PRIORITY QUEUES & HEAPS

Lecture 14

CS2110 Spring 2019

JavaHyperText Topics

- Interface, implements
- Stack, queue
- Priority queue
- Heaps, heapsort

Interface vs. Implementation

Interface: the operations of an ADT

- What you see on documentation web pages
- Method names and specifications
- Abstract from details: what to do, not how to do it
- Java syntax: **`interface`**

Implementation: the code for a data structure

- What you see in source files
- Fields and method bodies
- Provide the details: how to do operation
- Java syntax: **`class`**

Could be many implementations of an interface

e.g. List: ArrayList, LinkedList

ADTs (interfaces)

ADT	Description
List	Ordered collection (aka sequence)
Set	Unordered collection with no duplicates
Map	Collection of keys and values, like a dictionary
Stack	Last-in-first-out (LIFO) collection
Queue	First-in-first-out (FIFO) collection
Priority Queue	*Later this lecture!*

Implementations of ADTs

Interface	**Implementation (data structure)**
List	ArrayList, LinkedList
Set	HashSet, TreeSet
Map	HashMap, TreeMap
Stack	*Can be done with a LinkedList*
Queue	*Can be done with a LinkedList*
Priority Queue	*Can be done with a heap — later this lecture!*

Efficiency Tradeoffs

Class:	ArrayList	LinkedList
Backing storage:	array	chained nodes
prepend(val)	O(n)	O(1)
get(i)	O(1)	O(n)

Which implementation to choose depends on expected workload for application

Priority Queues

Priority Queue

- Primary operation:
  - Stack: remove newest element
  - Queue: remove oldest element
  - Priority queue: remove highest priority element
- Priority:
  - Additional information for each element
  - Needs to be **`Comparable`**

Priority Queue

Priority	Task
	Practice for swim test
	Learn the Cornell *Alma Mater*
	Study for 2110 prelim
	Find Eric Andre ticket for sale

java.util.PriorityQueue<E>


```
class PriorityQueue<E> {
 boolean add(E e); //insert e.
 E poll(); //remove&return min elem.
 E peek(); //return min elem.
 boolean contains(E e);
 boolean remove(E e);
 int size();
 ...
}
```

Implementations

LinkedList

`add()` put new element at front – O(1)

`poll()` must search the list – O(n)

`peek()` must search the list – O(n)

LinkedList that is always sorted

`add()` must search the list – O(n)

`poll()` highest priority element at front – O(1)

`peek()` same – O(1)

Balanced BST

`add()` must search the tree & rebalance – O(log n)

`poll()` same – O(log n)

`peek()` same – O(log n)

Can we do better?

Heaps

A Heap..

Is a binary tree satisfying 2 properties:

