CS 412
Introduction to Compilers

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Lecture 5: Top-down parsing
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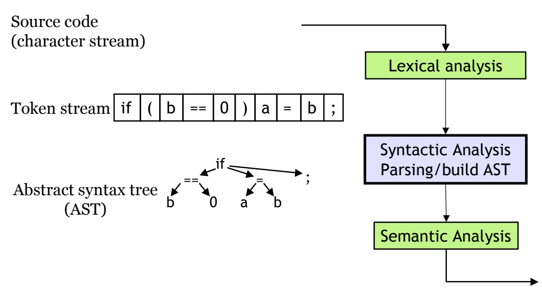
Outline

- More on writing CFGs
- Top-down parsing
- LL(1) grammars
- Transforming a grammar into LL form
- Recursive-descent parsing - parsing made simple

Where we are



Review of CFGs

- Context-free grammars can describe programming-language syntax
- Power of CFG needed to handle common PL constructs (e.g., parens)
- String is in language of a grammar if derivation from start symbol to string
- Ambiguous grammars a problem

if-then-else

- How to write a grammar for if stmts?

  $S \rightarrow$ if (E) S

  $S \rightarrow$ if (E) S else S

  $S \rightarrow X = E$ | if (E) S else S

Is this grammar ok?

No—Ambiguous!

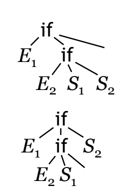
- How to parse?

  if (E_1) if (E_2) S_1 else S_2

$S \rightarrow$ if (E) S
$S \rightarrow$ if (E) S else S
$S \rightarrow$ *other*

$S \rightarrow$ if (E) S
$\rightarrow$ if (E) if (E) S else S

$S \rightarrow$ if (E) S else S
$\rightarrow$ if (E) if (E) S else S



Which "if" is the "else" attached to?

Grammar for Closest-if Rule

- Want to rule out if (E) if (E) S else S
- Problem: unmatched if may not occur as the "then" (consequent) clause of a containing "if"

statement	→ *matched* | *unmatched*
matched	→ if (E) *matched* else *matched*
	| *other*
unmatched	→ if (E) *statement*
	| if (E) *matched* else *unmatched*

Top-down Parsing

- Grammars for top-down parsing
- Implementing a top-down parser (recursive descent parser)
- Generating an abstract syntax tree

Parsing Top-down

$S \rightarrow E + S \mid E$
$E \rightarrow$ **num** $\mid (S)$

Goal: construct a leftmost derivation of string while reading in token stream

Partly-derived String	Lookahead	parsed part unparsed part
S	(	(1+2+(3+4))+5
→ E+S	(	(1+2+(3+4))+5
→ (S) +S	1	(1+2+(3+4))+5
→(E+S)+S	1	(1+2+(3+4))+5
→ (1+S)+S	2	(1+2+(3+4))+5
→ (1+E+S)+S	2	(1+2+(3+4))+5
→ (1+2+S)+S	2	(1+2+(3+4))+5
→(1+2+E)+S	(	(1+2+(3+4))+5
→(1+2+(S))+S	3	(1+2+(3+4))+5
→(1+2+(E+S))+S	3	(1+2+(3+4))+5

Problem

$S \rightarrow E + S \mid E$
$E \rightarrow$ **num** $\mid (S)$

- Want to decide which production to apply based on next symbol

(1) $S \rightarrow E \rightarrow (S) \rightarrow (E) \rightarrow (1)$

(1)+2 $S \rightarrow E + S \rightarrow (S) + S \rightarrow (E) + S \rightarrow (1)+E \rightarrow (1)+2$

- *Why is this hard?*

Grammar is Problem

- This grammar cannot be parsed top-down with only a single look-ahead symbol
- Not **LL(1)**
- **L**eft-to-right-scanning, **L**eft-most derivation, **1** look-ahead symbol
- Is it LL(k) for some k?
- Can rewrite grammar to allow top-down parsing: create LL(1) grammar for same language

Making a grammar LL(1)

