Prelim 1 review #2:

- Diagonalization (other than sets)
  \[ \mathbb{N} \rightarrow \{0,1\} \] is unctbl

- How to debug proofs

- Functions that operate on \( f^n \) (ex: #2 on HW)
  \( \#3 \) on rec 2

- Negation of statements
  \( \#3 \) on sample
  \( 1(d) \) on rec 4

- Proving surjectivity
  (and injectivity)
  \( \#2 \) on rec 2

- Definitions using set notation
  \( \#2(b), \) rec 2
Claim: $\mathbb{N} \rightarrow \{0,1\}$ is uncountable.

Proof: Assume (for sake of contra.) that $\mathbb{N} \rightarrow \{0,1\}$ is countable. Then $\exists$ a surjection $f: \mathbb{N} \rightarrow \{0,1\}$.

In general, let $f_D: \mathbb{N} \rightarrow \{0,1\}$ be given by $f_D(n) = \begin{cases} 0 & \text{if } f(n)(n) = 1 \\ 1 & \text{if } f(n)(n) = 0 \end{cases}$. 

Then $f_D$ is not $f(n)$ for any $n$, because $f_D(n) \neq f(n)(n)$, so $f_D \neq f(n)$. This contradicts the fact that $f$ was surjective, so initial assumption (that $\mathbb{N} \rightarrow \{0,1\}$ is countable) must be false.
(a) Give an element of the set:
\[
\left\{ (x, y) \mid x \in \{2, 3, 5\}, y \in \{2, 3, 11\} \right\}
\]
\([x \rightarrow y]\) is the set of all \(f:\) s \(f:\) s \(x\) \(\rightarrow\) \(y\),
\(\text{dom} \ X \subseteq \text{cod} \ Y \).

\[
\begin{align*}
(x \rightarrow 1) & \quad \text{by } x \quad \rightarrow \quad 1 \\
(y \rightarrow 0) & \quad \text{by } y \quad \rightarrow \quad 0 \\
(3 \rightarrow 1) & \quad \text{by } 3 \quad \rightarrow \quad 1
\end{align*}
\]
\[
\left\{ (x, y) \mid x \in \{2, 3, 5\}, y \in \{2, 3, 11\} \right\} = [x \rightarrow y] = [A \rightarrow B]
\]

(b) \(A = \{x, y\}\), \(B = \{0, 1\}\)

give a element of \([CA \rightarrow B] \rightarrow (A \times B)\)
\([x \rightarrow y]\)

\[
\begin{align*}
(x, 0) & \quad \text{f} \\
(x, 1) & \quad \text{f} \\
y, 0 & \quad \text{f} \\
y, 1 & \quad \text{f}
\end{align*}
\]

What is \(F(x, 0)\)?

\[
\begin{align*}
[f(x) = 0] & = F(x, 0) \\
F(x, 0) & = f_x \\
f(x) & = 0 \\
f(y) & = 1
\end{align*}
\]

It is not enough to say \(F : (A \times B) \rightarrow [A \rightarrow B]\)

We given by \(F((x, y)) = f_x\) where \(f : A \rightarrow B\)

\(f_x = \begin{cases} 0 & \text{if } x \in A \\ 1 & \text{if } x \notin A \end{cases}\)
How to debug proofs:

1. Go line by line, ask: what is proof saying, is it checking the things it needs to?

2. Work through examples as you're reading a proof.

3. Work with a specific counterexample to the claim, see when the proof says something false.