2600 Prelim I Review session

- WS 2, Q3: function from $2^{X \times Y} \to 2^X \times 2^Y$. Is it inj./surjective?

- 9(c) rev.: $F: X \to [2 \to X]$ ... is it surj./inj.?

- 5 rev.: which are ctdbl/unctdbl?

- HW 20(a): countability of finite subsets argument.

- HW 20(b): give bij. $F: [X \times [Y \to 2]] \to [(X \times Y) \to 2]$. 2 Bézout coeffs.

- Every $n \in \mathbb{N}$ has prime factorization.

- 3 rev.: Negation of $\exists x, \forall y, ...$

- HW 3 #4(c): reachability rel equivalence classes.

- HW 3 #3(c): times not well-defined.

- HW 3 #4(d) well-defined w/ reachability Base b rep.
If $P(n)$ is true for all $k$ such that $1 \leq k \leq n-1$, and $n$ is not prime, then $P(n)$ is true for all $k$ such that $1 \leq k \leq n-1$.  

If $n$ is prime, then $P(n)$ is true for all $k$ such that $1 \leq k \leq n-1$.  

If $n$ is composite, then $n = kl$ for some $k,l \geq 2$.  

Since $k,l$ are both $< n$, we have $P(k)$ and $P(l)$, so  

If $k = p_1^{e_1}p_2^{e_2} \cdots p_m^{e_m}$ is all primes,  

If $l = p_1^{f_1}p_2^{f_2} \cdots p_m^{f_m}$ is all primes,  

Then $n = kl = p_1^{e_1 + f_1}p_2^{e_2 + f_2} \cdots p_m^{e_m + f_m}$ is all primes.
Proved that if \( n \), \( n \geq 2 \), \( \exists \) a base \( b \) representation of \( n \): 

write \( \frac{(17)}{10} \) in base \( \frac{3}{6} \):

key idea: last digit is \( \text{rem}(n, b) \)
other digits are digits of \( \text{quot}(n, b) \).

\[ d_0 = 2 = \text{rem}(17, 3) \]
rest of digits: digits of \( \text{quot}(17, 3) = 5 \)

write 5 in base 3:

\[ d_0' = \text{rem}(5, 3) = 2 \]
rest: digits of \( \text{quot}(5, 3) = 1 \)
\[ d_1' = 1 \]

5 = (012)_3

17 = (0122)_3 = 1 \cdot 3^2 + 2 \cdot 3 + 2
9 + 16 + 1 = 17 ✓
$P(0)$: \text{wts } v = \exists s,t \text{ with } g(s,t) = 0$

well, $g(0,0) = \alpha$ by def. of $g$.

Let $s = 1$ and $t = 0$

then $g(0,0) = \alpha = \frac{1}{\varepsilon} \alpha + 0$

$P(b)$, assuming $P(b-1), P(b-2), \ldots, P(0)$

$P(b)$ says $\forall \gamma \in \mathbb{S}, t \text{ with } g(\gamma, t) = sa + tb$.

Choose arb. $a$, let $s = t'$ and $t = \frac{s'-t'g}{\varepsilon}$

g(\gamma, t) = g(b, r).

where $r = \text{rem}(\gamma, b)$ so $a = \frac{a'}{b} + r$

Choose $\gamma' < \varepsilon$ for some $s', t'$, by $P(\gamma')$

Assumed because $r < b$.

to find $s, t$ for $a, b$

- Find $s', t'$ for $a, b, r$.

- Let $s = t'$

- Let $t = s' - t'g$

quotient of $a$ over $b$. 

\[ g(\gamma, t) = g(b, r) = \frac{s'}{b} + \frac{t'}{b} \cdot \frac{a'}{b} \text{ algebra} \]

\[ = sa + tb \]
\[ a \rightarrow b \rightarrow c \rightarrow d \]

\[ a \sim b \quad \text{and} \quad a \sim d \]

\[ v / ~ = \{ [a], [b], [c], [d] \} \]

\[ \gamma = \{ a, b, c, d \} \]

\[ \Gamma = \{ a, b, c \} \]

\[ \Gamma = \{ a, b, c \} \]

\[ (\{a\}, \{b\}) \in E' \quad \text{and} \quad (\{b\}, \{c\}) \in E' \]

\[ a \sim b \quad \text{and} \quad b \sim c \]

\[ x \in \{ y \in \mathbb{R} \mid x > 0 \} \]

\[ \text{and} \quad 2 = 2 + 0 \]
from A sample Qn:

Find logical negation of $\exists x (\forall y, \exists z, \forall P(x, y, z))$

$P \rightarrow \text{negation of } \exists x, P \rightarrow \forall x, \neg P$

$\neg \exists x, P \rightarrow \forall x, \neg P$

$\forall x, \forall y, \forall z, F(x, y, z)$

$Q$ is $\forall z, \forall F(x, y, z)$

the same as $\forall z, \forall F(x, y, z)$

$\forall x, \forall y, \forall z, F(x, y, z)$
**Df:** $X$ is countable if

- $\left| X \right| \leq \aleph_0$, or
- $\left| \mathbb{N} \right| = \aleph_0$.

**Q:** How many are countable / uncountable?

- $\mathcal{P}(\mathbb{N})$, set of strings of 0s and 1s
- $\mathbb{Q}$, set of rationals
- $\mathbb{R}$, set of reals

**Q:** Which of these are countable / uncountable?

- $\mathbb{N}$, set of naturals

**Q:** To find $f(n)$, traverse grid in this pattern.

- Example: $f(0) = (0,0)$,
- $f(4) = (2,0)$, ...

**f:** $\mathbb{N} \to \mathbb{N} \to \mathbb{R}$

- $f(0) = (0,0)$
- $f(1) = (0,1)$
- $f(2) = (1,0)$
- $f(3) = (1,1)$
- $f(4) = (0,2)$
- $f(5) = (1,2)$
- $f(6) = (2,0)$
- $f(7) = (2,1)$
- $f(8) = (0,0)$
- $f(9) = (0,1)$
- $f(10) = (1,0)$
- $f(11) = (1,1)$
- $f(12) = (0,2)$
- $f(13) = (1,2)$
- $f(14) = (2,0)$
- $f(15) = (2,1)$

- $2^{\mathbb{N}}$ to give 1st function

- $\mathbb{N} \times \mathbb{N}$ to give 2nd function

- $\mathcal{P}(\mathbb{N})$ to give 3rd function

- $\mathbb{R}$ to give 4th function

**Q:** $2^{\aleph_0}$ is uncountable?

- $2^{\aleph_0}$ > $\aleph_0$
\( X = \{1, 2, 3\} \), \( Y = \{2, 3, 3\} \)

(b): Write down 
\( (A, B) \) with \( 2 \in A \), \( 2 \notin B \).
\( (\{2\}, \emptyset) \).

\( 2 \notin Y \).
If \( \{2\} = S \cap X \), then \( 2 \in S \).
So \( 2 \in S \) since \( 2 \in Y \).

But \( 2 \notin \emptyset \).
So \( (\{2\}, \emptyset) \neq f(S) \) for any \( S \).

WTS \( f \) not surjective, i.e. \( \exists y \in 2^X \times 2^Y \) s.t. \( A \subseteq 2^X \), \( y \notin f(A) \).
Assumption: \( X \cup Y \neq \emptyset \), so \( \exists a \in X \cup Y \).