A deterministic finite automata $M$ is a 5-tuple $M = (Q, \Sigma, \delta, q_0, A)$ where
- $Q$ is a finite set of states
- $\Sigma$ is an alphabet
- $\delta : Q \times \Sigma \rightarrow Q$ is the transition function
- $q_0 \in Q$ is the start state
- $A \subseteq Q$ is the set of accepting states

The extended transition function $\hat{\delta} : Q \times \Sigma^* \rightarrow Q$ gives the ending state after processing an entire string starting in the given state
- $\hat{\delta}(q, \varepsilon) := q$ and $\hat{\delta}(q, xa) := \delta(\hat{\delta}(q, x), a)$
- $M$ accepts $x$ if $\hat{\delta}(q_0, x) \in A$
- The language of $M$ (written $L(M)$) is given by $L(M) := \{ x \mid \hat{\delta}(q_0, x) \in A \}$

Announcements:
- Course grade estimates out
- Homework 6 out today; due next week
- Homework 7 due before break; slip day allows submission after break
\[\begin{align*}
\hat{\delta}(q, \varepsilon) &:= q \quad \text{and} \quad \hat{\delta}(q, xa) := \delta(\hat{\delta}(q, x), a) \\
L(M) &:= \{x \mid \hat{\delta}(q_0, x) \in A\}
\end{align*}\]

Let \(M := \begin{array}{ccc}
& 0 & 1 \\
0 & q_0 & q_1 \\
1 & q_1 & q_0
\end{array}\)  
Let \(L := \{x \mid x \text{ has an even number of } 1\text{'s}\}\)

Claim: \(L(M) = L\).

- Need to prove \(L(M) \subseteq L\) and (later) \(L \subseteq L(M)\)
- Let \(P(x)\) be “if \(\hat{\delta}(q_0, x) = q_0\) then \(x\) has an even number of 1’s” : FAIL
- Need a specification for every state!
- Let \(P(x)\) be “if \(\hat{\delta}(q_0, x) = q_0\) then \(x\) has an even number of 1’s
  AND if \(\hat{\delta}(q_0, x) = q_1\) then \(x\) has an odd number of 1’s.”

Defn: A language \(L\) is **DFA-recognizable** if \(\exists\) a DFA \(M\) with \(L = L(M)\)

Claim: If \(L_1\) and \(L_2\) are DFA-recognizable then so is \(L_1 \cap L_2\)

- Convention: \(L_1\) recognized by \(M_1 := (Q_1, \Sigma, \delta_1, q_{01}, A_1)\) with ext. trans. fn. \(\hat{\delta}_1\), etc.
- Idea: \(M\) should simulate both \(M_1\) and \(M_2\)