Last time:

Claim: if $L$ is $\varepsilon$-NFA recognizable then $L$ is NFA recognizable
The set $RE$ of regular expressions is given by

$$r \in RE ::= a \mid \varepsilon \mid \emptyset \mid r_1r_2 \mid r_1 + r_2 \mid r^* \quad a \in \Sigma$$

$L : RE \rightarrow 2^{\Sigma^*}$ is given inductively by

$$L(a) := \{a\} \quad L(\varepsilon) := \{\varepsilon\}$$
$$L(\emptyset) := \emptyset \quad L(r_1r_2) := \{xy \mid x \in L(r_1) \text{ and } y \in L(r_2)\}$$
$$L(r_1 + r_2) := L(r_1) \cup L(r_2) \quad L(r^*) := \{x_1x_2 \cdots x_n \mid x_i \in L(r)\}$$

$x$ matches $r$ means $x \in L(r)$

A language $L \subseteq \Sigma^*$ is regular if there exists $r \in RE$ with $L = L(r)$

Claim: $L$ is regular if and only if $L$ is NFA-recognizable

Announcements:

Course evals due with homework (Sat. at noon); no late submission
Claim (Kleene’s theorem): the following statements are equivalent:

- $L$ is DFA-recognizable
- $L$ is NFA-recognizable
- $L$ is regular

Defn: a generalized NFA $N$ is like an NFA, but we allow regular expression transitions.

A string $x$ is accepted if $x$ can be written as $x = x_1x_2 \cdots x_k$ and there is a path $\text{start} \xrightarrow{r_1} \cdots \xrightarrow{r_n} \text{accept}$ in $N$ with $x_i \in L(r_i)$ for each $i$.

Claim: if $L$ is recognizable by a generalized NFA, then $L$ is regular

- Step 1: make a new accept state $q_a$, add $\varepsilon$-transitions from old accept states to $q_a$
- Step 2: remove non-start non-accept states
- Read off the RE