$S \rightarrow E + S$
$S \rightarrow E$
$E \rightarrow$ **num**
$E \rightarrow (S)$

⇩

$S \rightarrow ES'$
$S' \rightarrow \varepsilon$
$S' \rightarrow + S$
$E \rightarrow$ **num**
$E \rightarrow (S)$

- **Problem**: can't decide which S production to apply until we see symbol after first expression
- **Left-factoring**: Factor common S prefix, add new non-terminal S' at decision point. S' derives $(+E)^*$
- **Also:** convert left-recursion to right-recursion

Parsing with new grammar

$S \to ES' \qquad S' \to \varepsilon \mid +S \qquad E \to \textbf{num} \mid (S)$

S	(	(1+2+(3+4))+5
$\to E\,S'$	(	(1+2+(3+4))+5
$\to (S)\,S'$	1	(1+2+(3+4))+5
$\to (E\,S')\,S'$	1	(1+2+(3+4))+5
$\to (1\,S')\,S'$	+	(1+2+(3+4))+5
$\to (1{+}E\,S')\,S'$	2	(1+2+(3+4))+5
$\to (1{+}2\,S')\,S'$	+	(1+2+(3+4))+5
$\to (1{+}2 + S)\,S'$	(	(1+2+(3+4))+5
$\to (1{+}2 + E\,S')\,S'$	(	(1+2+(3+4))+5
$\to (1{+}2 + (S)\,S')\,S'$	3	(1+2+(3+4))+5
$\to (1{+}2 + (E\,S')\,S')\,S'$	3	(1+2+(3+4))+5
$\to (1{+}2 + (3\,S')\,S')\,S'$	+	(1+2+(3+4))+5
$\to (1{+}2 + (3 + E)\,S')\,S'$	4	(1+2+(3+4))+5

Predictive Parsing

- **LL(1)** grammar:
  - for a given non-terminal, the look-ahead symbol uniquely determines the production to apply
  - top-down parsing = predictive parsing
  - Driven by ***predictive parsing table*** of
    non-terminals $\times$ input symbols $\to$ productions

Using Table

$S \to E\,S'$
$S' \to \varepsilon \mid +S$
$E \to \textbf{num} \mid (S)$

S	(	(1+2+(3+4))+5
$\to E\,S'$	(	(1+2+(3+4))+5
$\to (S)\,S'$	1	(1+2+(3+4))+5
$\to (E\,S')\,S'$	1	(1+2+(3+4))+5
$\to (1\,S')\,S'$	+	(1+2+(3+4))+5
$\to (1 + S)\,S'$	2	(1+2+(3+4))+5
$\to (1{+}E\,S')\,S'$	2	(1+2+(3+4))+5
$\to (1{+}2\,S')\,S'$	+	(1+2+(3+4))+5

	num	+	(	)	\$ / EOF
S	$\to E\,S'$		$\to E\,S'$		
S'		$\to +S$		$\to \varepsilon$	$\to \varepsilon$
E	$\to$ **num**		$\to (S)$		

How to Implement?

- Table can be converted easily into a ***recursive-descent parser***

	num	+	(	)	\$
S	$\to E\,S'$		$\to E\,S'$		
S'		$\to +S$		$\to \varepsilon$	$\to \varepsilon$
E	$\to$ **num**		$\to (S)$		

- Three procedures: parse_S, parse_S', parse_E

Recursive-Descent Parser

lookahead token

```
void parse_S () {
   switch (token) {
        case num: parse_E(); parse_S'(); return;
        case '(': parse_E(); parse_S'(); return;
        default: throw new ParseError();
   }
}
```

	number	+	(	)	\$
S	$\to ES'$		$\to ES'$		
S'		$\to +S$		$\to \varepsilon$	$\to \varepsilon$
E	$\to$ number		$\to (S)$		

Recursive-Descent Parser

```
void parse_S'() {
   switch (token) {
        case '+': token = input.read(); parse_S(); return;
        case ')': return;
        case EOF: return;
        default: throw new ParseError();
   }
}
```

	number	+	(	)	\$
S	$\to ES'$		$\to ES'$		
S'		$\to +S$		$\to \varepsilon$	$\to \varepsilon$
E	$\to$ number		$\to (S)$		

