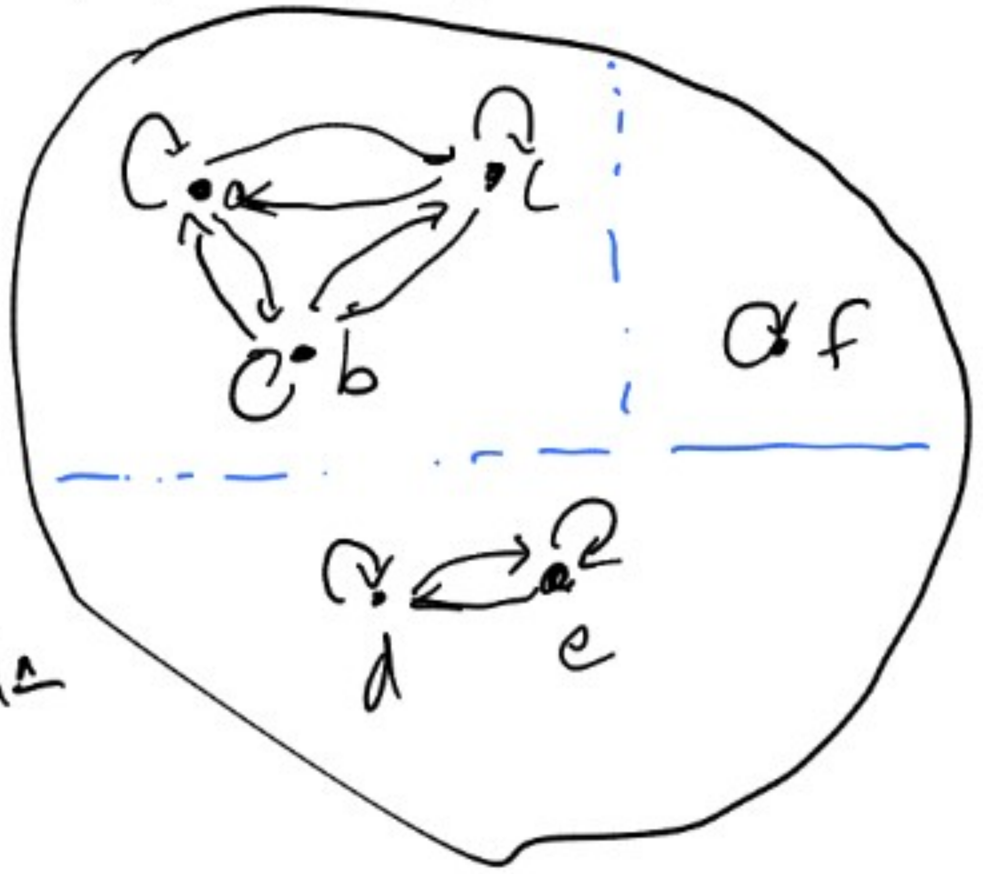
Lecture 11 : Equivalence classes

- Def$^{\underline{n}}$s : $[a]_R$, A/R, representative
- A/R partitions A
- Defining f$^{\underline{n}}$s $f : A/R \to X$

$A := \{a, b, \ldots, f\}$

R $\rightarrow$ is a eqv. reln



$[a] = \{a, b, c\} = [b] = [c]$

$[d] = \{d, e\} = [e]$

$[f] = \{f\}$

In general, aRb is the same as $[a] = [b]$

(pf next slide!)

$A/R = \{[a], [e], [f]\}$

<u>Defn</u>: if $a \in A$ and R is an <u>eqv relation</u> on A, the <u>equivalence class</u> of a (by R) is the set of all elts $b \in A$ with aRb. It is written $[a]_R$.

sometimes leave off if clear

i.e. $[a]_R := \{b \in A \mid aRb\}$

<u>Defn</u>: A/R "A mod R" is the set of all equivalence classes of A by R.

i.e. $A/R := \{[a]_R \mid a \in A\}$

<u>Term</u>: if $a \in A$, $c \in A/R$, we say a is a <u>representative</u> of c if $a \in c$.

