Lecture 4: proofs

- propositions & logical connectives
  - and, or, if…then, for all, there exists
- how to prove, use, disprove each.

- A proposition is something that is true or false (e.x. $\{1,2\} \cup \{2,3\} = \{1,2,3\}$).

- A prop. with "free variables" is a predicate: (e.g. $A \cup B \subseteq C$).

- logical connectives build more complicated props/preds from simpler ones.

  e.g. if P & Q are propositions, then "P and Q" is also, as is

  ex: "P $A \subseteq B$ or Q", $B \subseteq A$, $A = B$ means $A \subseteq B$ <u>and</u> $B \subseteq A$

  (pred.) (pred.) (pred.)

To prove something:
- plug in definitions
- apply something you know
- apply rules of reasoning

WTS ...
...
...

↑ know
rule of reasoning ↓ ↓
↓ want to show.

		to prove	to use	to disprove
$P \overset{\text{"and"}}{\wedge} Q$	P and Q	both: - prove P - prove Q.	may use either P or Q	either - disprove P or - disprove Q.
$P \overset{\text{"or"}}{\vee} Q$	P or Q	either - prove P <u>or</u> - prove Q	to prove R, must prove R in "P" case also prove R in "Q" case.	<u>both</u>: - disprove P <u>and</u> - disprove Q
"implies" $P \Rightarrow Q$	if P then Q.	assume P is true, (possibly use that assumption), prove Q.	if you prove P, can use it to conclude Q. Note: can't conclude P if you know Q.	assume P, disprove Q.
$\forall$	for all x, P usually "for all $x \in \mathbb{N}$, P" think of this: "for all x, if $x \in \mathbb{N}$, then P.	choose "arbitrary" x, prove P about that x. choose arb. $x \in \mathbb{N}$, prove P.	can conclude P for any particular x. can conclude P for any $x \in \mathbb{N}$.	come up with an <u>example</u> x, disprove P for that x.
"there exists" $\exists$	there exist an x <u>such that</u> (s.t.) P	tell me what x is, prove P for that x. "let x = ..., then..."	can say "let x be the thing that exists, we know P (for that x)"	disprove P for all x.
	"not P" "P is false"	disprove P	if know P and not P, can conclude anything you want.	prove P.

does not mean an internal relationship.

in any world where P is true, Q is also.

arbitrary means you know nothing about x, <u>not</u> a specific example.

specific, not arbitrary.

arbitrary

"every # has a square root"

$\forall x \in \mathbb{R}^+, \exists y \in \mathbb{R}, y^2 = x$