Claim: \( A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C) \)

Proof: Choose arbitrary sets \( A, B, \) and \( C \). Let \( LHS := A \cup (B \cap C) \), and let \( RHS := (A \cup B) \cap (A \cup C) \).

We want to show that every \( x \in LHS \) is also in \( RHS \).

Choose an arbitrary \( x \in LHS \). Then either \( x \in A \) or \( x \in (B \cap C) \).

In the former case (when \( x \in A \)), we have \( x \in A \cup B \) by definition of union, and similarly \( x \in A \cup C \). Thus \( x \in RHS \).

In the latter case (when \( x \in (B \cap C) \)), we have \( x \in B \) and \( x \in C \). We can therefore conclude that \( x \in A \cup B \) and \( x \in A \cup C \), so \( x \in RHS \), as required.
A proposition is a (fully defined) statement that is either true or false
- E.g: it is raining outside, $3 < 7$, you will get an A in the class, $0 \in \mathbb{N}$

A predicate is a proposition with variables
- its truth depends on the value of the variables
- E.g: $P(x) := "x < 7"$ or $Q(x, y) := "x will get a y in the class"

If $P$ and $Q$ are propositions (or predicates) then so are:
- "$P$ and $Q$" (written $P \land Q$)
- "$P$ or $Q$" (written $P \lor Q$)
  - implicitly: “or both”
- "if $P$ then $Q$" (written $P \implies Q$)
  - note that this is automatically true if $P$ is false
  - there is not necessarily any other relationship between $P$ and $Q$
- "$P$ is false" (or “not $P$, written $\neg P$)
- “for all $x \in A$, $P(x)$” (written $\forall x \in A, P(x)$)
  - You can think of this as “$\forall x$, if $x \in A$ then $P(x)$”
- “there exists an $x \in A$ such that $P(x)$” (written $\exists x \in A, P(x)$)
  - “s.t.” is an abbrev. for “such that”
  - You can think of this as “$\exists x$ such that $x \in A$ and $P(x)$”

Prefer words to notation where possible
## Proof outlines / proof techniques

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<td>$P$ is false $(\neg P)$</td>
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