# CS 2800: Discrete structures

http://courses.cs.cornell.edu/cs2800/wiki

Announcements:

- Have your clickers ready
- No vertical screens please
- HW 1 out Monday, due Friday 2/7 at 5PM
- Office hours start Wednesday; calendar up soon
- Make sure you join Piazza
- Canvas up soon with links to everything

## Set definition

"in" or "is an element of"

**Defn:** A *set* $S$ is a collection of things. Every thing $x$ is either *in* $S$ (written $x \in S$) or not in $S$ (written $x \notin S$).

Examples:

- let P be the set of people in the room.
- Let $A := \{1,2,3\}$ $1 \in A$, $2 \in A$, $3 \in A$

  nothing else in A.

  is defined as.

  $::=$

  $A = \{1,2,3\}$

- Let $\mathbb{N} := \{0,1,2,\ldots\}$ $0 \in \mathbb{N}$ $1 \in \mathbb{N}$ $17 \in \mathbb{N}$ $35 \in \mathbb{N}$

  set of natural numbers.

- Let $C := \{1,3,5,7 \ldots, 23\}$

  = set of odds between 1 & 23.

## Set definition

**Defn:** A *set* $S$ is a collection of things. Every thing $x$ is either *in* $S$ (written $x \in S$) or not in $S$ (written $x \notin S$).

if you answer this question unambiguously, you've defined a set

? : can't answer yet, terms not defined (yet)

Questions:

- Do duplicates matter — is $\{1,1,1\}$ a set? ?   A: yes.
- let $A := \{1,2,3\}$, $B := \{1,2,3\}$ can I conclude $A = B$? ?
- Does order matter? ?
- Can we operations on sets? A: yes if you give a defn for your operation.
- is an empty set allowed?, A: yes, for all $x$, $x \notin S$   $\emptyset$   $\{\}$
- do sets need to be $\boxed{\text{finite}}$ ?
- can sets contain sets as elements?. A: yes, sets are things. $\{\emptyset\}$
- are sets $\boxed{\text{bounded}}$ ?
- is $\{\emptyset, \emptyset, \emptyset\}$ empty?

## Set equality

**Defn:** A *set* $S$ is a collection of things. Every thing $x$ is either *in* $S$ (written $x \in S$) or not in $S$ (written $x \notin S$).

Questions:

- Do duplicates matter? A: No
- Does order matter? A: No

**Defn:** Sets $A$ and $B$ are *equal* (written $A = B$) if:

- Every $x \in A$ is also in $B$, and
- Every $x \in B$ is also in $A$.

$x_1, x_2$ and $x_3$

1 in position #2

only $x_1$

nothing in position #2

- Question: is $\overset{A}{\{1, 1, 1\}} = \overset{B}{\{1\}}$ ?

correct

A: yes, equal ~80%

B: No, not equal ~20%

correct.

$1 \in A$, ok, $1 \in B$
$1 \in A$, ok, $1 \in B$
$1 \in A$, ok, $1 \in B$ } every elt. $x \in A$ is also in B.

$1 \in B$, ok, $1 \in A$ ✓ every elt. of B is also in A.

- Question: $\{1, 2\} = \{2, 1\}$

A: yes. equal

B: no not equal?

## Set comprehension notation

$\{ \underline{p} \mid \underline{p \text{ is a person in the room}} \}$

"the set of all" ____ "such that" ____

$\{ x \mid x = y^2 \text{ for some } y \in \mathbb{N} \}$

$= \{ 0^2, 1^2, 2^2, \ldots \}$

$= \{ 0, 1, 4, 9, 16, \ldots \}$

Ex: $\{ \underline{x \in \mathbb{N}} \mid \underline{x > 5} \}$

$\{6, 7, 8, \ldots\}$ $\cancel{\{6 \in \mathbb{N}, 7 \in \mathbb{N}, 8 \in \mathbb{N}, \ldots\}}$

Ex: $\{ \underbrace{x + y} \mid \underline{x > 0 \text{ and } y < 12} \}$

## Clicker question

$x, y$ are integers.

Question: is $-3+4 \in \{x+y \mid x>0\}$?

correct

- A: yes $\sim 65\%$
- B: no $\sim 35\%$

$-3+4=1$

asking: $1 \in \{x+y \mid x>0\}$

yes $1 = 1+0 \in \{x+y \mid x>0\}$

$x=4$, $y=-3$.

in fact $\{x+y \mid x>0\}$ is equal to the set of all integers.

<u>Defn</u>: The set $\mathbb{Z} := \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ is the set of integers.

<u>Claim/ Exercise</u>: $\{x+y \mid x>0 \text{ and } x, y \in \mathbb{Z}\} = \mathbb{Z}$

## The empty set

**Defn:** $x \in S$ means $x$ is in $S$. $A \subseteq B$ means for every $x \in A$, $x \in B$.

**Defn:** The *empty set* (written $\emptyset$ or $\{\}$) is the set containing no elements. In other words, for all $x$, $x \notin \emptyset$.

let $x = \emptyset$, then $x \notin \emptyset$, so $\emptyset \notin \emptyset$.

Question: is $\emptyset \boxed{\in} \emptyset$?

A: yes B: no

$\Rightarrow$ $\emptyset$ is a thing, $\emptyset$ doesn't contain anything

$\emptyset \notin \emptyset$

Question: is $\emptyset \subseteq \emptyset$?

$\subseteq$ defined next time

everything not in $\emptyset$ is also not in $\emptyset$