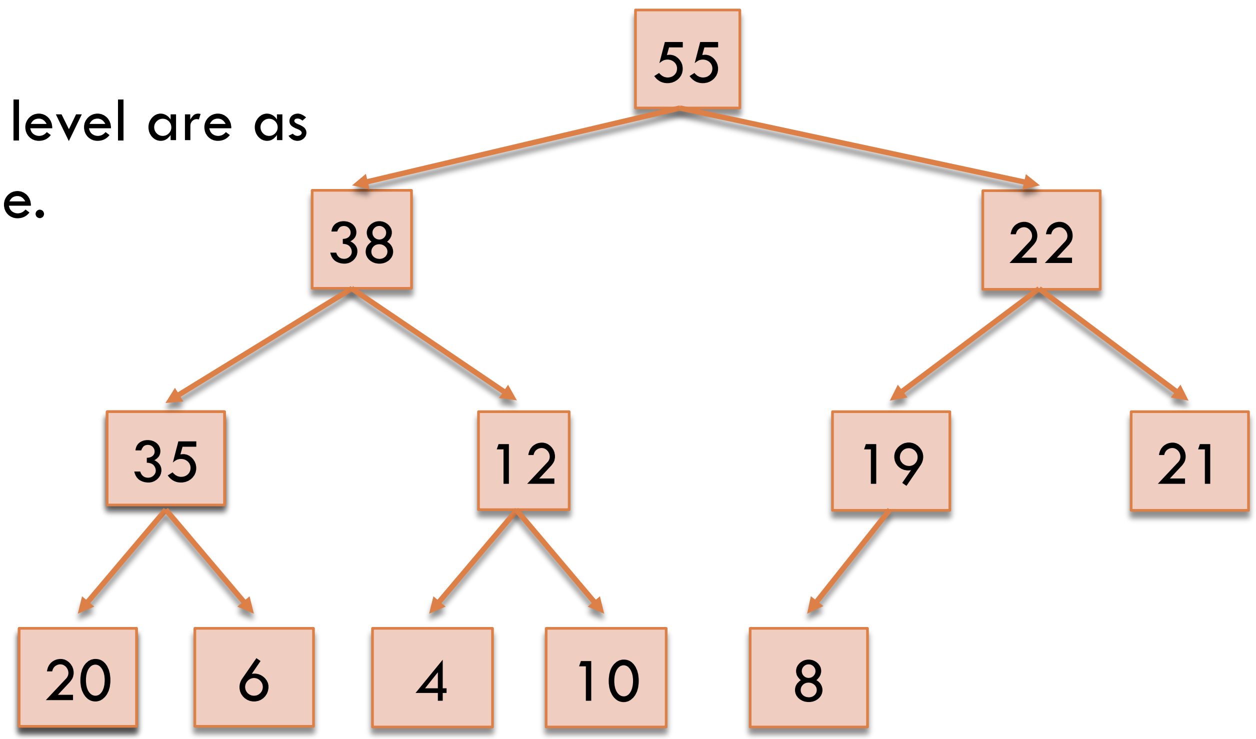
1) **Completeness.** Every level of the tree (except last) is completely filled, and on last level nodes are as far left as possible.

Do not confuse with heap memory – different use of the word heap.

Completeness

Every level (except last) completely filled.

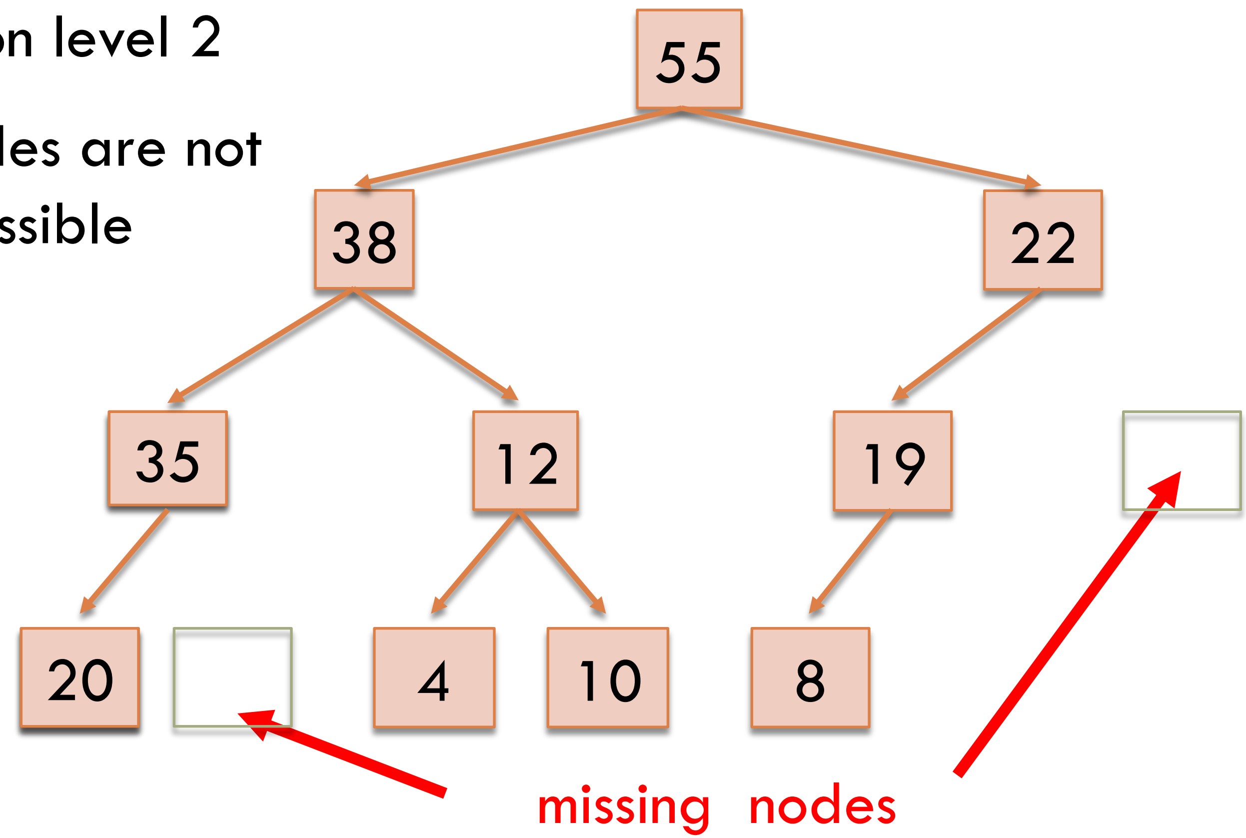
Nodes on bottom level are as far left as possible.