Def<u>n</u>: Sets $A_1, A_2, \dots, A_n$ partition a set B

if ① $B = A_1 \cup A_2 \cup \dots \cup A_n$, and

② $A_i \cap A_j = \emptyset$ for all $i \neq j$.

contrapositive

Claim: A/R partitions A

Pf: ① WTS $A = [a_1] \cup [a_2] \cup \dots$, i.e. WTS $\forall a \in A, \exists c \in A/R$ such that $a \in c$. (note: this would show $A \subseteq \bigcup A/R$, other dir is obvious). Choose arb. $a \in A$, let $c = [a]$. Then $a \in c$ because aRa. (true b/c R is reflexive).

② WTS if $[a] \cap [b] \neq \emptyset$ then $[a] = [b]$.

Assume $[a] \cap [b] \neq \emptyset$. So $\exists c \in [a] \cap [b]$.

We WTS $[a] = [b]$, i.e. ① $[a] \subseteq [b]$ & $[b] \subseteq [a]$.

Choose arb $d \in [a]$, WTS $d \in [b]$.

know aRc since $c \in [a]$.

also bRc since $c \in [b]$.

also aRd since $d \in [a]$.

cRa by symmetry

bRa by trans. (since bRc & cRa)

bRd by trans (since bRa & aRd)

therefore $d \in [b]$.

Similarly, $[b] \subseteq [a]$.

Note: really ~~two~~ intermingled proofs.

① if $[a] \cap [b] \neq \emptyset$ then aRb

② if aRb then $[a] = [b]$

Often want to define $\underline{f}^{n}$s $f: A/R \to X$ for some set X. Tempting to note that every $c \in A/R$ is $[a]$ for some $a \in A$, we might write

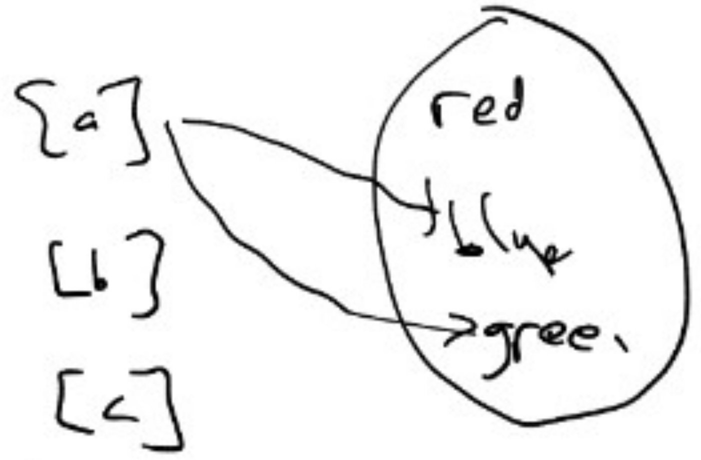
$$f: A/R \to X \quad \text{be given by } f([a]) := \dots a \dots$$

Ex.

Let f: Families $\to$ Colors be give by

$f([a]) :=$ a's eye color.

Ambiguous! $[a]=[b]$. But maybe a has blue eyes & b has green eyes.

