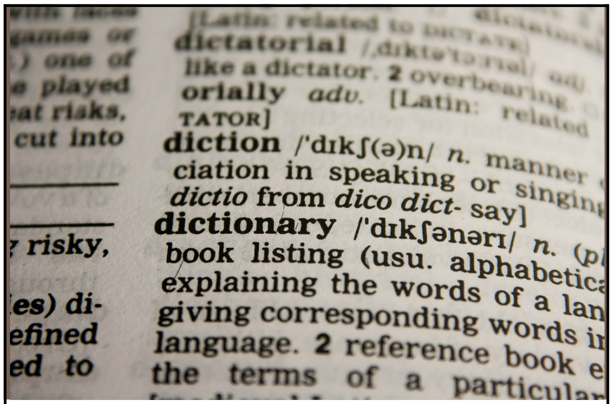

HASHING

CS2110

Announcements

2

- Submit Prelim 2 conflicts by Wednesday (tomorrow) night
- A6 is due April 18 (*Thursday!*)
- Prof Clarkson diagnosed with a concussion and is staying home this week. Don't send him email —he's supposed to stay away from his computer.

Material in for Hashing

3

- Tutorial on hashing:
  in lower navigation bar in JavaHyperText
- Entry hash in JavaHyperText
- Specific to Java. API documentation for:
  hashCode() and function equals(Object ob)
- Lecture notes page of course website.
  Demo code for hashing with chaining and hashing with open addressing

Ideal Data Structure

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Table gives expected times, not worst-case times

Data Structure	add(val x)	get(int i)	contains(val x)
ArrayList [2|1|3|0]	$O(n)$	$O(1)$	$O(n)$
LinkedList 2→1→3→0	$O(1)$	$O(n)$	$O(n)$
Goal:	$O(1)$	$O(1)$	$O(1)$

Also known as: add, lookup, search

New Data Structure : Hash Set

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Table gives expected times, not worst-case times

Data Structure	add(val x)	get(int i)	contains(val x)
ArrayList [2|1|3|0]	$O(n)$	$O(1)$	$O(n)$
LinkedList 2→1→3→0	$O(1)$	$O(n)$	$O(n)$
HashSet [3| |1|2] (0 1 2 3)	$O(1)$	$O(1)$	$O(1)$

Expected time
Worst-case: $O(n)$

AKA add, lookup, search

Notion of hashing

6

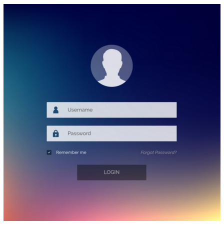
Hash: to chop to pieces; to make a confused muddle of; to jumble; to dice, chop, mince.

In computing: Produce a relatively small number or string from something a lot bigger, like a file, or an Java object.

Submitted	Date	By	Size	MD5 What's this?
A6GUI	April 10, 2018 04:28PM		10.82 kB	ca62dd8fc1273f51baa6f507efac1d2b

Look at CMS page for A2 submission. Md5 is a hash function. Given your A2.java file, it produces a 128-bit number from it. Sometimes called a checksum. Compare the Md5 number for your for your file to the MD5 number of the one that was uploaded. If different, uploading corrupted the file.

Application: Password Storage



- Hash functions are used to store passwords
- Could store plaintext passwords
  - Problem: Password files get stolen

h(password): h is the hash function.
It produces some jumbled version of the password.

Hashing history

We will use hashing —a hash function— to implement sets of values in a hash table.

1953. Hand Peter Luhn wrote an internal IBM memorandum that used hashing with chaining.

A few others did it roughly the same time.

Ershov (Russian) and Amdahl independently invented hashing with open addressing and linear probing.

Intuition behind a Hash Set

Idea: finding an element in an array takes constant time when you know which index it is.
So… let's place elements in the array based on their starting letter! (A=0, B=1, …)

add("CA") CA → # of 1st letter → 2

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	…	25
b			CA										MA	NY	OR	PA			

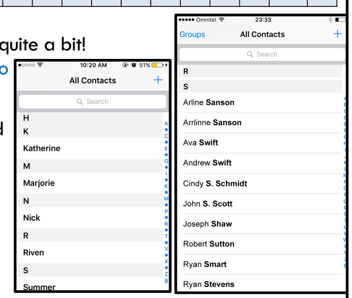
contains("DE") DE → # of 1st letter → 3

What could go wrong?

