

## Dominators and loops

- $A \operatorname{dom} B$ if $B$ is reachable only by going through $A$
- Defn of loop: set of stronglyconnected nodes with single entry point: loop header node
- loop header dominates all other nodes in loop
- Loop must contain back edge w/ respect to dominance relationship: $n \rightarrow h$ where $h \operatorname{dom} n$

back edge?
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## Control tree

- Nest loops based on subset relationship between natural loops
- Exception: natural loops may share same header; merge them into larger loop.
- Build control tree using nesting relationship



## Outline

- Loop optimizations
- Loop-invariant code motion
- Strength reduction
- Loop unrolling
- Array bounds checks
- Loop tiling ...
- Eliminating null checks


## Completing control-flow analysis

- Dominator analysis identifies all back edges
- Each back edge $n \rightarrow h$ has an associated natural loop with $h$ as its header: all nodes reachable from $h$ that reach $n$ without going through $h$
- For each back edge $n \rightarrow h$, find its natural loop:
$\left\{n^{\prime} \mid n\right.$ reachable from $n^{\prime}$ in $\left.G-h\right\} \cup$ \{h\}



## Redundant computation

for (int $\mathrm{i}=0 ; \mathrm{i}<\mathrm{a}$.length; $\mathrm{i}++$ ) \{

$$
a[i]=a[i]+1 ;
$$

\}


## Loop-invariant hoisting

- Idea: move computations that always give the same result out of the loop: only compute once!
- Hoisting $\mathrm{a}+\mathrm{b}$ : a and b must be loop-invariant:
- constant,
- only defined outside loop (use reaching definitions),
- or only one definition inside loop whose expression is computed on loop-invariant variables
- Can identify all loop-invariant exprs (\& dependencies) in one pass



## Identifying induction variables

- Basic induction variables: only one definition of the form $\mathrm{j}=\mathrm{j}+K$
- Derived (or dependent) induction variables: value is $\mathrm{j} * M+N$ for some b.i.v. j
( $K, M, N$ loop invariants)
$\mathrm{j}=3$; z = 0;
for $(\mathrm{i}=0 ; \mathrm{i}<\mathrm{n} ; \mathrm{i}+\mathrm{+})$ \{
$j=j+1 ; z=z+2$;
$\mathrm{k}=\mathrm{j}^{*} 4+8$;
$m=k^{*} n$;
\}


## Loop unrolling

- Loop unrolling: creates $N$ copies of loop in


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## Induction variables

- Induction variables are variables with value $A^{*} i+B$ on the $i^{\text {th }}$ iteration of a natural loop, for loop invariants $A \& B$
- Several optimizations can exploit information about induction variables:
-strength reduction
-bounds-check elimination
-loop unrolling


## Strength reduction

- Derived induction variable $k$ can be written as $A^{*} i+B, i$ some basic induction variable stepping by $A_{i}$
- For all distinct $(A, B)$ pairs:
- insert before loop header: $k=A^{*} i+B$
- insert after assignment to $i: k=k+\left(A^{*} A_{i}\right)$
- Replace definition of any $k^{\prime}$ whose formula is also $A^{*} i+B$ by $k^{\prime}=k$
- Effect: multiplication(s) replaced by single addition

$$
\begin{array}{lll}
\mathrm{t} 1=\mathrm{a}+\mathrm{i}^{*} 4 & \Rightarrow & \mathrm{t} 1=\mathrm{t} 1+A_{i}^{*} 4 \\
\mathrm{M}=\mathrm{k}^{*} \mathrm{n} & \Rightarrow \quad \mathrm{M}=\mathrm{M}+\mathrm{t}_{\mathrm{M}} \quad\left(\mathrm{t}_{\mathrm{M}}=A_{k}^{*} \mathrm{n}\right)
\end{array}
$$

- Other optimizations facilitated: constant propagation, algebraic simplification, copy propagation, dead variable elimination, dead code elimination
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## Using induction variables

- Idea: use one loop test to ensure that entire unrolled loop ( $N$ copies) will succeed
- Loop test must depend on induction variable: e.g., $\mathrm{i}<\mathrm{n}$
- $\mathfrak{i}+K^{*}(N-1)<n: n o$ interior loop tests needed
- Additional loop needed to "finish up" 0.. N -1 iterations
- Best if loop is small, straight-line code
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Useful unrolling

## Array bounds checks

- Iota ${ }^{+}$: On every expression a[i] , must ensure $\mathrm{i}<$ length $\mathrm{a}, \mathrm{i} \geq 0$ ( $\mathrm{i}<\mathrm{u}$ length a)
- Checking array bounds is expensive
- Array indices are often induction variables -can use induction variable information to avoid the bounds check entirely!



