Last time: Proof outlines / proof techniques

Proposition	To prove it	To use it	Logical negation (to disprove)
P and Q ($P \wedge Q$)	Prove both P and Q	Use either P or Q	P is false or Q is false
P or Q ($P \vee Q$)	Prove either P or Q	Prove R in the P and Q cases to conclude R (case analysis)	P is false and Q is false
if P then Q ($P \Rightarrow Q$)	Assume P and prove Q	If you know P, you can conclude Q	P is true but Q is false
for all $x \in A$, $P(x)$ ($\forall x, P(x)$)	Prove $P(y)$ for an arbitrary $y \in A$	Conclude $P(z)$ for any specific $z \in A$	There is some $x \in A$ for which P is false (counterexample)
there exists an $x \in A$ such that $P(x)$ ($\exists x, P(x)$)	Give a specific $y \in A$ and prove $P(y)$	Use $P(z)$ for an unknown z	for any $x \in A$, P is false
P is false ($\neg P$)	Disprove P	Contradiction	P

- Contradiction: if you prove contradictory facts, an assumption must have been wrong
- Defn: n is *composite* if $\exists i, j > 1$ with $n = ij$.
- Defn: n is *prime* if it is not composite
- Claim: there are infinitely many primes

- "for all" and "there exists" are *quantifiers*
- quantifiers can be nested, order matters

- Defn: We say $\lim_{x \to a} f = \ell$ if $\forall \epsilon > 0, \exists \delta > 0, \forall x' \in \mathbb{R},$
  if $0 < |x - x'| < \delta$ then $|f(x) - a| < \epsilon$
- Claim: $\lim_{x \to 0} x = 0$
- Claim: $\lim_{x \to 0} x \neq 1$

- Defn: We say f has a limit at x if $\exists a$ such that $\lim_{x \to a} f = a$
  - $\exists a \in \mathbb{R}, \forall \epsilon > 0, \exists \delta > 0, \forall x' \in \mathbb{R}, \ldots$
- Claim: f is continuous if $\forall x$, f has a limit at x
  - $\forall x, \exists a, \forall \epsilon, \exists \delta, \forall x', \ldots$