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	…	25
b	AL		CA	DE		FL	GA						MA	NY	OR	PA			

- Some buckets get used quite a bit!
  Connecticut, Colorado
  - called **Collisions**
- Not all buckets get used

bucket: one of the array elements



Hash Function

0	1	2	3	4	5	6	7	8	9
AL		CA	DE		FL	GA			

Given a value to be put into the table, a hash function returns an index where to put it.

E.g. hash function(stateName) could return value depending on first character:
0 for A, 1 for B, 2 for C, etc.

The hash function knows nothing about the table size.
Therefore, always take the hash-function values mod the table size in order to get an index into the table.

Example: hashCode("Oregon") mod 10 = 14 mod 10 = 4
So put "Oregon" in bucket 4.

Example: hashCode()

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- hashCode() defined in java.lang.Object
- Default implementation: uses memory address of object
  - If you override equals, you must override hashCode!!! We'll explain why later.
- String overrides hashCode:

  s. hashCode() :
  $= s[0] * 31^{n-1} + s[1] * 31^{n-2} + \ldots + s[n-1]$

Do we like this hashCode?

Can we have perfect hash functions?

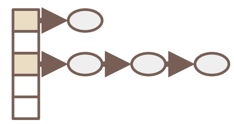
- A perfect hash function will map each value to a different index in the hash table
- Impossible in practice
  - Don't know size of the array
  - Number of possible values far far exceeds the array size
  - No point in a perfect hash function if it takes too much time to compute

Forget about perfect hash functions!

Collision Resolution

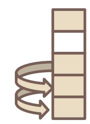
Two ways of handling collisions:

1. Chaining



A bucket contains a linked list of items that hash to it

2. Open Addressing

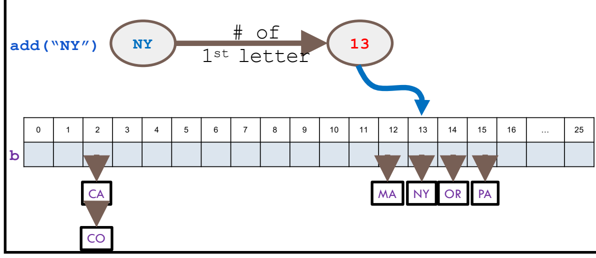


A bucket contains one item of the set. Look in successive array elements to find a place for a new item

Chaining (1)

add("NY")

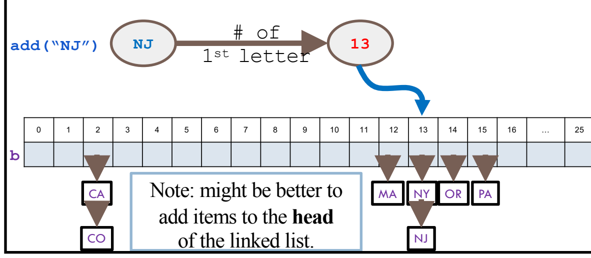
Each bucket is the beginning of a Linked List



Chaining (2)

add("NY")
add("NJ")

Each bucket is the beginning of a LinkedList

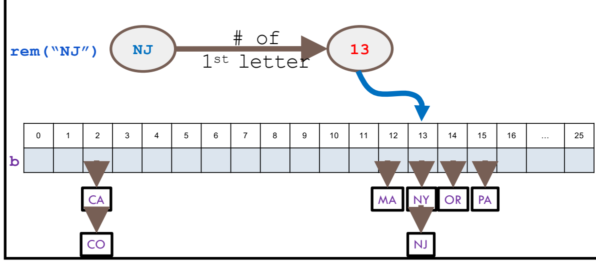


Chaining (3)

add("NY")
add("NJ")
rem("NJ")

