Definition: An alphabet \( \Sigma \) is any finite set
- Elements of \( \Sigma \) are called characters
- Typical alphabets: \( \{0, 1\} \), \( \{a, b, c, \ldots, z\} \), Unicode

Strings: \( x \in \Sigma^* ::= \varepsilon \mid xa \quad a \in \Sigma \)
- \( \varepsilon \) is called the empty string
- We usually leave off the leading \( \varepsilon \); so "abc" stands for \( \varepsilon abc \)

Definition: A language \( L \) is a set of strings (i.e. \( L \subseteq \Sigma^* \))

To define \( f : X \rightarrow Y \) where \( X \) is inductively defined, define \( f(x) \) for \( xs \) formed using each rule; you may apply \( f \) to any substructure of \( x \)
- Strings: define \( f(\varepsilon) \) and \( f(xa) \) (in terms of \( f(x) \))
- Example: Let \( \text{prepend} : \Sigma \times \Sigma^* \rightarrow \Sigma^* \) be given by \( \text{prepend}(a, \varepsilon) := \varepsilon a \) and \( \text{prepend}(a, yb) := \text{prepend}(a, y)b \)
- Can't define \( \text{prepend}(a, yb) := ayb \) because \( ayb \) is not defined
- If \( x \) and \( y \) are strings, we'll avoid using \( xy \) just to stick with inductive structure
  - You can define concatenation inductively (exercise) let \( \text{cat} : \Sigma^* \times \Sigma^* \rightarrow \Sigma^* \) be \ldots
A deterministic finite automata $M$ is a 5-tuple $M = (Q, \Sigma, \delta, q_0, A)$ where

- $Q$ is a finite set of states
- $\Sigma$ is an alphabet
- $\delta : Q \times \Sigma \to Q$ is the transition function
- $q_0 \in Q$ is the start state
- $A \subseteq Q$ is the set of accepting states

Example: $Q := \{q_0, q_1\}$, $\Sigma = \{0, 1\}$, $q_0 := q_0$, $A := \{q_1\}$

The extended transition function $\hat{\delta} : Q \times \Sigma^* \to Q$ gives the ending state after processing an entire string starting in the given state. 

- $\hat{\delta}(q, \varepsilon) := q$ and $\hat{\delta}(q, xa) := \delta(\hat{\delta}(q, x), a)$
- $M$ accepts $x$ if $\hat{\delta}(q_0, x) \in A$
- The language of $M$ (written $L(M)$) is given by $L(M) := \{x \mid \hat{\delta}(q_0, x) \in A\}$