

Lecture 2: Sets and functions

Today:

- ▶ Definition of set, examples
- ▶ Set notation
- ▶ Ramifications of equality
- ▶ Definition of function, partial function

Announcements:

- ▶ Homework 1 out today, due 9/13 at 5PM

Set definition

Defn: A set S is a collection of things. Every thing x is either in S (written $x \in S$) or not in S (written $x \notin S$).

Examples:

- the set \mathbb{N} of natural #'s:

$$\mathbb{N} := \{0, 1, 2, \dots\}$$

$$0 \in \mathbb{N}, 1 \in \mathbb{N}, 17 \in \mathbb{N}, \text{purple} \notin \mathbb{N}, 3.5 \notin \mathbb{N}$$

- the set of integers $\mathbb{Z} := \{\dots, -2, -1, 0, 1, 2, \dots\}$

- enumerated set: $B := \{\text{red, green, blue}\}$

$\text{red} \in B, \text{green} \in B, \text{blue} \in B$
nothing else is in B

- Set comprehension: $A := \{m \in \mathbb{Z} \mid \text{there is some } k \in \mathbb{Z} \text{ with } m = k^2\}$

set of all "such that" rule for whether
me this set.

$16 \in A?$ yes because there is a k
(namely $k=4$) such that
 $k^2 = 16$ ✓

- Let J be the set of all Java programs that compile.

- Let H be the set of Java programs that never go into an infinite loop.

Set definition

Defn: A set S is a collection of things. Every thing x is either *in* S (written $x \in S$) or not in S (written $x \notin S$).

Questions:

• Can elements be repeated?

✓ Can you have no elements?

✓ do sets have subsets?
(subsets: later)

✓ can sets be in sets?

• does order matter?

✓ can a set have diff. types of elements?

A: yes

A: yes

Notation: the empty set is written \emptyset or $\{\}$

yes:
example:
 $S := \{\{1,2\}, \{3,4\}\}$

$\{1,2\} \in S$

$\{3,4\} \in S$

nothing else is in S .

$1 \in S?$

$1 \neq \{1,2\}$

$1 \neq \{3,4\}$

so $1 \notin S$.

Set equality

Defn: A set S is a collection of things. Every thing x is either *in* S (written $x \in S$) or not in S (written $x \notin S$).

Questions:

- ▶ Do duplicates matter?
- ▶ Does order matter?

$\{1, 2, 3, 1+2\}$

$\{1, 2, 3, 3\} \stackrel{?}{=} \{1, 2, 3\}$

yes: check using defⁿ of equality.

Defn: Sets A and B are equal (written $A = B$) if:

- ▶ Every $x \in A$ is also in B , and
- ▶ Every $x \in B$ is also in A .

equivalently:

① every $x \in A$ is in B

② every $x \notin A$ is not in B .

"if $x \in B$, then $x \in A$ "

contrapositive

"if $x \notin A$ then $x \notin B$ "

$\{1, 2\} \stackrel{?}{=} \{2, 1\}$ yes: check using defⁿ.

$\{1, 3, 3, 3, \dots\}$ infinite $\stackrel{?}{=} \{1, 3\}$ finite yes

Function definition

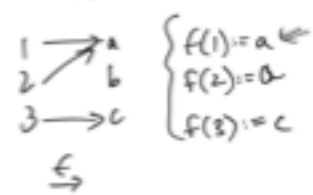
Defn: A function f from A to B (written $f: A \rightarrow B$) gives, for every input $x \in A$, an unambiguous output $f(x) \in B$.

Defn: A is the domain of f . B is the codomain of f .

Examples:

$$A = \{1, 2, 3\}$$

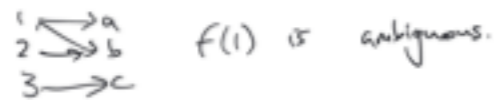
$$B = \{a, b, c\}$$



$$\begin{cases} f(1) = a \\ f(2) = b \\ f(3) = c \end{cases}$$

$f(1) = f(2)$ so not a f^n ?

$f(1)$ is unambiguously a
 $f(2)$ is unambiguously b
 actually f is a f^n



$$f(\{1, 2\}) = \{f(1), f(2)\}$$

$$f(\{1, 2, 2\}) = \{a, b\}$$

ambiguous.

Defn: two f^n s are equal if for all $x \in A$, $f(x) = g(x)$.

if $f \neq g$ have diff. domains or codomains, they can't be equal.

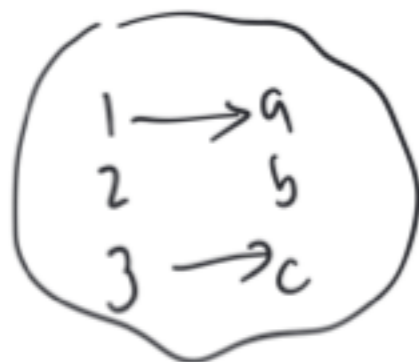
other ways to write down f^n s:

table $f: \{1, 2, 3\} \rightarrow \{a, \dots, z\}$

x	$f(x)$
1	a
2	a
3	c

Codomain is not clear.

(used to write it down)



doesn't output for every input, only some.

is called a partial function

Partial functions

Defn: A *partial function* f from A to B gives, for every input $x \in A$, at most one output $f(x) \in B$. If f gives no output, we say that $f(x)$ is undefined.