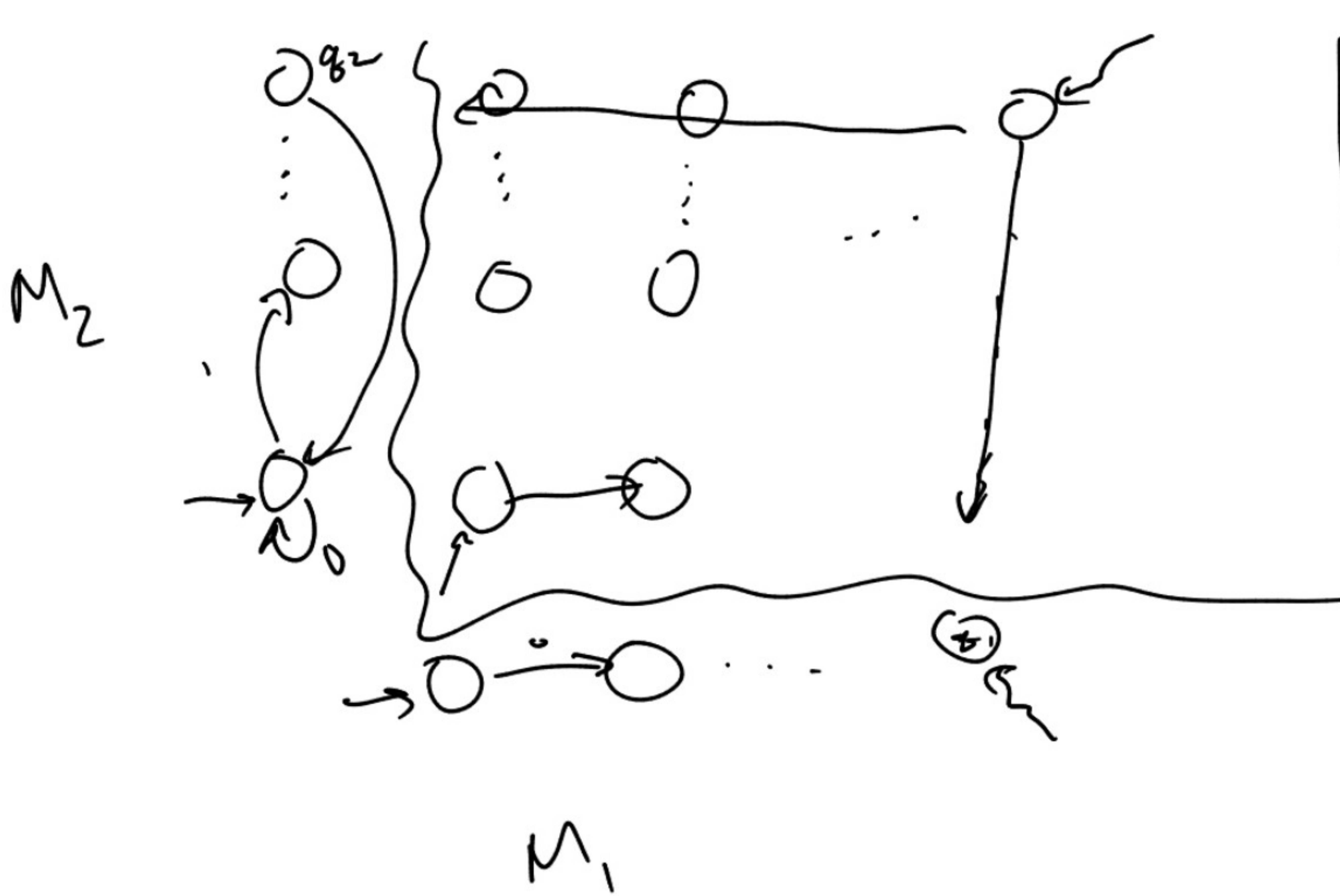
Lecture 23: Unrecognizable languages

- $\exists$ urecog. languages
- example: $0^n 1^n$
- pumping lemma



if

$$\hat{\delta}_M(q_{0M}, x) = (q_1, q_2)$$

then

$$\hat{\delta}_{M_1}(q_{0M_1}, x) = q_1$$

$$\hat{\delta}_{M_2}(q_{0M_2}, x) = q_2$$

$L \subseteq \Sigma^*$ is DFA-recognizable if $\exists$ a DFA M with $L(M) = L$

__Claim:__ there are unrecognizable languages.

__Question:__ how many languages are there?

A: uncountably many.

binary interp. is a bijection $\mathbb{N} \to \{0,1\}^*$

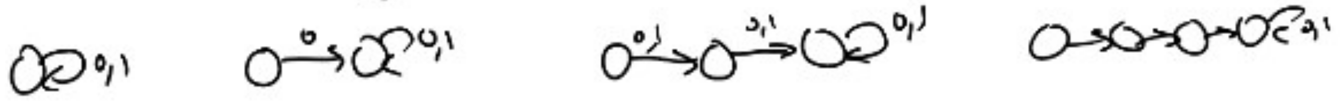
so $2^{\mathbb{N}}$ is bijective with 2^{Σ^*}.

Σ^* where $\Sigma = \{0,1\}$

__Question:__ how many strings exist?

A: countably many.

