Last time:

- Defn: a $T$-valued random variable $X$ is a function $X : S \to T$
  - we usually work with $\mathbb{R}$-valued random variables $X : S \to \mathbb{R}$
- Defn: if $x \in T$ then $(X = x)$ is $(X = x) := \{ s \in S \mid X(s) = x \}$
- Defn/claim: $E(X) := \sum_{s \in S} X(s) \cdot Pr(\{s\}) = \sum_{x \in \mathbb{R}} x \cdot Pr(X = x)$
- Claim (linearity of $E$): $E(X + Y) = E(X) + E(Y)$ and $E(cX) = cE(X)$
- Defn: $X$ and $Y$ are independent if for all $x, y$, the events $(X = x)$ and $(Y = y)$ are independent (i.e. $Pr((X = x) \cap (Y = y)) = Pr(X = x) Pr(Y = y)$)

No announcements today
Defn/claim: $E(X) := \sum_{s \in S} X(s) \cdot Pr\{s\} = \sum_{x \in \mathbb{R}} x \cdot Pr(X = x)$

Claim (linearity of $E$): $E(X + Y) = E(X) + E(Y)$ and $E(cX) = cE(X)$

If $c \in \mathbb{R}$ then there is a corresponding RV $C : S \to \mathbb{R}$ with $C(s) := c$
  
  We’ll usually use $c$ for both $c$ and $C$

Claim: if $c$ is a constant RV (i.e. $c \in \mathbb{R}$) then $E(c) = c$
  
  In particular, $E(E(X)) = E(X)$ since $E(X) \in \mathbb{R}$

Defn: $X$ and $Y$ are independent if for all $x, y$, the events $(X = x)$ and $(Y = y)$ are independent (i.e. $Pr((X = x) \cap (Y = y)) = Pr(X = x) Pr(Y = y)$)

Claim: if $X$ and $Y$ are independent, then $E(XY) = E(X)E(Y)$

Defn: The variance of $X$ is $Var(X) := E((X - E(X))^2)$

Claim: $Var(X) = E(X^2) - E(X)^2$