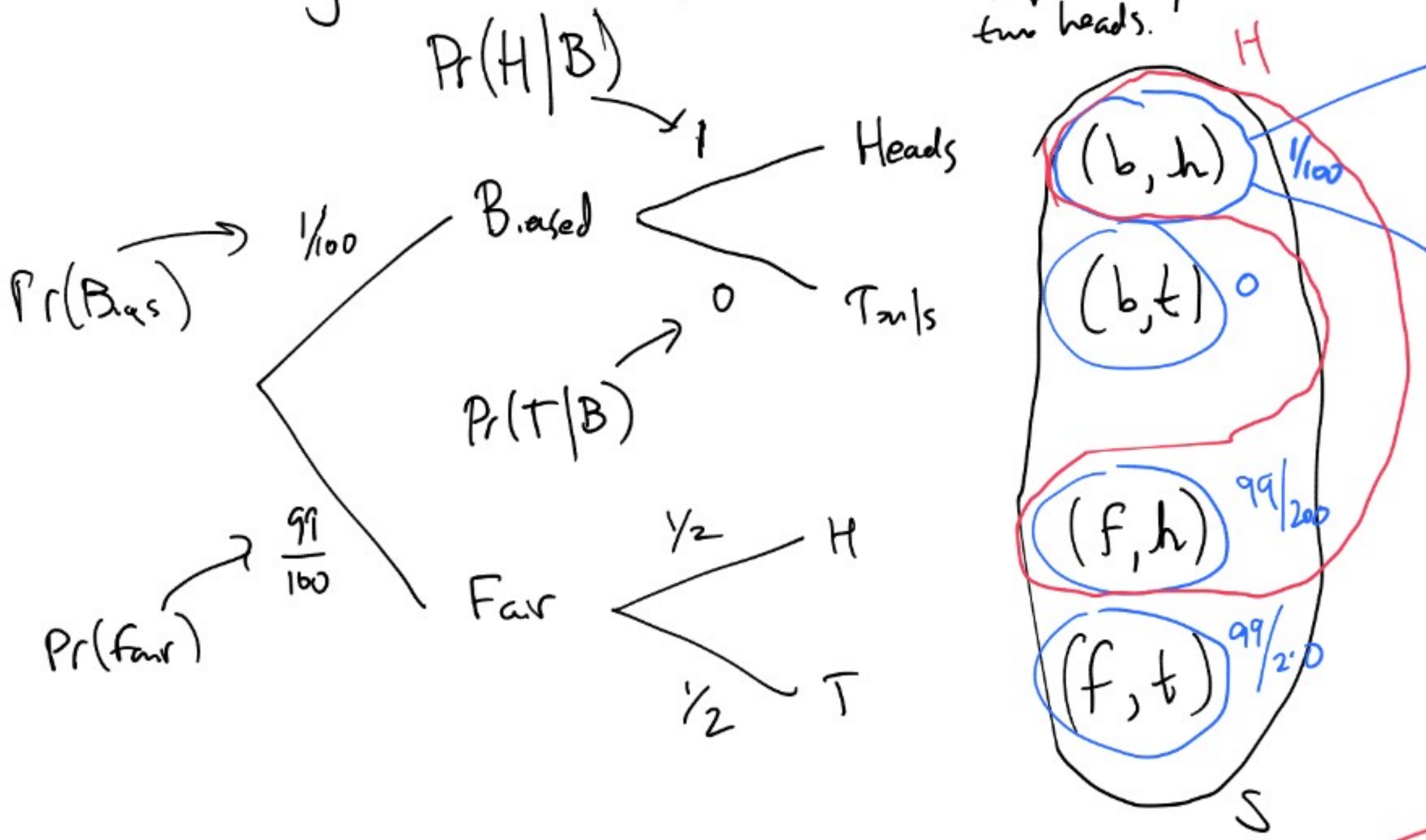
Lecture 33: random variables

- Conditional prob. coda:
  - Bayes' rule
  - Law of total prob.
- Random variables
  - Def$^{\underline{n}}$s, examples
  - Combining RVs
  - Converting RVs $\longleftrightarrow$ Events

