

Proofs are written using a small set of basic building blocks. **All** proofs should be completed using the following proof techniques¹. This is the **most** important material to understand in CS 2800: it is necessary for everything else we will do. Luckily, you'll get lots of practice this semester.

Each sentence in a proof should be:

- Clear (all terms have definitions)
- True
- Justified (using proof techniques listed below)

At each point in a proof, there is

- a **goal** proposition (what you're trying to prove)
- and some propositions that you **know** (assumptions, definitions, or things already proven)

Never confuse them! If you make a statement, you are telling the reader that it is something you have proved, so you better have already proved it! Say "we want to show..." to reiterate the goal without claiming you've proven it.

Valid proof steps:

- **Use** something you know.
- **Reduce** your goal to a simpler goal (the "to prove it" column below)

The following table shows how to do this for each kind of proposition:

Proposition	To prove it	To use it	To disprove it, prove
P and Q	Prove both P and Q	Use either P or Q	P is false or Q is false
P or Q	Prove P . Alternatively, prove Q	To prove R , prove R in the P and Q cases (case analysis)	P is false and Q is false
for all $x \in A$, $P(x)$	Prove $P(y)$ for an arbitrary $y \in A$	Conclude P for any specific $y \in A$	There is some $x \in A$ for which P is false (counterexample)
if P then Q	Assume P and prove Q	If you know P , you can conclude Q	P is true but Q is false
there exists an $x \in A$ such that P	Give a specific $y \in A$ and prove $P(y)$	Use an arbitrary $y \in A$ that satisfies $P(y)$	for any $x \in A$, P is false
P is false	Prove the logical negation of P (also, you always know P is either true or false)	If you can also prove P , you can conclude R (contradiction)	P

¹Mostly: we'll add a few more techniques during the semester.