

## Lecture 4: proofs

- propositions & logical connectives
  - and, or, if-then, for all, there exists
- how to prove, use, disprove each.

- A proposition is something that is true or false  
(e.x.  $\{1,2\} \cup \{2,3\} = \{1,2,3\}$ )

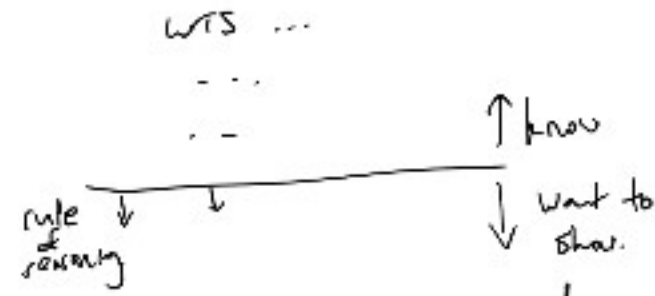
- A prop. with "free variables" is a predicate:  
(e.g.  $A \cup B \subseteq C$ ).

- logical connectives build more complicated  
props/preds from simpler ones.

e.g. if  $P$  &  $Q$  are propositions, then  
"P and Q" is also, as is

e.x. " $A \subseteq B$  &  $Q$ ",  $B \subseteq A$  ,  $A=B$  means  
(pred.) (pred.) ,  $A \subseteq B$  and  $B \subseteq A$   
(pred.)

- To prove something:
- plug in definitions
  - apply something you know
  - apply rules of reasoning



$P \wedge Q$  "and"

$P \vee Q$  "or"

$P \Rightarrow Q$  "implies"

$\forall$

"every # has a square root"

$$\forall x \in \mathbb{R}^+, \exists y \in \mathbb{R}, y^2 = x$$

"there exists"  $\exists$

does not mean an internal relationship  
in any world where P is true, Q is also.

arbitrary means you know nothing about x, not a specific example.

	to prove	to use	to disprove
$P \wedge Q$	both: - prove P - prove Q.	may use either P or Q	either - disprove P or - disprove Q.
$P \vee Q$	either - prove P or - prove Q	to prove R, must prove R in "P" case also prove R in "Q" case.	both - disprove P and - disprove Q
$P \Rightarrow Q$	assume P is true, (possibly use that assumption), prove Q.	if you prove P, can use it to conclude Q. <u>Note: can't conclude P if you know Q.</u>	assume P, disprove Q.
$\forall$	for all x, P  usually "for all $x \in \mathbb{N}$ , P" think of this "for all x, if freely then P."	choose "arbitrary" x, prove P about that x.  choose arb. $x \in \mathbb{N}$ , prove P.  can conclude P for any particular x.  can conclude P for any $x \in \mathbb{N}$ .	Come up with an <u>example</u> x, disprove P for that x.
"there exists" $\exists$	there exist an x such that P (S.E.)	can say "let x be the thing that exists, we know P (for that x)"	disprove P for all x.
"not P" "P is false"	disprove P	if know P and not P, can conclude anything you want.	prove P.

specific, not arbitrary.

arbitrary