Deterministic Finite Automata (DFA)

Formally, a DFA $M$ has:

- A (finite) set of states $Q$. 
  $Q = \{ q_0, q_1, q_2 \}$ 

- An alphabet $\Sigma$. 

- A transition function $\delta : Q \times \Sigma \rightarrow Q$ 
  Ex: $\delta(q_0, a) = q_1$ 
  $\delta(q_0, b) = q_0$ 

- A start state $q_0 \in Q$ 
  Ex: $q_0 = q_0$ 

- A set of accepting states $A \subseteq Q$ 
  Ex: $A = \{ q_1, q_2 \}$

$E = \{0, 1\}$

$M = (Q, \Sigma, \delta, q_0, A)$

Input $x = 100110$.

$(start) q_0 \xrightarrow{(1)} q_1 \xrightarrow{(0)} q_0 \xrightarrow{(1)} q_1 \xrightarrow{(1)} q_0 \xrightarrow{(1)} q_1 \xrightarrow{(0)} q_0 \xrightarrow{(1)} q_0$ 

Does $x$ accept?

$x = 100110$

A: No, we end in $q_0$, not an accept state.

Q: What strings does $M$ accept?

A: Strings with an odd #s.
Exercise: Build an DFA $M$ that recognizes the language $L = \{ x \mid x \text{ does not contain } 010 \text{ as a substring} \}$

$L = \{ 0, \epsilon, 11, 01, 001, 010, \ldots \}$

$Q = \{ q_0, q_1, q_2, \text{reject} \}$

$\Sigma = \{ 0, 1 \}$

$\delta : (q_i, a) \rightarrow q_j$

$q_0 : q_0$

$A : \{ q_0, q_1, q_2 \}$

$\delta(q_0, a) = \begin{cases} q_1 & a = 1 \\ q_2 & a = 0 \\ \text{reject} & \text{otherwise} \end{cases}$

$q_1 : \begin{cases} q_1 & a = 1 \\ \text{reject} & a = 0 \end{cases}$

$q_2 : \begin{cases} q_1 & a = 0 \\ q_2 & a = 1 \\ \text{reject} & a = \text{otherwise} \end{cases}$

$q_0 : x \text{ doesn't end with } 0 \text{ or } 01$

$q_1 : x \text{ ends with } 0$

$q_2 : x \text{ ends with } 01$

$\text{Reject: } x \text{ does contain } 010$
Formal definitions

**Defn**: A deterministic finite automaton (or DFA) $M$ is a 5-tuple $M = (Q, \Sigma, \delta, q_0, A)$ where:

- $Q$ is a finite set (elements $q \in Q$ are called states)
- $\Sigma$ is a finite set (elements $a \in \Sigma$ are called characters)
- $\delta : Q \times \Sigma \rightarrow Q$ is called the transition function
- $q_0 \in Q$ is called the start state
- $A \subseteq Q$ is the set of accepting states

**Defn**: $\delta$ tells us where $M$ ends up after processing a string.

$$\hat{\delta} : Q \times \Sigma^* \rightarrow Q$$

is given by

$$\hat{\delta}(q, \epsilon) := q.$$  

$$\hat{\delta}(q, ax) := \hat{\delta}(\hat{\delta}(q, x), a) \text{ (using } \hat{\delta}(q, x))$$

$\hat{\delta}$ called "extended transition function".

**Defn**: We say $M$ accepts $x$ if

$$\hat{\delta}(q_0, x) \in A.$$

**Defn**: The language of $M$ (written $L(M)$) is

$$L(M) = \{ x \in \Sigma^* | M \text{ accepts } x \} = \{ x \in \Sigma^* | \hat{\delta}(q_0, x) \in A \}.$$

**Defn**: A language is a subset of $\Sigma^*$.

**Defn**: If $L$ is a language, and $L = L(M)$, then we say $M$ "recognizes" $L$.

**Defn**: If $\exists M$ with $L(M) = L$, we say $L$ is DFA-recognizable.