## Null checks

- Java, Iota+ : need null checks on every
- field access or assignment (except on this)
- method invocation (except on this)
- array element access
- string operation
- Idea: Once we've checked for null, shouldn't need to check again


## Boolean propagation

- Augment constant propagation with special propagation of booleans
- Almost fits into standard dataflow analysis model
- Different information leaves on outedges of if quadruples



## Eliminating checks

- Given reference $\mathrm{a}[\mathrm{k}]$ where k is an induction variable with value $a^{*} i+b$ : find a conditional test on some induction variable j
- test terminates the loop
- test dominates the reference to $\mathrm{a}[\mathrm{k}]$
- test is against a loop-invariant expression that is ensures $\mathrm{k}<_{\mathrm{u}}$ a.length
- When to perform optimization?
- AST? Need domination analysis, other optimizations not done.
- Quadruples? Hard to recognize array length, array accesses, checks. Solution: propagate annotations

| Example |  |
| :---: | :---: |
| $\mathrm{u}=\mathrm{p} \cdot \mathrm{x}+\mathrm{p} \cdot \mathrm{y}$ |  |
| $\Rightarrow \quad t 1=p!=0$ | $\mathrm{t} 1=\mathrm{p}$ ! $=0$ |
| if t 1 goto L 1 else L2 | if t 1 goto L1 else L2 |
| L2: abort | L2: abort |
| L1: $a x=p+4$ | L1: $\mathrm{ax}=\mathrm{p}+4$ |
| tx $=\mathrm{M}[\mathrm{ax}]$ | tx $=M[\mathrm{ax}]$ |
| $\mathrm{t} 2=\mathrm{p}!=0 \quad \text { CSE: } \mathrm{t} 2=\mathrm{t} 1$ <br> if t 2 goto L else L4 | $\mathrm{t} 2=\mathrm{t} 1$ <br> Copy: if t 1 goto ... |
| L3: abort | L3: abort |
| L4: $\mathrm{ay}=\mathrm{p}+8$ | L4: $a y=p+8$ |
| ty $=$ M[ay] | ty $=$ M[ay] |
| $\mathrm{u}=\mathrm{tx}+\mathrm{ty}$ | $\mathrm{u}=\mathrm{tx}+\mathrm{ty}$ |
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## Finishing optimization

| $\mathrm{t} 1=\mathrm{p}!=0$ <br> if t 1 goto L 1 else L 2 | $\mathrm{t} 1=\mathrm{p}!=0$ <br> if t 1 goto L 1 else L2 |
| :---: | :---: |
| L2: abort | L2: abort |
| L1: $\mathrm{ax}=\mathrm{p}+4$ | L1: $a x=p+4$ |
| $t \mathrm{tx}=\mathrm{M}[\mathrm{ax}]$ | $t \mathrm{tx}=\mathrm{M}[\mathrm{ax}]$ |
| $\mathrm{t} 2=\mathrm{t} 1$ |  |
| L3: abort |  |
| L4: $\mathrm{ay}=\mathrm{p}+8$ | ay $=\mathrm{p}+8$ |
| ty $=$ M[ay] | $\mathrm{ty}=\mathrm{M}[\mathrm{ay}]$ |
| $\mathrm{u}=\mathrm{tx}+\mathrm{ty}$ | $\mathrm{u}=\mathrm{tx}+\mathrm{ty}$ |
|  | $\mathrm{u}=\mathrm{p} \cdot \mathrm{x}+\mathrm{p} \cdot \mathrm{y}$ |