Selecting a coin (99 fair, 1 $\underbrace{\text{biased}}_{\text{two heads.}}$), flip it



simple event
(one with only one outcome)

$$Pr(B \cap H) = Pr(H|B)\,Pr(B) = 1/100.$$

$$B := \{(b,h),(b,t)\}$$

$$H := \{(b,h),(f,h)\}$$

3rd axiom

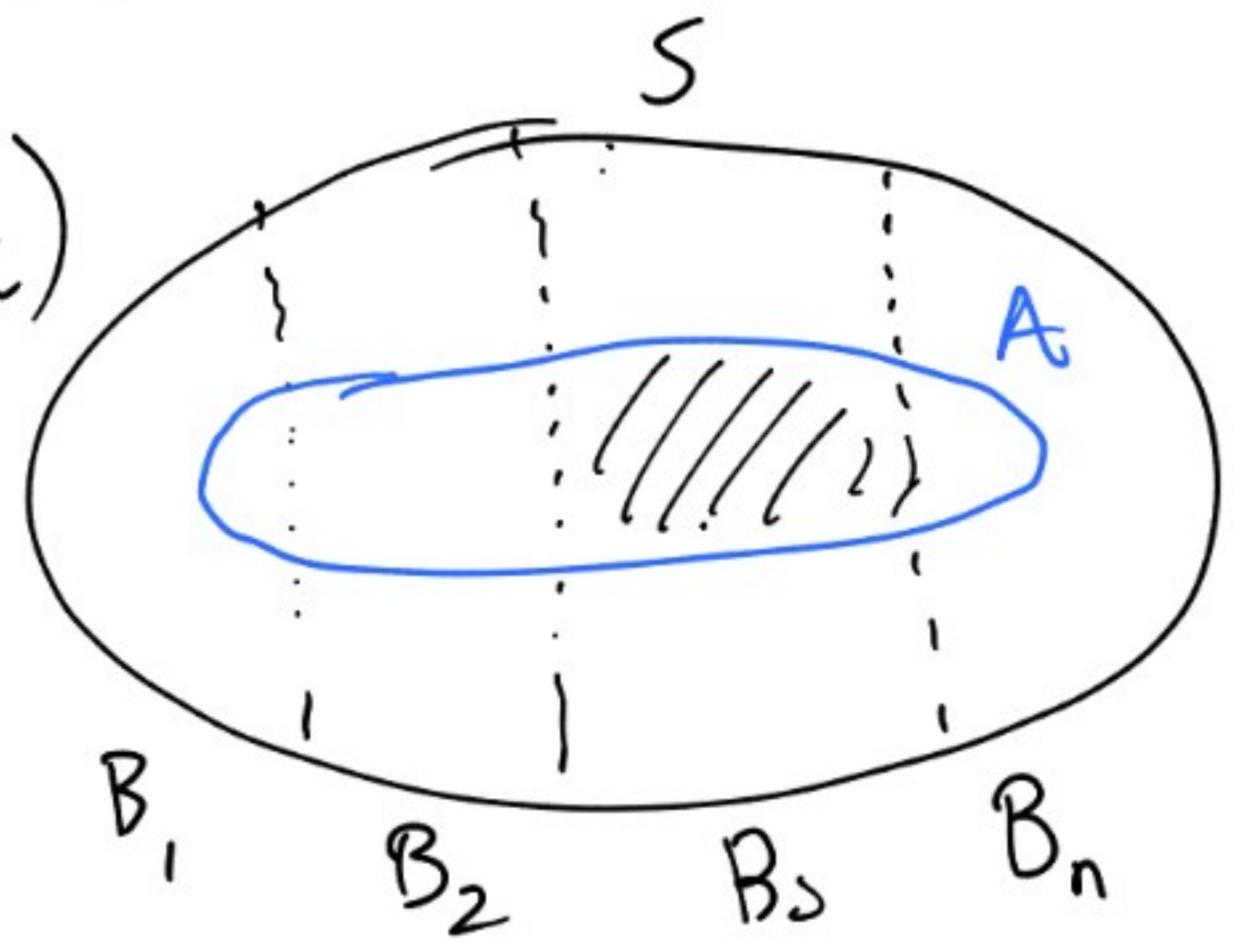
$$Pr(H) = Pr(\{(b,h)\}) + Pr(\{(f,h)\})$$

$$= \frac{101}{200}.$$

Law of total prob.

if $B_1, B_2, \ldots, B_n$ partition S.

