

Lecture 4: proofs

- propositions & logical connectives
 - and, or, if-then, for all, there exists
- how to prove, use, disprove each.

- A proposition is something that is true or false
(e.x. $\{1,2\} \cup \{2,3\} = \{1,2,3\}$)

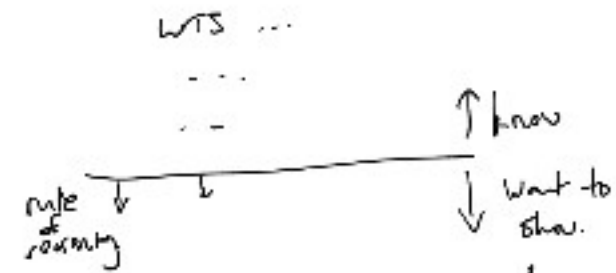
- A prop. with "free variables" is a predicate:
(e.g. $A \cup B \subseteq C$).

- logical connectives build more complicated
props/preds from simpler ones.

e.g. if P & Q are propositions, then
"P and Q" is also, as is

e.x. " $A \subseteq B$ & Q ", $B \subseteq A$, $A=B$ means
(pred.) (pred.) , $A \subseteq B$ and $B \subseteq A$
(pred.)

- To prove something:
- plug in definitions
 - apply something you know
 - apply rules of reasoning



$P \wedge Q$

$P \vee Q$

$P \Rightarrow Q$

\forall

"every # has a square root"
 $\forall x \in \mathbb{R}^+, \exists y \in \mathbb{R}, y^2 = x$

	to prove	to use	to disprove
$P \wedge Q$	both: - prove P - prove Q.	may use either P or Q	either - disprove P or - disprove Q.
$P \vee Q$	either - prove P or - prove Q	to prove R, must prove R in "P" case also prove R in "Q" case.	both - disprove P and - disprove Q
$P \Rightarrow Q$	assume P is true, (possibly use that assumption), prove Q	if you prove P, can use it to conclude Q <u>Note: can't conclude P if you know Q.</u>	<u>assume P, disprove Q.</u>
\forall	for all x, P usually "for all $x \in \mathbb{N}, P$ " "for all x , if exactly then P"	choose "arbitrary" x , prove P about that x . can conclude P for any particular x . can conclude P for any $x \in \mathbb{N}$.	Come up with an <u>example</u> x , disprove P for that x .
"there exist an x such that P (S.E.)"	tell me what x is, prove P for that x "let $x = \dots$, then..."	can say "let x be the thing that exists, we know P (for that x)"	disprove P for all x .
"not P" "P is false"	disprove P	if know P and not P can conclude anything you want.	prove P.

does not mean an internal relationship
 in any world where P is true, Q is also.

arbitrary means you know nothing about x
 not a specific example.

specific, not arbitrary.

arbitrary

oops! should say "prove P, disprove Q".

Ex. To disprove $\forall x \in \mathbb{R},$
 if $x^2 > 25$ then $x > 5$, I need
 to give example x (to disprove $\forall \dots$),
 and for that x , I need to
 disprove "if $x^2 > 25$ then $x > 5$ ".
 I would pick $x = -6$; then check
 (i.e. prove) that $x^2 > 25$ and
 $x \not> 5$. It wouldn't
 make sense to
assume $(-6)^2 > 25$.