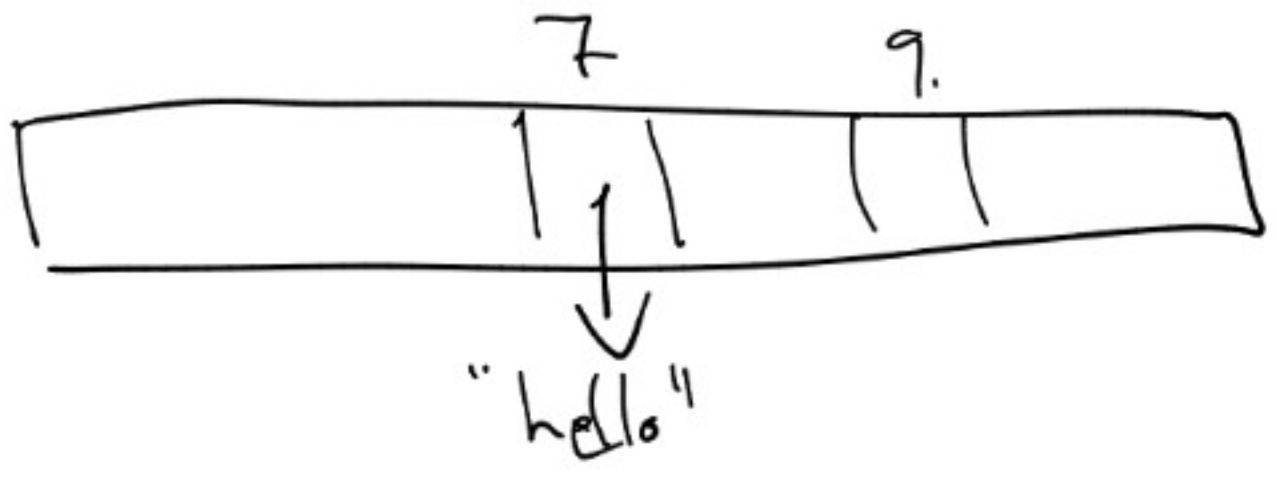


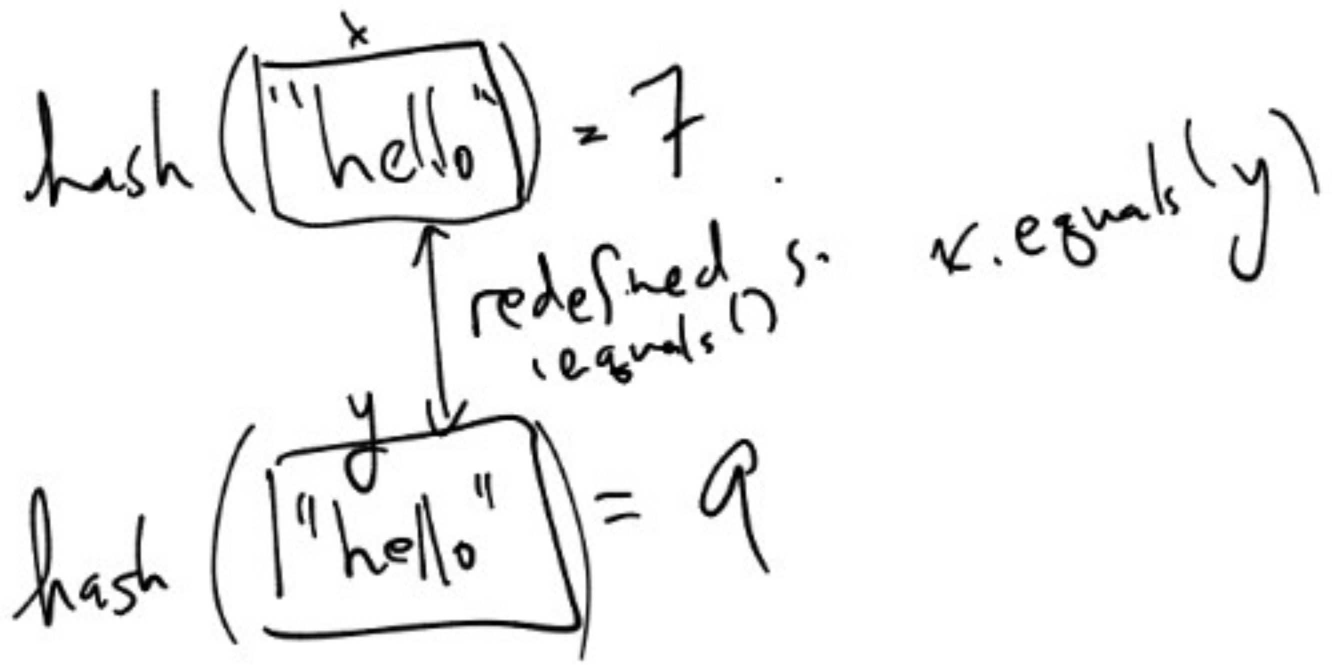
In general, check that if $[a]=[b]$ then $f([a])=f([b])$ before we call f a function.

In Java, can redefine ".equals(...)"

<u>doc</u>: if you redefine .equals(), must redefine hashCode(...).

object $\xrightarrow{\text{hash}}$ #





Want hashCode to be <u>well-defined</u> on equivalence classe of Objects.

⇒ ought to relefine (at least check) that all functions are still functions.