then $Pr(A) = \sum Pr(A|B_i)Pr(B_i)$



Pf: $Pr(A|B_i)Pr(B_i) = Pr(A \cap B_i)$

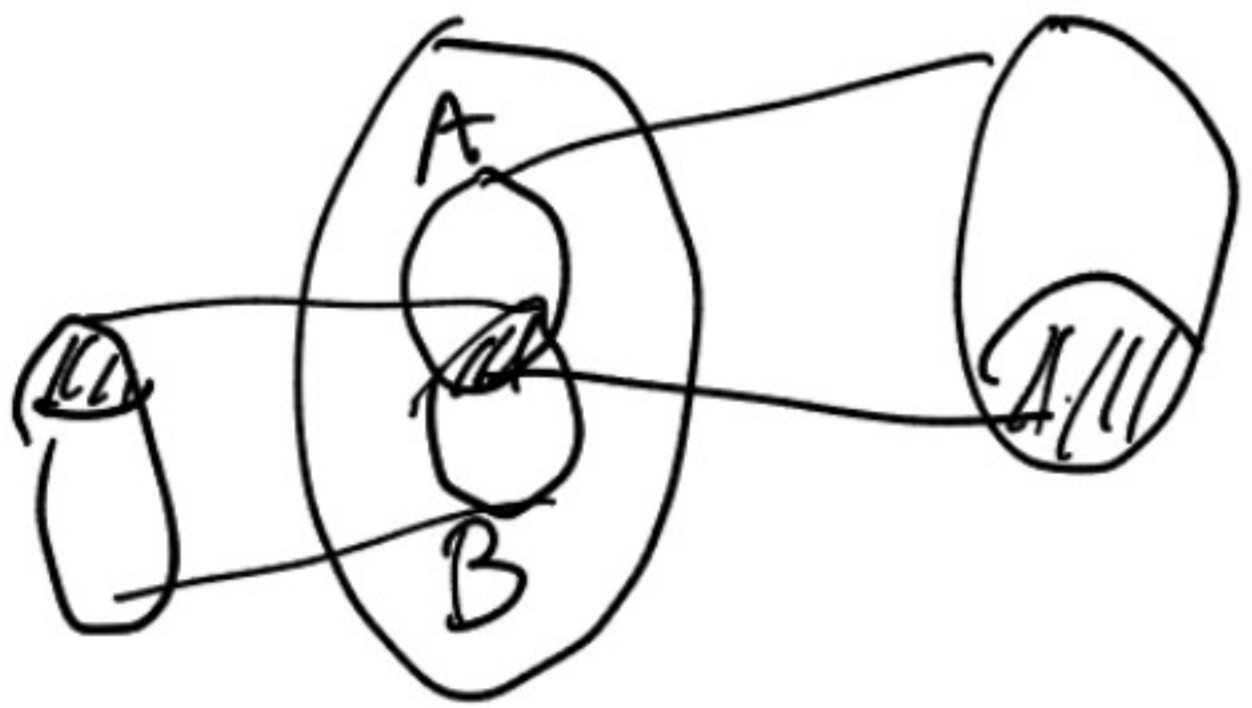
Using 3rd axiom

$$Pr(A) = Pr((A \cap B_1) \cup (A \cap B_2) \cup \ldots) \quad \text{(since } B_i \text{ partition } S\text{)}$$

$$= \sum Pr(A \cap B_i) \quad \text{since } B_i \text{ are disjoint, } A \cap B_i \text{ are disj.}$$

$$= \sum Pr(A|B_i)Pr(B_i) \quad \checkmark$$

<u>Bayes' rule</u>: $\Pr(B|A) = \dfrac{\Pr(A|B)\Pr(B)}{\Pr(A)}$.