Each bucket is the beginning of a LinkedList

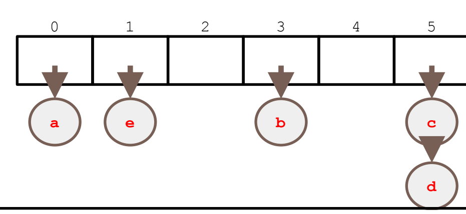
Rem: remove and return



Chaining in Action

Insert the following elements (in order) into an array of size 6:
Use (`hashCode % n_buckets`)

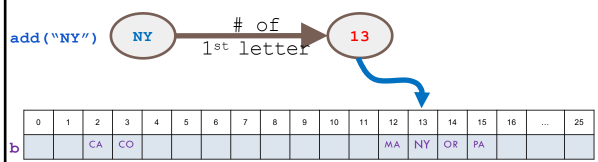
element	a	b	c	d	e
hashCode	0	9	17	11	19



Open Addressing (1)

`add("NY")`

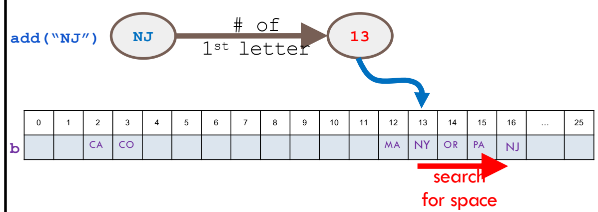
Probe: One test in finding space for a new item or when searching for an item



Open Addressing (2)

`add("NY")`
`add("NJ")`

Probe: One test in finding space for a new item or when searching for an item

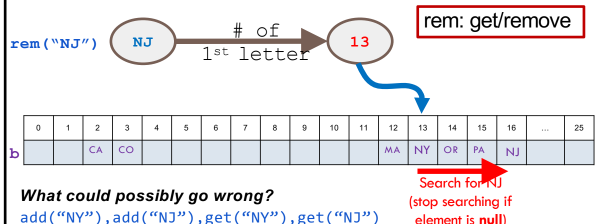


Open Addressing (3)

`add("NY")`
`add("NJ")`
`. . .`
`rem("NJ")`

Probe: One test in finding space for a new item or when searching for an item

rem: get/remove



What could possibly go wrong?

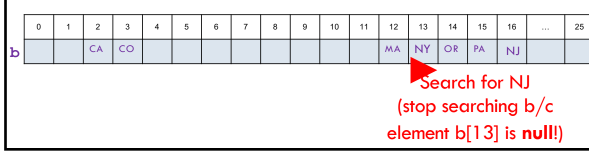
`add("NY"),add("NJ"),get("NY"),get("NJ")`

Deletion Problem w/Open Addressing

`add("NY")`
`add("NJ")`
`rem("NY")`
`rem("NJ")`

Probe: One test in finding space for a new item or when searching for an item

rem: get/remove



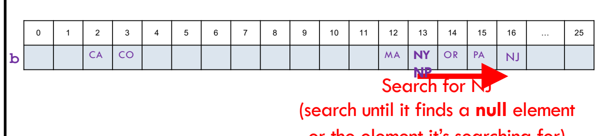
Deletion Solution for Open Addressing

`add("NY")`
`add("NJ")`
`get("NY")`
`get("NJ")`

Probe: One test in finding space for a new item or when searching for an item

to mark element as "not present"

Indicates to search that it should keep looking



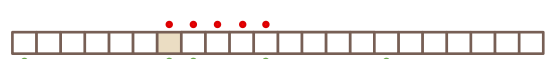
Different probing strategies

When a collision occurs,
how do we search for an empty space?

clustering: problem where nearby hashes have similar probe sequences so we get more collisions

linear probing: search the array in order: **i, i+1, i+2, i+3** . . .

quadratic probing: search the array in this sequence: **i, i+1^2, i+2^2, i+3^2** . . .



Quadratic probing requires the size of the array to be a prime in order to have access to every bucket.

Linear Probing in Action

Insert the following elements (in order) into an array of size 5:

element	a	b	c	d
hashCode	0	8	17	12

0	1	2	3	4
a		c	b	d
		probe #1 insert d: i full!	probe #2 insert d: i+1 full!	probe #3 insert d: i+2 has space!

