CS 2800: Discrete structures

http://courses.cs.cornell.edu/cs2800/wiki

Announcements:

► Have your clickers ready
► No vertical screens please
► HW 1 out Monday, due Friday 2/7 at 5PM
► Office hours start Wednesday; calendar up soon
► Make sure you join Piazza
► Canvas up soon with links to everything
Set definition

**Defn:** A set $S$ is a collection of things. Every thing $x$ is either in $S$ (written $x \in S$) or not in $S$ (written $x \notin S$).

Examples:

- Let $P$ be the set of people in the room.
- Let $A := \{1, 2, 3\}$ 1 is $A$, 2 is $A$, 3 is $A$
  \[ \text{is defined as.} \quad \therefore \]
  \[ A = \{1, 2, 3\} \]

- Let $N := \{0, 1, 2, \ldots\}$ \[ \text{set of natural numbers.} \]
- Let $C := \{1, 3, 5, 7, \ldots, 23\}$
  \[ = \text{set of odds between } 1 \text{ and } 23. \]
Set definition

Defn: A set S is a collection of things. Everything x is either in S (written $x \in S$) or not in S (written $x \notin S$).


Questions:

1. Can it answer yet?

2. Does order matter? Yes, I have defined a set synonymously.

3. If you give a defn for your operation, terms not defined yet.
**Set equality**

**Defn:** A set $S$ is a collection of things. Every thing $x$ is either in $S$ (written $x \in S$) or not in $S$ (written $x \notin S$).

Questions:
- Do duplicates matter? $A: No$
- Does order matter? $A: No$

**Defn:** Sets $A$ and $B$ are equal (written $A = B$) if:
- Every $x \in A$ is also in $B$, and
- Every $x \in B$ is also in $A.$

**Question:** $\{1, 2\} \stackrel{?}{=} \{2, 1\}$

$A: Yes, equal$

$B: No, not equal$

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**Question:** $\{1, 2\} = \{2, 1\}$

$A: Yes, equal$

$B: No, not equal$
Set comprehension notation

\[ \{ p \mid p \text{ is a person in the room} \} \]

"the set of all " "such that"

\[ \{ x \mid x = y^2 \text{ for some } y \in \mathbb{N} \} \]

= \{ 0^2, 1^2, 2^2, \ldots \} = \{ 0, 1, 4, 9, 16, \ldots \} \]

Ex: \[ \{ x \in \mathbb{N} \mid x > 5 \} \]

= \{ 6, 7, 8, \ldots \} \]

Ex: \[ \{ x + y \mid x > 0 \text{ and } y < 12 \} \]
Clicker question: \( x, y \) are integers.

Question: is \( -3 + 4 \in \{x + y \mid x > 0\} \)?

- A: yes \(~65\%~\)
- B: no \(~35\%~\)

\[-3 + 4 = 1\]

asking: \( 1 \in \{x + y \mid x > 0\} \)

yes \( 1 + 0 = \{x + y \mid x > 0\} \)

\( x = 1, \ y = -3 \).

In fact \( \{x + y \mid x > 0\} \) is equal to the set of all integers.

Def: The set \( \mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\} \) is the set of integers.

Claim/Exercise: \( \{x + y \mid x > 0 \text{ and } xy \in \mathbb{Z}\} = \mathbb{Z} \)
The empty set

**Defn:** \( x \in S \) means \( x \) is in \( S \). \( A \subseteq B \) means for every \( x \in A \), \( x \in B \).

**Defn:** The *empty set* (written \( \emptyset \) or \( \{\} \)) is the set containing no elements. In other words, for all \( x, x \notin \emptyset \).

Question: is \( \emptyset \subseteq \emptyset \)?

\[ \text{A: yes  B: no} \]

\[ \Rightarrow \emptyset \text{ is a thing, } \emptyset \text{ doesn't contain anything} \]

\[ \emptyset \subseteq \emptyset \]

**Answer:** everything not in \( \emptyset \) is also not in \( \emptyset \)

Question: is \( \emptyset \subseteq \emptyset \)?

\[ \subseteq \text{ defined next time} \]