Completeness

Not a heap because:

- missing a node on level 2
- bottom level nodes are not as far left as possible



A Heap..

Is a binary tree satisfying 2 properties:

1) **Completeness.** Every level of the tree (except last) is completely filled, and on last level nodes are as far left as possible.

2) **Heap-order.**

"max on top"

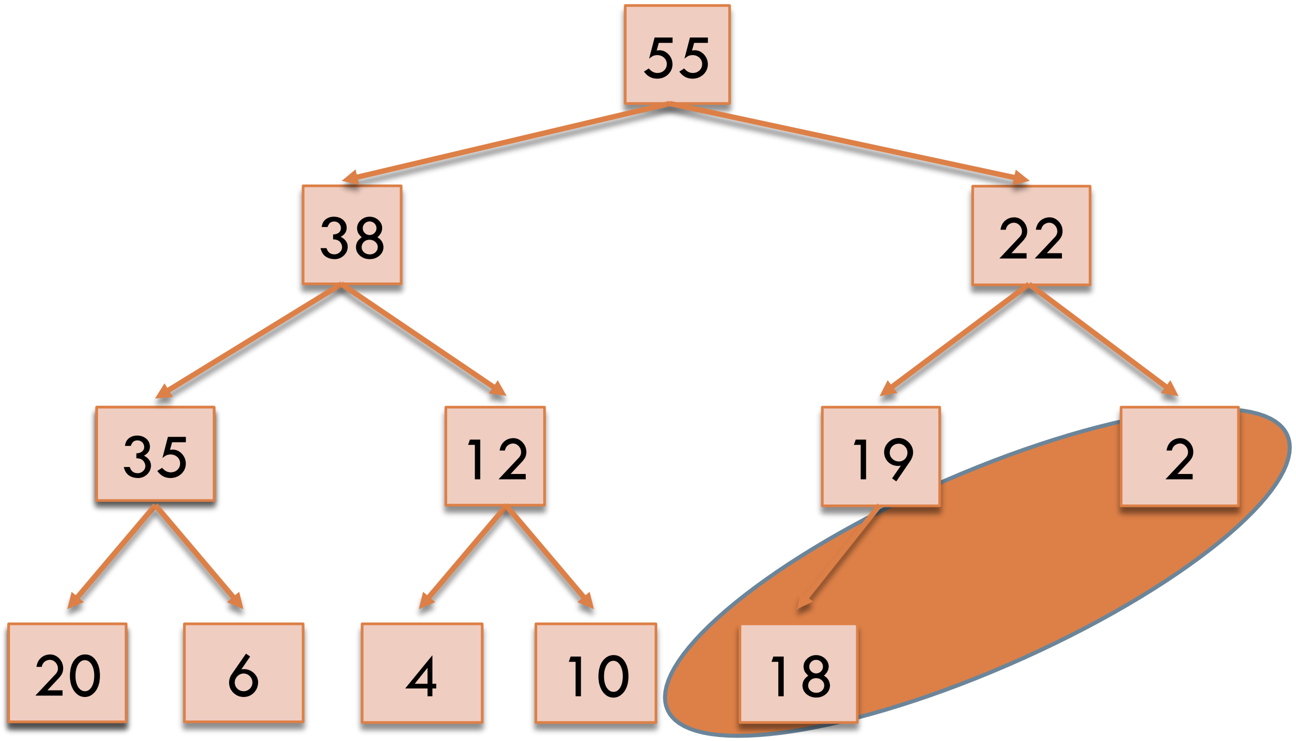
Max-Heap: every element in tree is <= its parent

