Lecture 4: proof techniques

- List of all "kinds" of statements

- For each:
  - how to prove
  - how to use if known
  - how to disprove

- Applications:
  - in course: doing all the problems
  - elsewhere: automatic program checking
"Analytic approach": draw pictures, try to understand what's going on, work through specific examples.

- extremely helpful, but need to be careful that you are general enough
- going between analysis and symbol manipulation is incredibly useful.
**Terminology**

- A **proposition** is a (fully defined) statement that is either true or false.
  - "I am in the room"

\[ P \land Q \quad 3 < 5 \]

\[ x < 5 \text{ not a prop. (called a predicate)} \]

- A **predicate** is a start with one or more variables, truth depends on values of vars.
  - e.g. \( P(x) := "x < 5" \)
  - \[ P(7) = "7 < 5" \]

- If \( P \) and \( Q \) are propositions or predicates, \( s. \) are:

\[ P \land Q \text{ means } "P \text{ and } Q" \]
\[ P \lor Q \text{ } \text{ (or } \text{ both) } \]
\[ P \Rightarrow Q \text{ } \text{ "if } P \text{ then } Q" \]
\[ \neg P \text{ } \text{ "P is false" (sometimes say "not } P" )} \]
\[ \forall x \in A, P(x) \text{ } \text{ "for all } x \in A, P(x)" \]
\[ \exists x \in A, P(x) \text{ } \text{ "there exists some } x \in A, P(x)" \]
\[ \exists x \in A \text{ s.t. } P(x) \text{ } \text{ "there exists } x \text{ with } P(x)" \]
"if P then Q" says nothing about the case where P is false (i.e. "if P then Q" still true whenever P is false).

Specification: "if input is positive then output is positive"

```python
def f(x):
    return x
```

```
f(-5) → -5
```

correct, even though f(-5) ≠ 0

```python
def g(x):
    return 3
```

```
g(-5)
```

correct, even though x ≠ 5

"P implies Q" is confusing; not necessarily a causal relationship.
Prove it's raining and I'm holding a pencil.

- need to check both things.

Prove it's raining or I'm holding a pencil.

- don't need to run outside!

holding a pencil is enough.

Prove if P then Q:

```python
def f(x): return x
```

WTS if input positive then output is pos.

- may use the fact that \( x > 0 \),
  since nothing to prove if \( x \leq 0 \).

To use if P then Q:

if I knew if \( x \) was pos.

then \( h(x) \) also pos.

Can prove \( y > 0 \), can conclude \( x > 0 \).