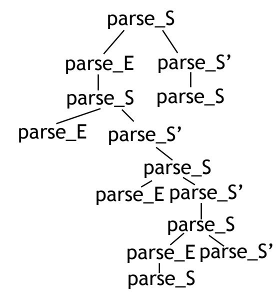
Recursive-Descent Parser

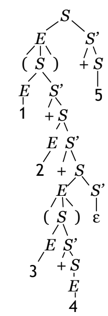
```
void parse_E() {
   switch (token) {
      case number: token = input.read(); return;
      case '(': token = input.read(); parse_S();
            if (token != ')') throw new ParseError();
            token = input.read(); return;
      default: throw new ParseError(); }
}
```

	number	+	(	)	$
S	$\to ES'$		$\to ES'$		
S'		$\to +S$		$\to \varepsilon$	$\to \varepsilon$
→ E	→ number		→ (S)		

Call Tree = Parse Tree

(1 + 2 + (3 + 4)) + 5





How to Construct Parsing Tables

- Needed: algorithm for automatically generating a predictive parse table from a grammar

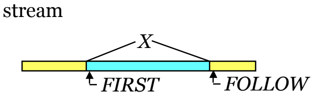
$S \to ES'$
$S' \to \varepsilon \mid +S$
$E \to$ number | (S)

?

	N	+	(	)	$
S	ES'		ES'		
S'		$+S$		ε	ε
E	N		(S)		

Constructing Parse Tables

- Can construct predictive parser if:
  For every non-terminal, every look-ahead symbol can be handled by at most one production
- FIRST(γ) for arbitrary string of terminals and non-terminals γ is:
  - set of symbols that might begin the fully expanded version of γ
- FOLLOW(X) for a non-terminal X is:
  - set of symbols that might follow the derivation of X in the input stream



Parse Table Entries

- Consider a production $X \to \gamma$
- Add $\to \gamma$ to the X row for each symbol in FIRST(γ)

	num	+	(	)	$
S	$\to ES'$		$\to ES'$		
S'		$\to +S$		$\to \varepsilon$	$\to \varepsilon$
E	→ num		→ (S)		

- If γ can derive ε (γ is *nullable*), add $\to \gamma$ for each symbol in FOLLOW(X)
- Grammar is LL(1) if no conflicting entries

Computing nullable, FIRST

- X is nullable if it can derive the empty string:
  - if it derives ε directly ($X \to \varepsilon$)
  - if it has a production $X \to YZ$... where all RHS symbols (Y, Z) are nullable
  - Algorithm: assume all non-terminals non-nullable, apply rules repeatedly until no change in status
- Determining FIRST(γ)
  - $FIRST(X) \supseteq FIRST(\gamma)$ if $X \to \gamma$
  - $FIRST(\mathbf{a}\,\beta) = \{ \mathbf{a} \}$
  - $FIRST(X\beta) \supseteq FIRST(X)$
  - $FIRST(X\beta) \supseteq FIRST(\beta)$ if X is nullable
  - **Algorithm:** Assume $FIRST(\gamma) = \{\}$ for all γ, apply rules repeatedly to build *FIRST* sets.

Computing FOLLOW

- *FOLLOW*(*S*) $\supseteq$ { \$ }
- If $X \to \alpha Y \beta$,
  FOLLOW(*Y*) $\supseteq$ *FIRST*(β)
- If $X \to \alpha Y \beta$ and β is nullable (or non-existent),
  FOLLOW(*Y*) $\supseteq$ *FOLLOW*(*X*)
- **Algorithm:** Assume *FOLLOW*(*X*) = { } for all *X*, apply rules repeatedly to build *FOLLOW* sets
- Common theme: iterative analysis. Start with initial assignment, apply rules until no change

