Non-determinism

0011 would be rejected.

1001 would be rejected

Idea: A *non-deterministic* finite automaton (NFA) always makes that "right" choice to get to an accept state. If it's possible to accept a string x, it accepts x.

(must process each char!)

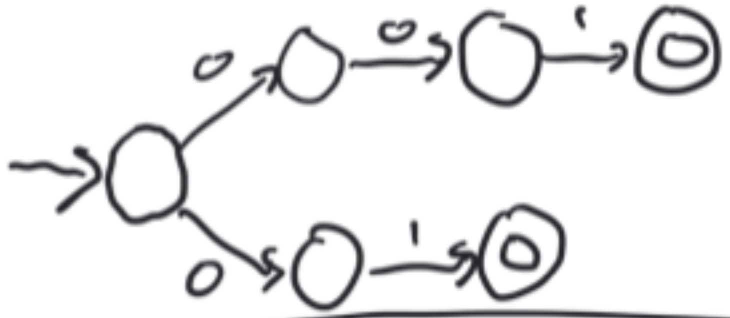
Question: What are the languages of the following NFA?

$(\Sigma := \{0,1\})$

$\{001\}$

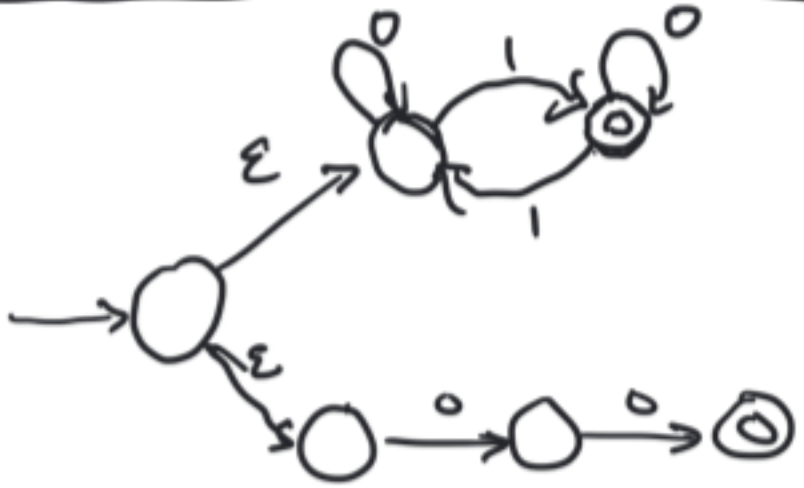
~~{}~~ {any string ~~containing~~ ~~substring~~ 001}

...0...0...1...



$\{001, 01\}$

11101 is rejected.



$\{00\} \cup \{x \mid x$ has odd ~~#~~ 1s$\}$

"ε transitions" may be taken "for free": without processing any chars.

11

0

ε01001

Formalizing ε-NFA

ε-NFA N

Defn: An ~~DFA M~~ consists of:

- A set Q of states ✓
- An alphabet Σ ✓
- A transition function $\delta : Q \times \Sigma \to \cancel{Q}\ 2^Q$
- A start state $q_0 \in Q$ ✓
- A set of accept states $A \subseteq Q$ ✓

- An ε-trans. fn
  $\varepsilon : Q \to 2^Q$

$\Sigma = \{0,1\}$

$\delta(q,0) = \cancel{q'?} = \{q', q''\}$

$= \cancel{q''?}$

$\delta(q,1) = \cancel{?}\ \emptyset$

$\varepsilon(q'') := \{q'\}$

$\varepsilon(q) := \emptyset$

Language of an ε-NFA

Defn: If M is a DFA, the language of M (written $L(M)$) is the set of strings M accepts

Defn: M accepts x if $\hat{\delta}(q_0, x) \in A$

Defn: $\hat{\delta} : Q \times \Sigma^* \to Q$ gives the state M ends in after processing x; it is given by $\hat{\delta}(q, \varepsilon) := q$ and $\hat{\delta}(q, xa) := \delta(\hat{\delta}(q, x), a)$

Defn: N is an NFA, $L(N)$ is the set of strings N accepts.

