Lecture 3: Set constructions and relationships

Today:

- Set comparison operations (subset and equals)
- Set constructions (union, intersection, difference, power set)
- Reasoning about sets (a first proof or two)

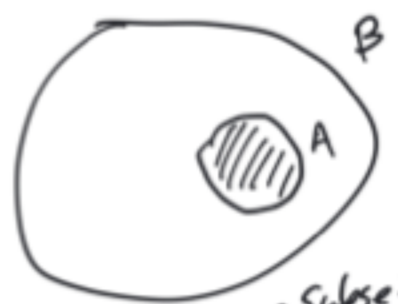
Announcements:

- HW1 out; start soon; procedure

Subsets

only makes [illegible] if A is a set.

Defn: A is a subset of B (written $A \subseteq B$) if, for all $x \in A$, x is also in B



Venn diagrams

$\{1,2\} \in \{1,2,\{1,2\}\}$

$\{1,2\} \subseteq \{1,2,\{1,2\}\}$

subset

$\{1,2,3\} \subseteq \mathbb{N}$

$\{1,2,3\} \notin \mathbb{N}$

element of

Q: $\underbrace{\{1,2\}}_{A} \overset{?}{\nsubseteq} \underbrace{\{\{1,2\},\{3,4\}\}}_{B}$ A: LHS $\nsubseteq$ RHS

Q: $2 \subseteq \{1,2\}$ A: No

Q: $\{\{1,2\}\} \subseteq \{\{1,2\},\{3,4\}\}$ A: Yes

Q: $\{1,2,3\} \subseteq \{1,2,3\}$ A: yes

Q: is $\emptyset$ a subset of every set

$\emptyset \subseteq \{1,2,3\}$ yes.

"For all $x \in \emptyset$, something" always true. (vacuous truth)

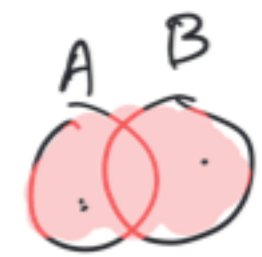
Q: is $\emptyset \in \{1,2,3\}$

Union, intersection, difference

<u>union</u>: A union B (written $A \cup B$) is given by

$$A \cup B := \{x \mid x \in A \text{ or } x \in B\}$$

(or both)



A union B

$A \cup B$

<u>intersection</u>: A intersect B (written $A \cap B$) is given by

$$A \cap B := \{x \mid x \in A \text{ and } x \in B\}$$

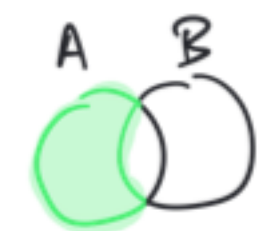
A intersect B

$A \cap B$

<u>Set difference</u>:

same as "and"

$$A \setminus B := \{x \mid x \in A \text{ but } x \notin B\}$$



A minus B

$A \setminus B$

(avoid) <u>complement</u> $\quad A^c \quad \bar{A}$

"universe" $\setminus$ A

Power set

The power set of A is the set of all subsets of A.

power set of $\{1,2\}$:

$$\{\{1\}, \emptyset, \{1,2\}, \{2\}\}$$

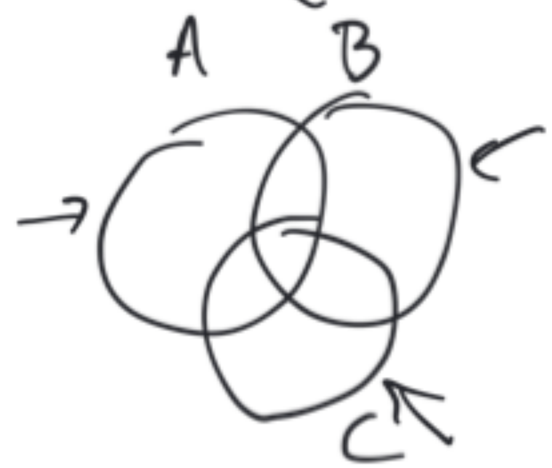
notation: the power set of A is written 2^A (other people: $\mathrm{pow}(A)$)

A first proof

For all A, B, C

Claim: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Proof: (version 1)



$B \cap C$

$A \cup B$

$A \cup C$

$A \cup (B \cap C)$

$= LHS$

same!

$(A \cup B) \cap (A \cup C)$

irst proof

Claim: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Proof: (version 2)

$x \in A$ or ($x \in B$ and $x \in C$).

$$A \cup (B \cap C) = \{x \mid x \text{ in } A \text{ or } (B \text{ and } C)\}$$

$$= \{x \mid x \text{ in } (A \text{ or } B) \text{ and in } (A \text{ or } C)\}$$

$$= \{x \mid x \text{ in } (A \cup B) \text{ and } x \text{ in } (A \cup C)\}$$

$$= (A \cup B) \cap (A \cup C)$$

- doesn't generalize
- uses facts that require proofs.

A first proof

Claim: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Proof: (version 3)

Reminders:

- **Defn:** $A \cup B := \{x \mid x \in A \text{ **or** } x \in B\}$
- **Defn:** $A \cap B := \{x \mid x \in A \text{ **and** } x \in B\}$
- **Defn:** $A \subseteq B$ if, for all $x \in A$, x is in B
- **Defn:** $A = B$ if $A \subseteq B$ and $B \subseteq A$

WTS for all $x \in LHS$, $x \in RHS$ and for all $x \in RHS$, $x \in LHS$.

$(LHS \subseteq RHS)$ $(RHS \subseteq LHS)$

- $(LHS \subseteq RHS)$

- $(RHS \subseteq LHS)$