<u>Pf</u>: $\Pr(B|A) = \dfrac{\Pr(B\cap A)}{\Pr(A)}$

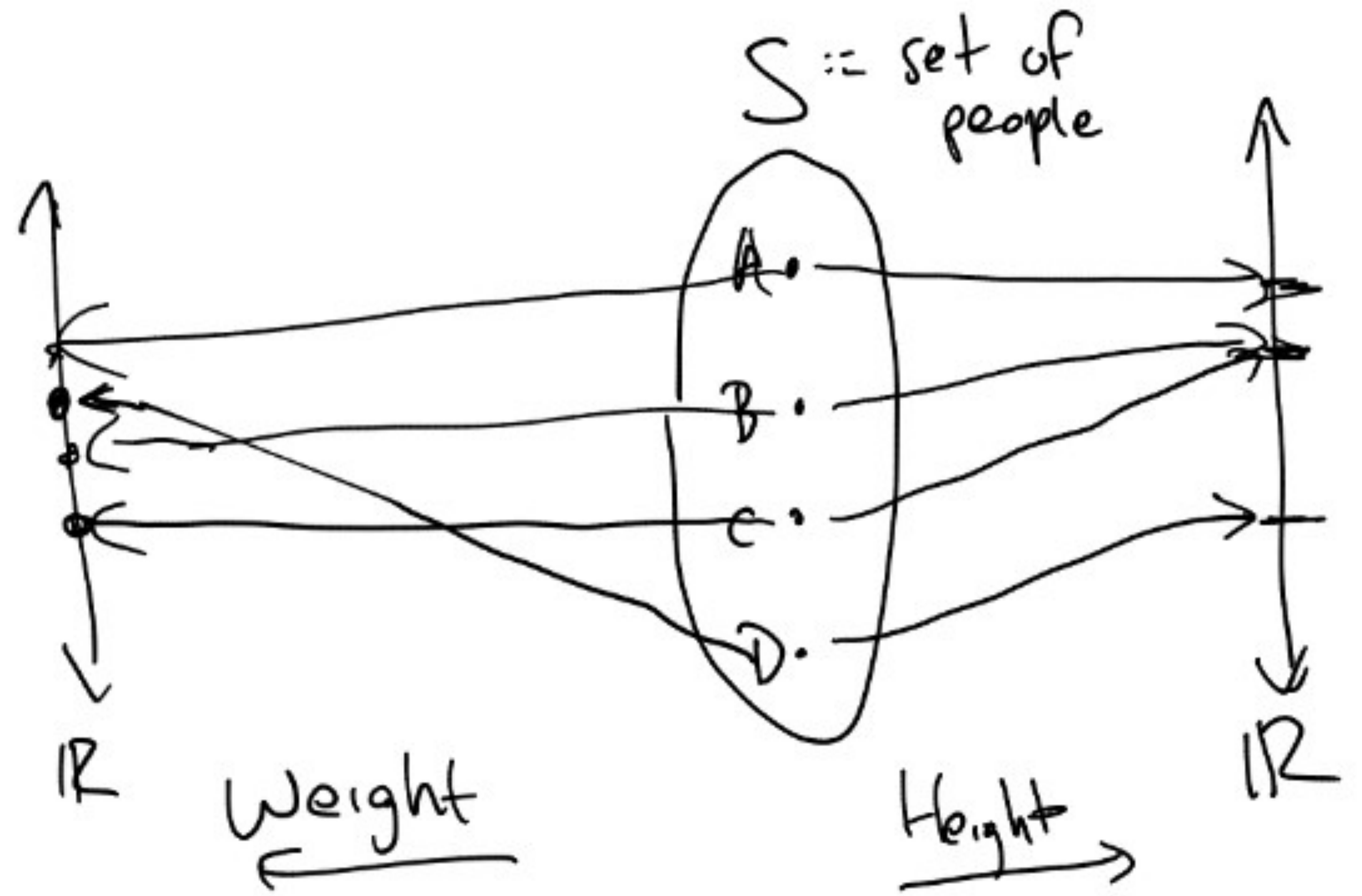
so $\Pr(A)\Pr(B|A) = \Pr(B\cap A)$.

sim. $\Pr(B)\Pr(A|B) = \Pr(B\cap A)$

So $\Pr(A)\Pr(B|A) = \Pr(B)\Pr(A|B)$

divide ✓

Experiment: - Select random person (uniformly)
- measure height.



Height is a random variable. ($\mathbb{R}$-valued)

Def$^{\underline{n}}$ An (X-valued) random variable is a function $V : S \to X$

By default, our random variables are $\mathbb{R}$-valued.

$$BMI := \text{Weight}/\text{Height}$$

we'll require X & Y have same sample space (i.e. domain).

<u>Def^n</u> if X & Y are (ℝ-valued) RV's then $X+Y$ is a <u>RV</u> defined by:

- a tuple? (height & width) (not f^n-s)
- given an outcome $s \in S$, $X(s)$ add to $Y(s)$.

$X+Y : S \to \mathbb{R}$ is given by

$$(X+Y)(s) := X(s) + Y(s)$$

similarly, all operations on RVs will be defined pointwise.

e.g. $\left(\sin(X)\right)(s) := \sin\left(X(s)\right)$.

If X, Y are RVs, then $(X=Y)$ is an event (i.e. a subset of S)

$\Pr(\underbrace{X=Y})$

Sim: $(X > Y) := \{ s \mid X(s) > Y(s) \}$

$\vdots$

given by: $(X = Y) := \{ s \in S \mid X(s) = Y(s) \}$

Ex: select 2 people ($S := \text{People} \times \text{People}$)

$H_1(p_1, p_2) := p_1$'s height

$H_2(p_1, p_2) := p_2$'s height

$(H_1 = H_2)$ is set of pairs of ppl with same height.

$\Pr(H_1 = H_2)$ is prob. 1st & 2nd person have same height.

If $c \in \mathbb{R}$, we can use c as a RV, by just taking the const. $f^{\underline{n}}$.

$c : S \to \mathbb{R}$ is given by

$c(s) := c$ e.g. $2(s) := 2$

c as a RV.

number c

$\Pr(\underline{\text{Height}} > \underline{5})$