Quadratic Probing in Action

Insert the following elements (in order) into an array of size 5:

element	a	b	c	d
hashCode	0	8	17	12

0	1	2	3	4
a	d	c	b	
	probe #3 insert d: $i+2^2$ has space!	probe #1 insert d: i full!	probe #2 insert d: $i+1^2$ full!	

In Java, functions hashCode and equals

HashSet, HashMap use functions hashCode(), equals(…)

c.HashCode() in class Object returns the address in memory of object c

c.equals(c1) in class Object is true iff c and c1 point to the same object

In Java, functions hashCode and equals

Elements of set HashSet have class type, e.g. Pt

```
Class Pt {
   int x;
   int y;
   …
}
```

Rewrite equals

```
/** Return true iff this and ob are of the same
  * class type, their x fields are equal, and
  * their y fields are equal. */
public boolean equals(Object ob) {…}
```

Because b and c are equal, only one of them should be put in the set

0	1	2	3	4
c			b	

b and c are different Pt objects but
b.x = c.x
b.y = c.y

In Java, functions hashCode and equals

What we learn from this

```
Class Pt {
   int x;
   int y;
   …
}
```

Function hashCode has to be defined so that:

if b.equals(c) is true,
then b.hashCode() == c.hashCode()

so that b and c hash to the same index.
The test for equality of c and b will show it's already in.

0	1	2	3	4
			c b	

In Java, functions hashCode and equals

Elements of set HashSet have class type, e.g. Pt

```
Class Pt {
   int x;
   int y;
   …
}
```

Rewrite equals

```
/** Return true iff this and ob are of the same
  * class type, their x fields are equal, and
  * their y fields are equal. */
public boolean equals(Object ob) {…}

public int hashCode() {
   return abs(x + y);
}
```

0	1	2	3	4
			c b	

b and c are different Pt objects but
b.x = c.x
b.y = c.y

Load Factor

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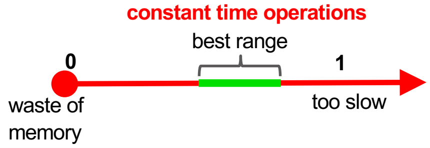
Load factor $\lambda = \frac{\textit{\# of entries}}{\textit{length of array}}$

If load factor = ½, expected # of probes is 2.
What happens when the array becomes too full?
i.e. load factor gets a lot bigger than ½?

no longer expected constant time operations



Chaining: Worst case time O(n)

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0	8999
8999 nulls, 1 list of size 6000	

Chaining worst case time

Suppose everything hashes to the last array element, so that all array elements are null except the last, and that last linked list has n elements in it ---the set has size n.

In this case, operations add, contains, and remove all take time O(n). That's the worst case.

Linear probing: Worst case time O(n)

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Chaining worst case time

	0	n	8999
b	n elements	null null … null	

Suppose everything hashes to 0, so that b[0..n-1] contains the set of elements and b[n..] are all null.

In this case, operations add, contains, and remove all take time O(n). That's the worst case.

Chaining: Expected time if load factor small: O(1)

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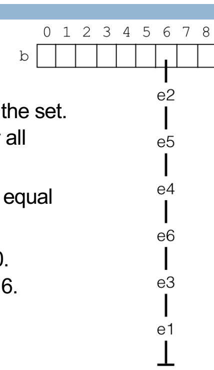
EXAMPLE. 6 elements, table size 9, load factor 6/9



Consider searching for e ---not in the set. Find average length of chain over all possibilities.

e hashes to a number in 0..8 with equal probability.

8 of the possibilities have length 0.
The other 1 possibility has length 6.

(8*0 + 1*6) / 9 = 6/9 (load factor)

Chaining: Expected time if load factor small: O(1)

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Example. 6000 elements, table size 9000, load factor 6/9

0	8999
8999 nulls, 1 list of size 6000	

Find average length of chain over all possibilities.

e hashes to a number in 0..8999 with equal probability.

8999 of the possibilities have length 0.
The other 1 possibility has length 6000.