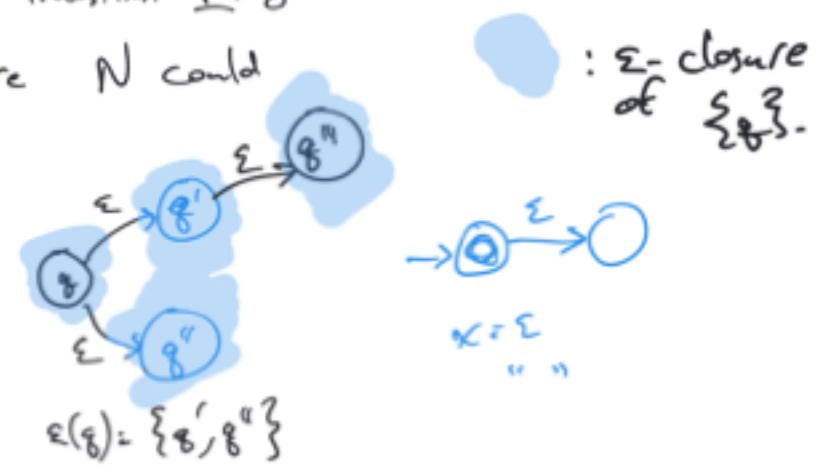
Defn: N accepts x if there is an accept state in $\hat{\delta}(q_0, x)$ i.e. $\hat{\delta}(q_0, x) \cap A \neq \emptyset$

idea: should accept if it is possible to get x to accepting state.

Defn. for an NFA, the extended transition fn. $\hat{\delta}$
$\hat{\delta} : Q \times \Sigma^* \to 2^Q$ tells us where N could reach on input x.

$$\hat{\delta}(q, \varepsilon) := \hat{\varepsilon}(\{q\})$$

$$\hat{\delta}(q, xa) :=$$



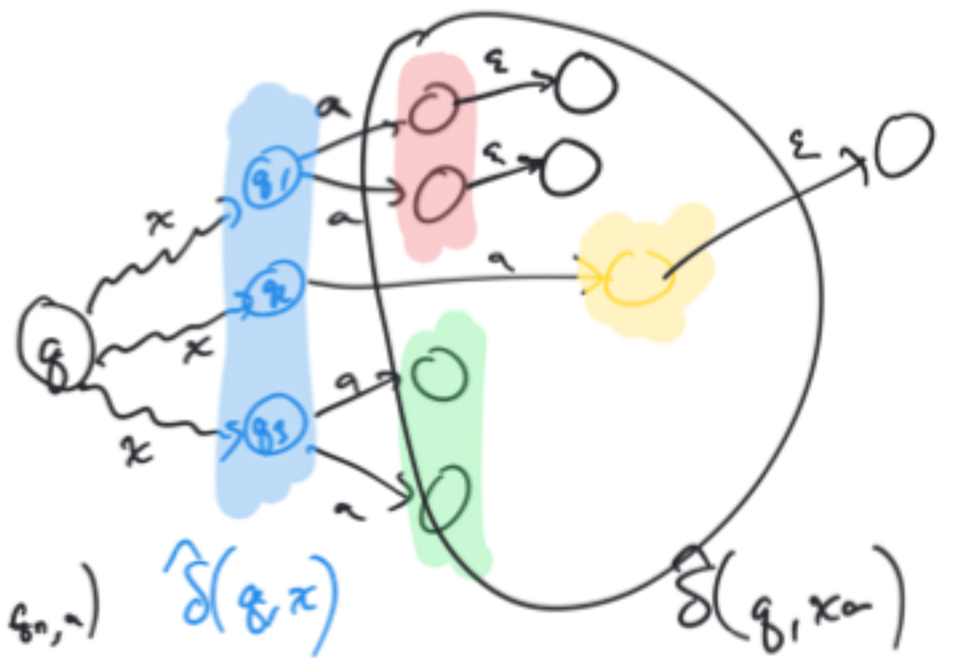
Defn: if S is a set of states, $\hat{\varepsilon}(S)$ is the set of states reachable from S with any # of ε-trans (no other transitions)

