Lecture 6: Quantifiers & "Jectivity

- Canvas / Gradescope up (HW1 posted soon)
- HW 2 out this week (overlap w/ HW1)
  - due Fri. 2/14 5:00

Question: Which of the following is true / false?

A. yes
B. no

(1) $\forall x \in \mathbb{R} \left(\exists y \in \mathbb{R}, x > y \right)$ — $\boxed{A}$ yes (~65), B. no (~30)

(2) $\exists y \in \mathbb{R}, \forall x \in \mathbb{R}, x > y$

A. both true
B. (1) true, (2) false
C. (1) false, (2) true
D. both false
E. unsure.

$\forall$: "for all"

$\exists$: "there exists"

$\underbrace{\exists y \in \mathbb{R}, x > y}$

predicate on x: true or false, possibly depending on x.

let $x = 5$, does $\exists y \in \mathbb{R}, x > y$

yes: let $y = 4$. Then $x > y$.

Claim: $\forall x \in \mathbb{R}, \left(\underbrace{\exists y, x < y}\right)$

Pf: Choose an arb. $x \in \mathbb{R}$. Let $y = x + 1$.

Then $x < y$ because $x < x + 1$.

$\exists y, \left(\underbrace{\forall x, x > y}_{\text{predicate on } y}\right)$

if $y = 0$, does $\forall x, x > y$?

no

Claim: $\exists y, \forall x, x > y$ — what's x? x is undefined.

(Bogus) Proof: let $y = x - 1$. Choose an arbitrary x.

Clearly $x > x - 1 = y$ so $x > y$

Claim: There does not exist $y \in \mathbb{R}$ such that $\forall x, x > y$.

$\boxed{\text{is } y \text{ arbitrary?}}$

Proof (by contradiction):

(opposite of our goal)

$\rightarrow$ Assume that $\exists y, \forall x, x > y$.

Let $x = y - 1$. Clearly $x \not> y$. But by assumption, every x is $> y$. So $x > y$ and $x \not> y$, clearly nonsense. So initial assumption must be false.

Logical negation:

log. neg. of P is what you would prove to disprove P.

To disprove $\forall x, P$,
give an example x, show P is false for that x.

in other words,
prove $\exists x$, P is false.

To disprove $\exists x, P$:
prove P is false for an arbitrary x,

in other words,
prove $\forall x$, P is false.

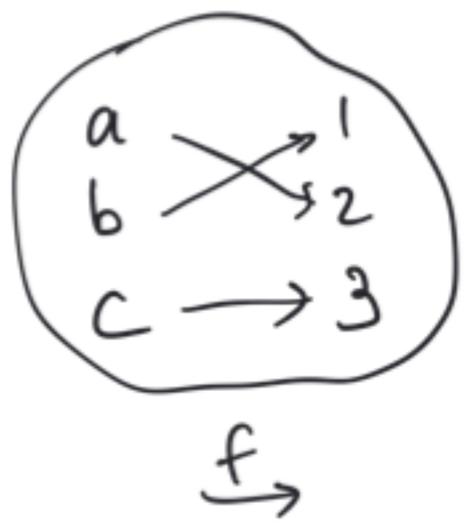
Claim: $\exists y, (\forall x,\ x > y)$ is false

Proof: We want to show $\forall y, ((\forall x, x > y)$ is false$)$

Choose arb. y; we WTS $\exists x, x \not> y$.

Let $x = y - 1$. Then $y - 1 \not> y$ so $x \not> y$.

Question:



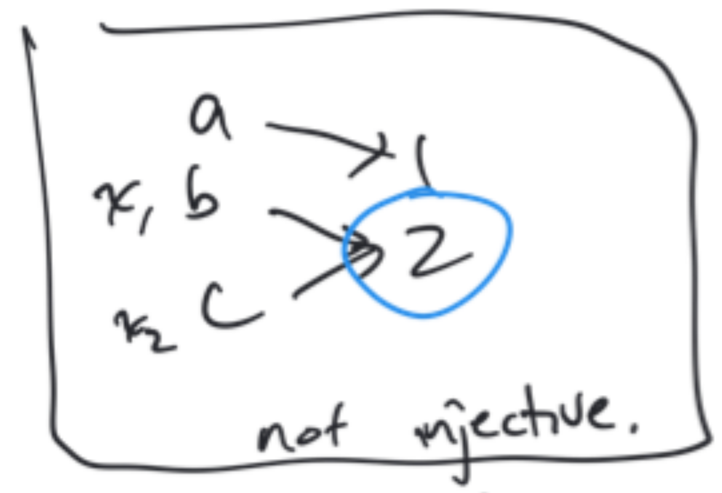
is f

$\boxed{\text{A.}}$ injective & surjective ✓ ✓

B. just injective

C. just surjective

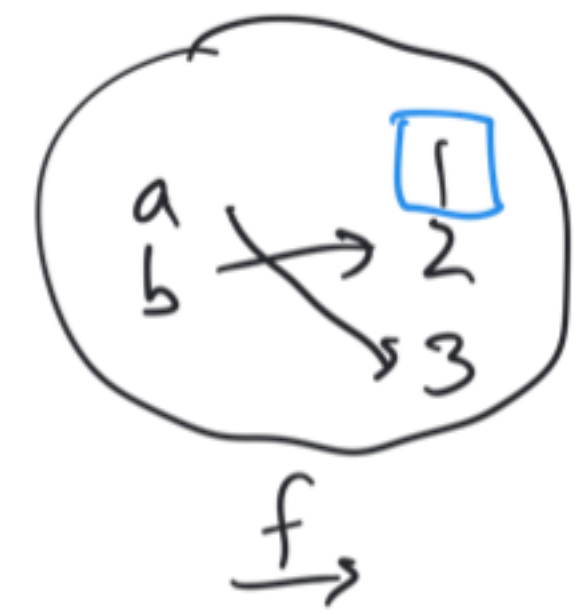
D. neither



Def$^{\underline{n}}$ $f: A \to B$ is injective means there does not exist x_1 and x_2 with $f(x_1) = f(x_2)$ and $x_1 \neq x_2$.

Exercise: prove these are same

Def$^{\underline{n}}$: $f: A \to B$ is injective if $\forall x_1$ and $x_2 \in A$, if $f(x_1) = f(x_2)$ then $x_1 = x_2$.



not surjective

Def$^{\underline{n}}$: $f: A \to B$ is surjective if $\forall y \in B$, $\exists x \in A$ with $f(x) = y$.