(8999*0 + 1*6000) / 9000 = 6/9 (load factor)

Chaining: Expected time if load factor small: O(1)

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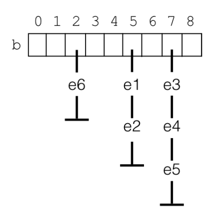
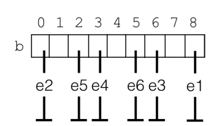
Example: 6 elements, table size 9, load factor 6/9
Consider *any* configuration of a set with load factor 6/9.
The average chain length is the load factor: 6/9





Average chain length: 6/9

Chaining: Expected time if load factor small: O(1)

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Searching for a value, whether in the set or not.

If the distribution of elements to buckets is sufficiently uniform, the average cost of a lookup depends only on the average number of elements per bucket.

That is: (size of set) / (size of array)

That's the load factor!

Load factor .75:	average of .75 elements per bucket
Load factor 1:	average of 1 element per bucket
Load factor 2:	average of 2 elements per bucket

Java HashMap uses chaining with load factor .75

Linear probing: Expected time, small load factor: O(1)

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This analysis is more complicated, harder.
State without proof:

The number of probes (buckets examined) to insert a value in a hash table with load factor lf is

1 / (1 - lf)

Choose lf = ½ and get average number of probes: 2

Resizing

When the load factor gets too big, create a new array twice the size, move the values to the new array, and then use the new array going forward

YOU DID THIS IN A5, method ensureSpace()!

Collections class ArrayList does the same.

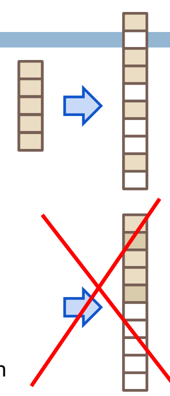
Collections classes HashSet and HashMap resize when the load factor becomes greater than .75, but you can change it.

Resizing

Solution: ***Dynamic resizing***

- double the size*
- reinsert / rehash all elements to new array
- Why not simply copy into first half?

  index for an item is:
  hash code mod table-size



*if using quadratic probing, use a prime >2n

Resizing takes constant amortized time

Say it cost $100.00

We bought a machine that makes fizzy water.
The machine cost $100.

Make one glass of fizzy water:
glass cost $100.00.

Make 100 glasses of fizzy water:
Each glass cost $1.00.

Make 1,000 glasses:
Each glass cost 10 cents.

Amortizing cost of machine over use of machine, over number of operations "make a glass …".

image taken from sodastreamusa.com

Amortizing the cost of resizing

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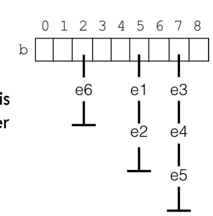
Each element of the array took at most constant time C (say) to add it to the set.

Double the size of the array:
Each element has to be rehashed into the new array, taking time at most C.

So we say that the time for each element is 2C —we amortize the cost of resizing over the time for the add operation.



Collision Resolution Summary

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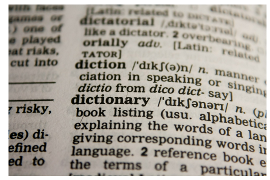
Chaining	Open Addressing
□ store entries in separate chains (linked lists) □ Uses more memory	□ store all entries in table □ use linear or quadratic probing to place items □ uses less memory □ clustering can be a problem — need to be more careful with choice of hash function

Application: Hash Map




```
Map<K,V>{

    void put(K key, V value);

    void update(K key, V value);

    V get(K key);

    V remove(K key);

}
```

- Use the **key** for lookups
- Store the **value**

Example: **key** is the word, **value** is its definition

HashMap in Java

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- Computes hash using key.hashCode()
  - No duplicate keys
- Uses chaining to handle collisions
- Default load factor is .75
- Java 8 attempts to mitigate worst-case performance by switching to a BST-based chaining!

Hash Maps in the Real World

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- Network switches
- Distributed storage
- Database indexing
- Heaps with the ability to change a priority
- Index lookup (e.g. Dijkstra's shortest-path algorithm)
- Useful in lots of applications…