Example

$S \to E\ S'$
$S' \to \varepsilon \mid + S$
$E \to$ **num** | **(** S **)**

- nullable
  - only S' is nullable
- FIRST
  - *FIRST*($E\ S'$) = {**num**, **(** }
  - *FIRST*($+S$) = { + }
  - *FIRST*(**num**) = {**num**}
  - *FIRST*(**(**S**)**) = { **(** }, FIRST(S') = { + }
- FOLLOW
  - *FOLLOW*(*S*) = { **\$**, **)** }
  - *FOLLOW*(S') = {**\$**, **)**}
  - *FOLLOW*(*E*) = { +, **)**, **\$**}

	num	**+**	**(**	**)**	**\$**
S	$\to E\ S'$		$\to E\ S'$		
S'		$\to +S$		$\to \varepsilon$	$\to \varepsilon$
E	$\to$ num		$\to (S)$		

Ambiguous grammars

- Construction of predictive parse table for ambiguous grammar results in *conflicts* (*but converse does not hold*)

$S \to S + S \mid S * S \mid$ **num**

FIRST($S + S$) = *FIRST*($S * S$) = *FIRST*(**num**) = { **num** }

	num	+	*
S	$\to$**num**, $\to S + S$, $\to S * S$		

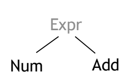
Completing the parser

Now we know how to construct a recursive-descent parser for an LL(1) grammar.

Can we use recursive descent to build an abstract syntax tree too?

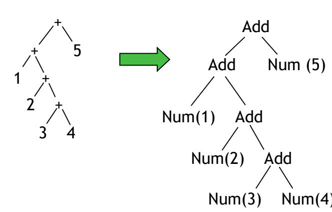
Creating the AST

```
abstract class Expr { }

class Add extends Expr {
   Expr left, right;
   Add(Expr L, Expr R) { left = L; right = R; }
}

class Num extends Expr {
   int value;
   Num (int v) { value = v);
}
```



AST Representation

(1 + 2 + (3 + 4)) + 5



How can we generate this structure during recursive-descent parsing?

Creating the AST

- Just add code to each parsing routine to create the appropriate nodes!
- Works because parse tree and call tree have same shape
- parse_S, parse_S’, parse_E all return an Expr:
  - void parse_E() ⇒ Expr parse_E()
  - void parse_S() ⇒ Expr parse_S()
  - void parse_S’() ⇒ Expr parse_S’()

AST creation code

```
Expr parse_E() {
   switch(token) {
     case num: // E → number
         Expr result = Num (token.value);
         token = input.read(); return result;
     case '(': // E → ( S )
         token = input.read();
         Expr result = parse_S();
         if (token != ')') throw new ParseError();
         token = input.read(); return result;
     default: throw new ParseError();
   }
}
```


parse_S

```
Expr parse_S() {
   switch (token) {
      case num:
      case '(':
         Expr left = parse_E();
         Expr right = parse_S'();
         if (right == null) return left;
         else return new Add(left, right);
      default: throw new ParseError();
   }
}
```

$S \rightarrow E\, S'$
$S' \rightarrow \varepsilon \mid + S$
$E \rightarrow \mathbf{num} \mid (S)$

Or…an Interpreter!

$S \rightarrow E\, S'$
$S' \rightarrow \varepsilon \mid + S$
$E \rightarrow \mathbf{num} \mid (S)$

```
int parse_E() {
   switch(token) {
      case number:
         int result = token.value;
         token = input.read(); return result;
      case '(':
         token = input.read();
         int result = parse_S();
         if (token != ')') throw new ParseError();
         token = input.read(); return result;
         default: throw new ParseError(); }}

int parse_S() {
   switch (token) {
      case number:
      case '(':
        int left = parse_E();
        int right = parse_S'();
        if (right == 0) return left;
        else return left + right;
      default: throw new ParseError(); } }
```


Summary

- We can build a recursive-descent parser for LL(1) grammars
  - Make parsing table from *FIRST*, *FOLLOW* sets
  - Translate to recursive-descent code
  - Instrument with abstract syntax tree creation
- Systematic approach avoids errors, detects ambiguities
- Next time: converting a grammar to LL(1) form, bottom-up parsing