Lecture 2: Sets and functions

Today:

- Definition of set, examples
- Set notation
- Ramifications of equality
- Definition of function, partial function

Announcements:

- Homework 1 out today, due 9/13 at 5PM

Set definition

Defn: A *set* S is a collection of things. Every thing x is either *in* S (written $x \in S$) or not in S (written $x \notin S$).

Examples:

- the set $\mathbb{N}$ of natural #'s:

  $\mathbb{N} := \{0, 1, 2, \ldots\}$

  $0 \in \mathbb{N}$, $1 \in \mathbb{N}$, $17 \in \mathbb{N}$, purple $\notin \mathbb{N}$, $3.5 \notin \mathbb{N}$

- the set of integers $\mathbb{Z} := \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

- enumerated set: $B := \{$red, green, blue$\}$ red $\in B$, green $\in B$, blue $\in B$
  nothing else is in B

- Set comprehension: $A := \{ n \in \mathbb{Z} \mid$ there is some $k \in \mathbb{Z}$ with $n = k^2 \}$

  set of all

  "such that"

  rule for whether in this set.

  $16 \in A$? yes because there is a k (namely $k = 4$) such that $k^2 = 16$ ✓

- Let J be the set of all Java programs that compile.

- Let H be the set of Java programs that never go into an infinite loop.

Set definition

Defn: A *set* S is a collection of things. Every thing x is either *in* S (written $x \in S$) or not in S (written $x \notin S$).

Questions:

- Can elements be repeated?
- ✓ Can you have no elements?

A: yes

Notation: the empty set is written $\emptyset$ or $\{\}$

- ✓ do sets have subsets? (subsets: later)
- ✓ can sets be in sets?

yes:

example: $S := \{\{1,2\}, \{3,4\}\}$

$\{1,2\} \in S$

$\{3,4\} \in S$

$\boxed{\text{nothing else is in } S}$

$1 \in S$?

$1 \notin \{1,2\}$

$1 \notin \{3,4\}$

so $1 \notin S$.

- does order matter?
- ✓ Can a set have diff. types of elements?

A: yes

Set equality

Defn: A *set* S is a collection of things. Every thing x is either *in* S (written $x \in S$) or not in S (written $x \notin S$).

$\{1, 2, 3, 1+2\}$

Questions:

- Do duplicates matter?
- Does order matter?

$\{1,2,3,3\} \stackrel{?}{=} \{1,2,3\}$ yes: check using defn of equality.

Defn: Sets A and B are *equal* (written $A = B$) if:

- Every $x \in A$ is also in B, and
- Every $x \in B$ is also in A.

"if $x \in B$, then $x \in A$"

equivalently:

1. every $x \in A$ is in B
2. every $x \notin A$ is not in B.

contrapositive

"if $x \notin A$ then $x \notin B$"

$\{1,2\} \stackrel{?}{=} \{2,1\}$ yes: check using defn.

$\underbrace{\{1,3,3,3,\ldots\}}_{\text{in}\cancel{\text{finite}}} \stackrel{?}{=} \underbrace{\{1,3\}}_{\text{finite}}$ yes

Function definition

Defn: A *function* f from A to B (written $f : A \to B$) gives, for every input $x \in A$, an unambiguous output $f(x) \in B$.

Defn: A is the *domain* of f. B is the *codomain* of f.

only one output.

Examples:

$A = \{1,2,3\}$
$B = \{a,b,c\}$

$1 \to a$, $2 \to a$, $3 \to c$

f

$\begin{cases} f(1) := a \\ f(2) := a \\ f(3) := c \end{cases}$

$f(1) = f(2)$ so not a f^n?

$f(1)$ is unambiguously a

$f(2)$ is unambiguously a

actually f is a f^n

other ways to write down f^ns:

∘ table $f: \{1,2,3\} \to \{a \dots z\}$

x	$f(x)$
1	a
2	a
3	c

Codomain is not clear.

(need to write it down)

$1 \to a$, $1 \to b$, $2 \to b$, $3 \to c$ $f(1)$ is ambiguous.

$f(\{1,2\}) = f(\{1,2,2\})$

$f(\{a_1, a_2, \dots, a_n\}) := n$ ambiguous.

Def^n: two f^ns $f: A \to B$ and $g: A \to B$ are equal if for all $x \in A$, $f(x) = g(x)$.

if f & g have diff. domains or codomains, they can't be equal.

$1 \to a$
$2 \quad b$
$3 \to c$

doesn't output for every input, only some.

is called a partial function

Partial functions

Defn: A *partial function* f from A to B gives, for every input $x \in A$, *at most one* output $f(x) \in B$. If f gives no output, we say that $f(x)$ is undefined.