P2 review session: Automata

- Reg. expr -> DFA.
- ε-NFA -> NFA.
- NFA -> RE
- NFA -> DFA

- Prove automaton correct.

DFA that accepts X if the 10th to last symbol is 1.
Give a DFA that accepts $x$ if the 10th to last digit is 1.

need to know what the last 10 chars were.

have a state for each possible ending of last 10 chars.

name them $q_x$ if $x$ is a 10-digit string or smaller.

\[
\delta(q_0, x) = q_y \text{ then last 10 chars of } \\
\text{are } y. (\text{if }\delta(y) < 10 \text{ then) ?}
\]

\[
\delta(q_0, a) = \delta \text{ for 10 chars of } \\
n \text{ then}
\]

\[
A = \{ q_x | y \text{ has 9 digits} \}
\]

\[
\epsilon = \{0, 1\}
\]

\[
B_0 = q_0
\]

NFA: 10 trans.

\[
\begin{align*}
\delta(q_0, x) &= q_y \text{ then last 10 chars of } \\
\text{are } y. (\text{if }\delta(y) < 10 \text{ then) ?}
\end{align*}
\]

\[
\delta(q_0, a) = \delta \text{ for 10 chars of } \\
n \text{ then}
\]

\[
A = \{ q_x | y \text{ has 9 digits} \}
\]

\[
\epsilon = \{0, 1\}
\]

\[
B_0 = q_0
\]
1. Given DFA, give Q, E, S, q0, A,
and, for δ: Q × E → Q,

- δ accepts state q up:
- δ(Q, ε) = Q
- δ(q, a) = δ(δ(q, a), a)

2. check that if δ(q, x) = q then x has "right" property.
   - Call this specification P(x), prove inductively.

3. Relation 'specification' to L(M), by plugging in
   - M = L(M)
   - in general, L(M) = \{ x | δ(M, x) ∈ A \}

Ex1:

Ex2:

Ex3:
\[ \delta(2, 3, 9) = \delta_N(2, a) \cup \delta_N(3, a) \]

\[ \epsilon \epsilon^3 \cup \emptyset = \epsilon 3 \]

idea: If \( M \) on \( \alpha \) gets to \( q_i \), then \( N \) could have gotten to any of \( 2 - 8 \).

QM = 2^Q0

\( \text{can take \( \epsilon \)-trans (if present)} \)
1. Add new accept state, make old not accept.
2. One-by-one, remove non-start non-accept states, replace with RE trans.
3. Consolidate pairs of edges thru same states.

\[ \Gamma = \Gamma_1 \epsilon \Gamma_2 \]
\[ \text{ε-NFA → NFA} \]

\[ \text{idea: remove ε-trans, replace w/ non-det. trans} \]

\[ \text{make \& accept if it can reach an accept state using ε-trans} \]

\[ \text{remove ε-trans, replace w/ non-det trans.} \]

\[ \text{Diagram of ε-NFA to NFA conversion steps.} \]
Convert RE to e-NFA:

Ex. give an e-NFA \( N \) with \( L(\epsilon^* + a \epsilon a^*) \)

\[
\text{ReRE ::= a | \epsilon | \epsilon^*}
\]

\[
N_{a^*} \rightarrow a \rightarrow \epsilon
\]

\[
N_{b} \rightarrow \epsilon
\]

\[
N_{c} \rightarrow \epsilon
\]

\[
N_{a^* + a \epsilon a^*}
\]