Last time:

- A function $f$ from a set $A$ to a set $B$ is a rule that, for every input $x \in A$ gives an unambiguous output $f(x) \in B$
- Notation: $f : A \rightarrow B$ means “$f$ is a function from $A$ to $B$”
- To give a function, make sure domain, codomain, and rule are all clear
- Defn: Two functions $f$ and $g : A \rightarrow B$ are equal if, for all $x \in A$, $f(x) = g(x)$

Announcements:

- None today
To give a function, make sure domain, codomain, and rule are all clear.

**Defn:** Two functions $f$ and $g : A \to B$ are *equal* if, for all $x \in A$, $f(x) = g(x)$.

**Defn:** $f : A \to B$ is *injective* if $\forall x_1, x_2 \in A$, if $f(x_1) = f(x_2)$ then $x_1 = x_2$.

**Defn:** $f : A \to B$ is *surjective* if $\forall y \in B$, $\exists x \in A$, $f(x) = y$.

**Defn:** $f$ is bijective if it is both injective and surjective.

**Defn:** if $f : B \to C$ and $g : A \to B$ then $f \circ g : A \to C$ is given by $(f \circ g)(x) := f(g(x))$.

**Defn:** for any $A$, $id_A : A \to A$ is given by $id_A(x) := x$.

**Defn:** if $f \circ g = id$ then $f$ is a left-inverse of $g$ and $g$ is a right-inverse of $f$.

**Defn:** $g$ is a *two-sided inverse* of $f$ if it is both a left- and right-inverse of $f$.

we write $g = f^{-1}$; need to check that it is unique.

**Claim:** $f$ is injective if and only if it has a left-inverse.

**Claim:** $f$ is surjective if and only if it has a right-inverse.

**Claim:** $f$ is bijective if and only if it has a two-sided inverse.