Subtype constraints

```
class SortedSet[T] where T <: Comparable {
  T[] elements;
  boolean contains(e: T) {
    for (int i=0; i < elements.length(); i++) {
      if (0 == e.compare(elements[i]))
        return true;
  }
}
class Comparable {
  int compare(Comparable x);
}
```


Implementation

```
SortedSet[T] where T { int compare(T); } {
  ... if (0 == e.compare(elements[i])) ...
}
```

- How to generate code once for all instantiations? Problem: don't know index of compare in DV of T
- Solution: separate dispatch vector for each instantiation of SortedSet[T]



Summary

- Overloading is not true polymorphism
- Parametric polymorphism helps write correct code conveniently
- Simple approach: templates. Breaks separate compilation, causes code bloat
- Unconstrained parametric polymorphism: simple to implement, may require boxing/unboxing
- Constrained parametric polymorphism: avoids more run-time errors, can be folded into dispatch vector