__Question:__ how many automata exist?



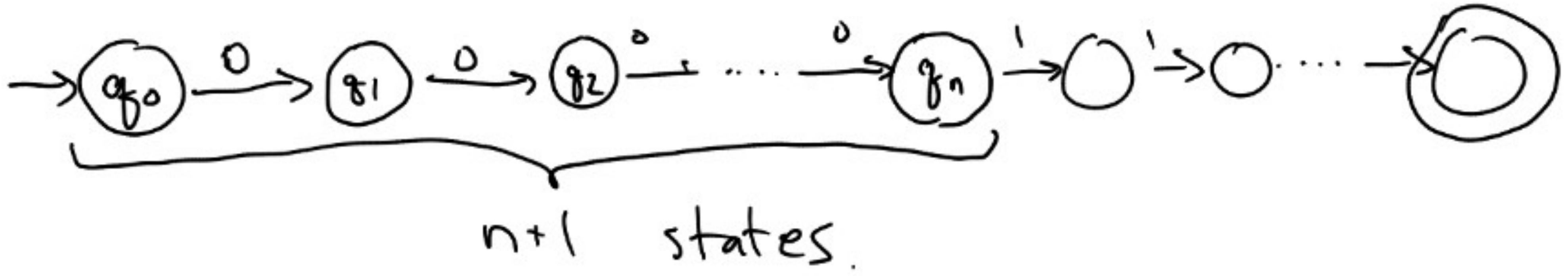
A: countably many.

__Pf:__ L: DFA, $\to$ Language can't be surjective

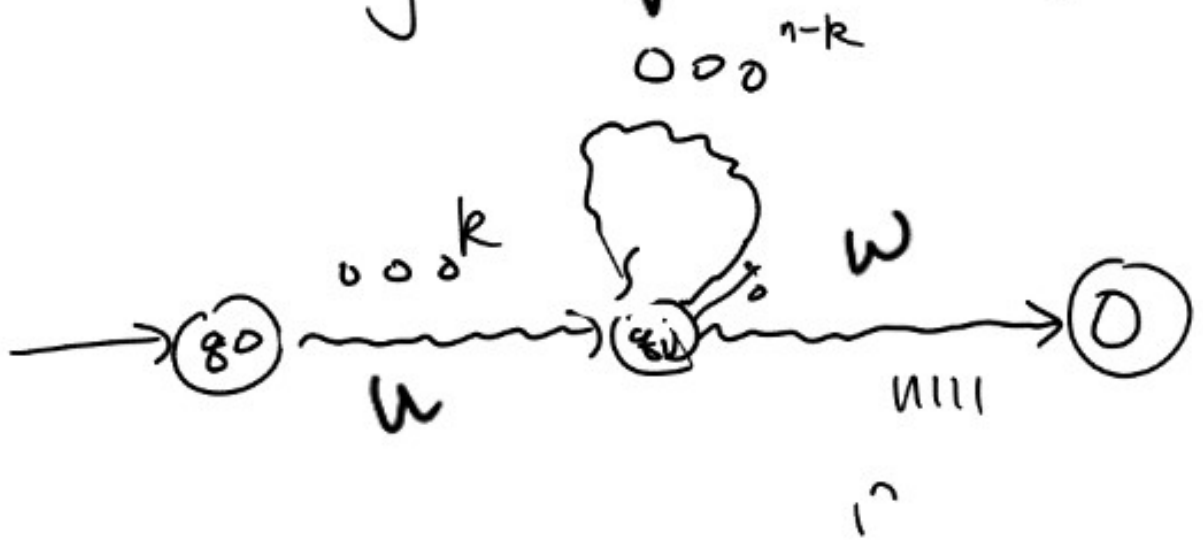
i.e. there are languages than automata.

let $n = \#$ of states of M.

let $x = 0^n 1^n$. Then $x \in L$, so M accept x.



Since only n states, $q_i = q_j$ for some $i \neq j$.



going around loop twice also gets to accept state!

i.e. $y = 0^{n+n-k} 1^n$ goes around twice,
So M accepts it!, but $y \notin L$,

Claim (Pumping lemma): if L is recognizable, then

$\exists\, n \in \mathbb{N}$ such that

$\forall\, x \in L$ with $\mathrm{len}(x) \geq n$,

$\exists\, u, v, w \in \Sigma^*$ such that

① $x = uvw$ ③ $v \neq \varepsilon$

② $\mathrm{len}(uv) \leq n$ ④ $\forall k,\ uv^k w \in L$

Ex: Claim: $L = \{0^n 1^n \mid n \in \mathbb{N}\}$ is unrec.

Pf: by contra, assume L is rec.

Then $\exists\, n \in \mathbb{N}$ as in pumping lemma.

Let $x = 0^n 1^n$ Then by pumping lemma,

since $x \in L$ (obvious), $\mathrm{len}(x) \geq n$ (obvs),

$\exists\, u, v, w$ as in PL.

Since $\mathrm{len}(uv) \leq n$, first n chars of x are 0

v can only have 0's, since $v \neq \varepsilon$, it

has $k \neq 0$ 0's in it.

But $uv^2w \in L$ by PL, but ← pumping lemma

uv^2w has $n+k$ 0's, only n 1's.

So $uv^2w \notin L$. Contradiction.