"ε-closure of S"

$$\hat{\delta}(q, xa) := \quad \text{(can use } \hat{\delta}(q, x)\text{)}$$

$$\cancel{\delta(\underbrace{\hat{\delta}(q,x)}_{\text{set}}, a)}$$

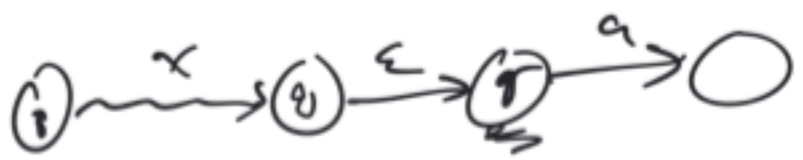
$$\delta : Q \times \Sigma \to 2^Q$$

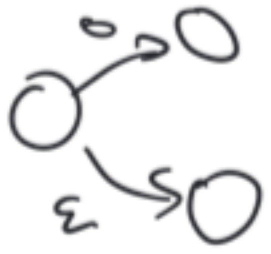


$\delta(q_1, a) \cup \delta(q_2, a) \cup \dots \cup \delta(q_n, a)$

elts of $\hat{\delta}(q, x)$

$$\boxed{\hat{\varepsilon}\left(\bigcup_{q' \in \hat{\delta}(q,x)} \delta(q', a)\right)}$$





$$\hat{\delta}(q, \varepsilon) := \hat{\varepsilon}(\{q\})$$

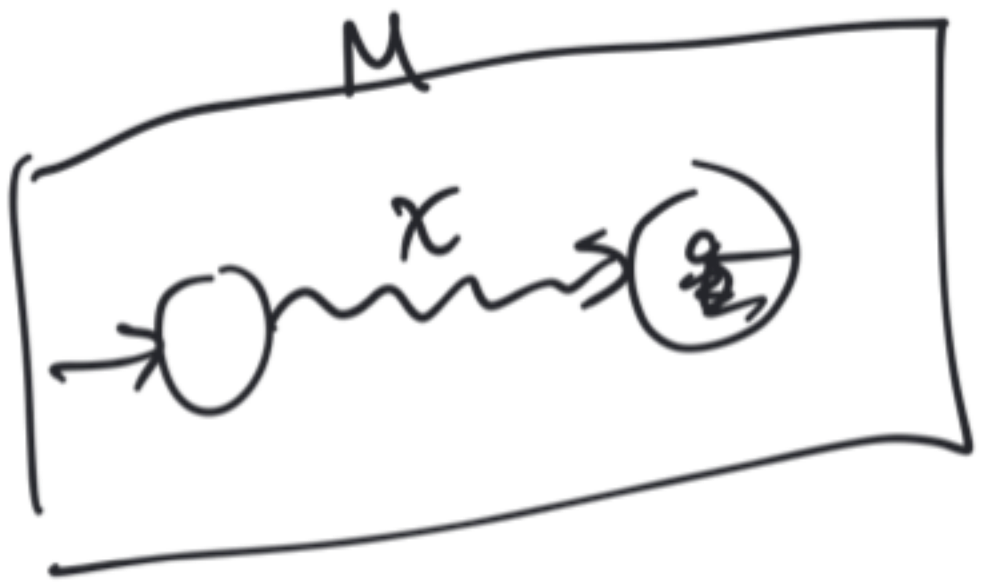
$$\hat{\delta}(q, xa) := \hat{\varepsilon}\left(\bigcup_{q' \in \hat{\delta}(q,x)} \delta(q', a)\right)$$

Removing non-determinism

Question: Are non-deterministic automata more powerful than deterministic automata?

Claim: For all NFA N there exists a DFA M with $L(M) = L(N)$

Proof: Choose an arbitrary N. Let M be given as follows:



idea: each state of M corresponds to a set of states of N.