CS 412
Introduction to Compilers

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Lecture 11: Static Semantics
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Administration

- Programming Assignment 2 due in 1 week

Static Semantics

- Can describe the types used in a program. How to describe type checking?
- Formal description: *static semantics* for the programming language
- Is to type-checking as grammar is to parsing
- Static semantics defines types for all legal language ASTs
- We will write ordinary language syntax to mean the corresponding AST

Type Judgements

- Static semantics defines how to derive *type judgments*

$E : T$ means "E is a well-typed expression of type T"

2 : int   2 * (3 + 4) : int

true : bool   "Hello" : string

if (b) 2 else 3 : int

Deriving a judgment

if (b) 2 else 3 : int

- What do we need to decide that this is a well-typed expression of type **int**?

- b must be an bool (b: bool)
- 2 must be an int (2: int)
- 3 must be an int (3: int)

Type Judgments

- Type judgment: $A \vdash E : T$
  - means "In the context A (symbol table), the expression E is a well-typed expression with the type T"
- Type context is set of type assignments *id* : *T*

b: bool, x: int $\vdash$ b : bool

b: bool, x: int $\vdash$ if (b) 2 else x : int

$\vdash$ 2 + 2 : int

Deriving a judgement

- To show

  b: bool, x: int ⊢ if (b) 2 else x : int

- Need to show:

  b: bool, x: int ⊢ b : bool

  b: bool, x: int ⊢ 2 : int

  b: bool, x: int ⊢ x : int

General Rule

- For *any* environment A, expression E, statements S_1 and S_2, the judgment

  $$A \vdash \text{if } (E)\ S_1 \text{ else } S_2 : T$$

  is true if

  $$A \vdash E : \text{bool}$$
  $$A \vdash S_1 : T$$
  $$A \vdash S_2 : T$$

As an Inference Rule

Premises

$$\frac{A \vdash E : \text{bool} \qquad A \vdash S_1 : T \qquad A \vdash S_2 : T}{A \vdash \text{if } (E)\ S_1 \text{ else } S_2 : T} \ \text{(name)}$$

Conclusion

- Holds for any choice of the syntactic meta-variables E, S_1, S_2, T

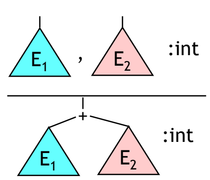
Why inference rules?

- Inference rules: compact, precise language for specifying static semantics (can specify languages in ~20 pages vs. 100's of pages of Java Language Specification)
- Inference rules correspond directly to recursive AST traversal that implements them
- Type checking is attempt to prove type judgments $A \vdash E$: T true by walking backward through rules

Meaning of Inference Rule

- Inference rule says: given that antecedent judgments are true
  - with some substitution for meta-variables A, E_1, E_2
- Then, consequent judgment is true
  - with a consistent substitution

$$\frac{A \vdash E_1 : \textbf{int} \qquad A \vdash E_2 : \textbf{int}}{A \vdash E_1 + E_2 : \textbf{int}} \ (+)$$



Proof Tree = Call graph

- Expression is well-typed if there exists a *type derivation* for a type judgment
- Type derivation is a *proof tree*

let A1 = b: bool, x: int

$$\dfrac{\dfrac{A1 \vdash b : \text{bool}}{A1 \vdash !b : \text{bool}} \qquad \dfrac{A1 \vdash 2 : \text{int} \quad A1 \vdash 3 : \text{int}}{A1 \vdash 2+3 : \text{int}} \qquad A1 \vdash x : \text{int}}{b: \text{bool}, x: \text{int} \vdash \text{if } (!b)\ 2+3 \text{ else } x : \text{int}}$$

Implementing a rule

- Work backward from goal:

```
class Add extends Expr {
   Expr e1, e2;
   Type typeCheck(SymTab A) {
        Type   t1 = e1.typeCheck(A),
               t2 = e2.typeCheck(A);
        if (t1 == Int && t2 == Int) return Int;
        else throw new TypeCheckError("+");
   }
```

$$T = E.\text{typeCheck}(A) \iff A \vdash E : T$$

$$\frac{A \vdash E_1 : \textbf{int} \quad A \vdash E_2 : \textbf{int}}{A \vdash E_1 + E_2 : \textbf{int}} \ (+)$$

More about Inference Rules

- Rules are implicitly universally quantified over free variables
- No premises: *axiom* $\quad \dfrac{}{A \vdash \text{true} : \textbf{bool}}$
- Same goal judgment may be provable in more than one way
- *Syntax-directed* rules: can prove judgements without searching

$$\frac{A \vdash E_1 : \textbf{float} \quad A \vdash E_2 : \textbf{float}}{A \vdash E_1 + E_2 : \textbf{float}} \qquad \frac{A \vdash E_1 : \textbf{float} \quad A \vdash E_2 : \textbf{int}}{A \vdash E_1 + E_2 : \textbf{float}}$$

While

- For statements that do not have a value, use the type **unit** to represent their result type (**unit** = completed succesfully)

$$\frac{A \vdash E : \textbf{bool} \quad A \vdash S : T}{A \vdash \text{while } (E) \ S : \textbf{unit}} \ (\text{while})$$

If statements

- Iota: the value of an if statement (if any) is the value of the arm that is executed.
- If no else clause, no value:

$$\frac{A \vdash E : \textbf{bool} \quad A \vdash S : T}{A \vdash \text{if } (E) \ S : \textbf{unit}} \ (\text{if})$$

Assignment

$$\frac{id : T \in A \quad A \vdash E : T}{A \vdash id = E : T} \ (\text{assign})$$

$$\frac{A \vdash E_3 : T \quad A \vdash E_2 : \text{int} \quad A \vdash E_1 : \text{array}[T]}{A \vdash E_1[E_2] = E_3 : T} \ (\text{array-assign})$$