Last time:
- Claim: if $L$ is DFA-recognizable then $L$ is NFA-recognizable
  - Proof: draw DFA $M$, squint at it
- Claim: if $L$ is NFA-recognizable then $L$ is DFA-recognizable
  - Proof: build a DFA $M$ with states $Q_M := 2^{\hat{Q}_N}$
  - Subclaim: for all $x$, $\hat{\delta}_M(q_{0M}, x) = \hat{\delta}_N(q_{0N}, x)$.
- $\varepsilon$-NFA have $\varepsilon$-transitions that may be followed without using any characters
- Claim: if $L$ is $\varepsilon$-NFA recognizable then $L$ is NFA recognizable (proof idea today)

Announcements:
- Course evals due with homework (Sat. at noon); no late submission
Claim: if $L$ is $\varepsilon$-NFA recognizable then $L$ is NFA recognizable

- Proof idea: remove $\varepsilon$-transitions, replace “lost ability” with nondeterministic transitions

The set $RE$ of regular expressions is given by

$$r \in RE ::= a \mid \varepsilon \mid \emptyset \mid r_1r_2 \mid r_1 + r_2 \mid r^* \quad \text{a} \in \Sigma$$

$L : RE \to 2^{\Sigma^*}$ is given inductively by

- $\mathcal{L}(a) := \{a\}$
- $\mathcal{L}(\varepsilon) := \{\varepsilon\}$
- $\mathcal{L}(\emptyset) := \emptyset$
- $\mathcal{L}(r_1r_2) := \{xy \mid x \in \mathcal{L}(r_1) \text{ and } y \in \mathcal{L}(r_2)\}$
- $\mathcal{L}(r_1 + r_2) := \mathcal{L}(r_1) \cup \mathcal{L}(r_2)$
- $\mathcal{L}(r^*) := \{x_1x_2\cdots x_n \mid x_i \in \mathcal{L}(r)\}$

$x$ matches $r$ means $x \in \mathcal{L}(r)$

A language $L \subseteq \Sigma^*$ is regular if there exists $r \in RE$ with $L = \mathcal{L}(r)$

Claim: $L$ is regular if and only if $L$ is NFA-recognizable