Last time:

- Defn: a $T$-valued random variable $X$ is a function $X : S \rightarrow T$
  - we usually work with $\mathbb{R}$-valued random variables
- Defn: if $X$ is a random variable and $x \in T$ then $(X = x)$ is the event
  $$(X = x) := \{s \in S \mid X(s) = x\} \quad \text{sim.} \quad (X > x) := \{s \in S \mid X(s) > x\}, \ldots$$
- Defn: The probability mass function of $X$ is $\text{PMF}_X : T \rightarrow \mathbb{R}$ given by
  $$\text{PMF}_X(x) := \Pr(X = x)$$
- Defn: If $X$ and $Y$ are RVs then $X + Y$ is the RV given by
  $$(X + Y)(s) := X(s) + Y(s) \quad \text{sim.} \quad (XY)(s) := X(s)Y(s)$$

No announcements today
- If \( c \in \mathbb{R} \) then there is a corresponding RV \( C : S \to \mathbb{R} \) with \( C(s) := c \)
  - We'll usually use \( c \) for both \( c \) and \( C \)
- The joint PMF of \( X \) and \( Y \) gives \( \text{PMF}_{X,Y}(x,y) := Pr((X = x) \cap (Y = y)) \)
- Defn: the expected value (or expectation) of a random variable \( X \) is
  \[
  E(X) := \sum_{s \in S} X(s)Pr(\{s\})
  \]
- Alternate defn:
  \[
  E(X) := \sum_{x \in \mathbb{R}} xPr(X = x)
  \]
- Claim: \( E(X + Y) = E(X) + E(Y) \) \( \{\text{linearity of expectation}\} \)
- Claim: if \( c \in \mathbb{R} \) then \( E(cX) = cE(X) \)
- Warning! \( E(XY) \) may not be \( E(X)E(Y) \) (later: it will be if \( X, Y \) independent)
- Defn: If \( A \subseteq S \) is an event, then the indicator variable of \( A \) is \( I_A : S \to \mathbb{R} \) given by
  \[
  I_A(s) := \begin{cases} 
  1 & \text{if } s \in A \\
  0 & \text{otherwise}
  \end{cases}
  \]