Lecture 4: proofs

- propositions & logical connectives
    - and, or, if…then, for all, there exists

- how to prove, use, disprove each.

- A proposition is something that is true or false (e.x. $\{1,2\} \cup \{2,3\} = \{1,2,3\}$).

- A prop. with "free variables" is a predicate: (e.g. $A \cup B \subseteq C$).

- logical connectives build more complicated props/preds from simpler ones.

  e.g. if P & Q are propositions, then "P and Q" is also, as is

  ex: "P $A \subseteq B$ or Q", $B \subseteq A$, $A = B$ means $A \subseteq B$ and $B \subseteq A$
  (pred.) (pred.) (pred.)

To prove something:
- plug in definitions
- apply something you know
- apply rules of reasoning

WTS …

…

…

↑ know

rule of reasoning ↓

↓ want to show.

	to prove	to use	to disprove
P and Q	both: - prove P - prove Q.	May use either P or Q	either - disprove P or - disprove Q.
P or Q	either - prove P or - prove Q	to prove R, must prove R in "P" case also prove R in "Q" case.	both: - disprove P and - disprove Q
if P then Q.	assume P is true, (possibly use that assumption) prove Q	if you prove P, can use it to conclude Q. Note: can't conclude P if you know Q.	assume P, disprove Q.
for all x, P	choose "arbitrary" x, prove P about that x.	can conclude P for any particular x.	Come up with an example x, disprove P for that x.
usually "for all $x \in \mathbb{N}$, P" think of this "for all x, if $x \in \mathbb{N}$, then P."	choose arb. $x \in \mathbb{N}$, prove P.	can conclude P for any $x \in \mathbb{N}$.	
there exist an x such that (s.t.) P	tell me what x is, prove P for that x. "let $x = \ldots$, then…"	Can say "let x be the thing that exists, we know P (for that x)"	disprove P for all x.
"not P" "P is false"	disprove P	if know P and not P, can conclude anything you want.	prove P.

P "and" $\wedge$ Q

P "or" $\vee$ Q

P "implies" $\Rightarrow$ Q

does not mean an internal relationship.

in any world where P is true, Q is also.

$\forall$

arbitrary means you know nothing about x, not a specific example.

specific, not arbitrary.

arbitrary

"every # has a square root"

$\forall x \in \mathbb{R}^+, \exists y \in \mathbb{R}, y^2 = x$

oops! should say "prove P, disprove Q".

Ex. To disprove $\forall x \in \mathbb{R}$, if $x^2 > 25$ then $x > 5$, I need to give example x (to disprove $\forall$…), and for that x, I need to disprove "if $x^2 > 25$ then $x > 5$". I would pick $x = -6$; then check (i.e. prove) that $x^2 > 25$ and $x \not> 5$. It wouldn't make sense to assume $(-6)^2 > 25$.