Min-Heap: every element in tree is >= its parent

"min on top"

Heap-order (max-heap)

Every element is <= its parent



Note: Bigger elements can be deeper in the tree!

Piazza Poll #1

A Heap..

Is a binary tree satisfying 2 properties

1) **Completeness.** Every level of the tree (except last) is completely filled. All holes in last level are all the way to the right.
2) **Heap-order.**

   Max-Heap: every element in tree is <= its parent

Primary operations:

1) add(e): add a new element to the heap
2) poll(): delete the max element and return it
3) peek(): return the max element

Priority queues

🎉🎉🎉

Heaps can implement priority queues

🎉🎉🎉

- Each heap node contains priority of a queue item
- (For values+priorities, see JavaHyperText)

Priority queues

🎉🎉🎉

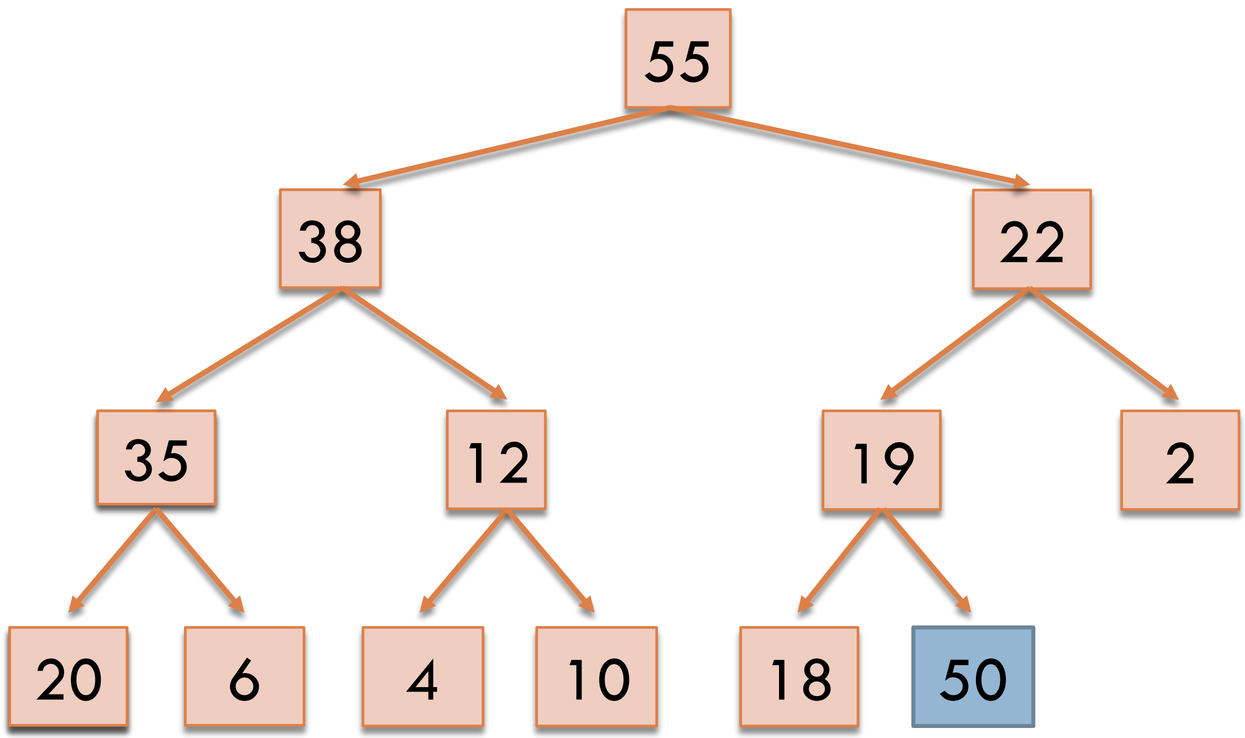
Heaps can implement priority queues

🎉🎉🎉

- Efficiency we will achieve:
  - add(): O(log n)
  - poll(): O(log n)
  - peek(): O(1)
- No linear time operations: better than lists
- peek() is constant time: better than balanced trees

Heap Algorithms

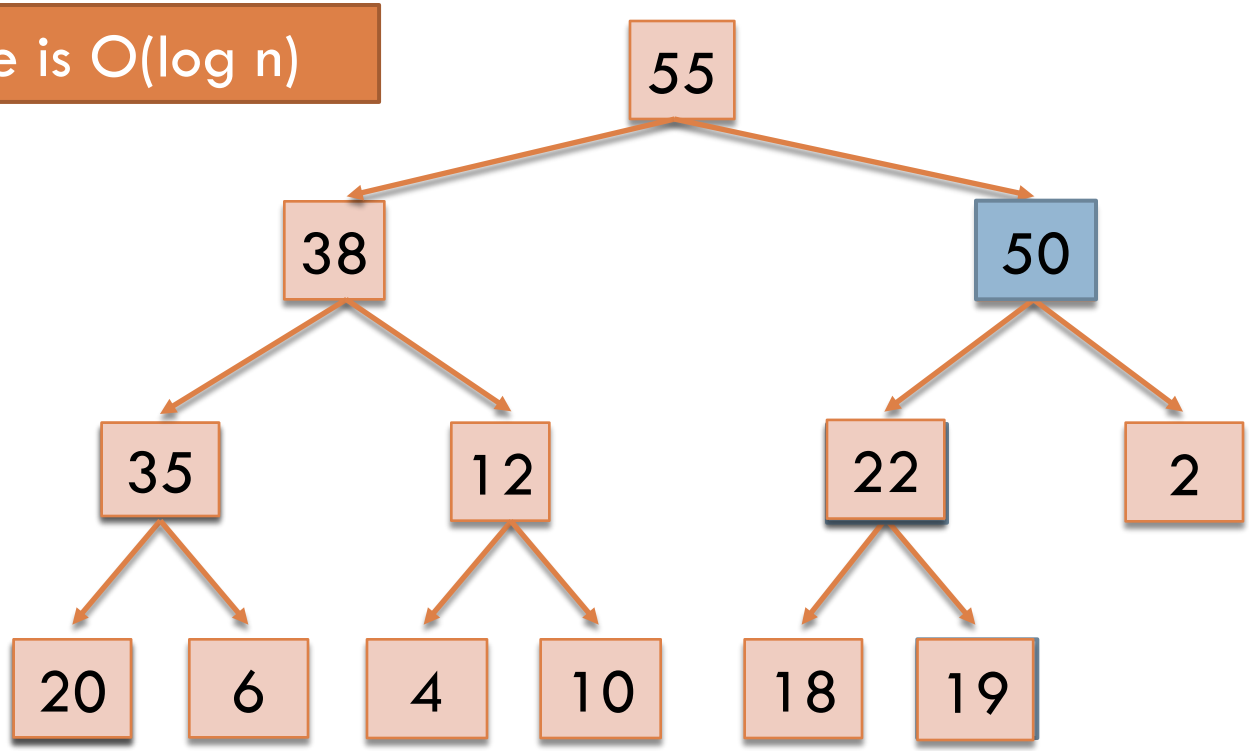
Heap: add(e)



1. Put in the new element in a new node (leftmost empty leaf)

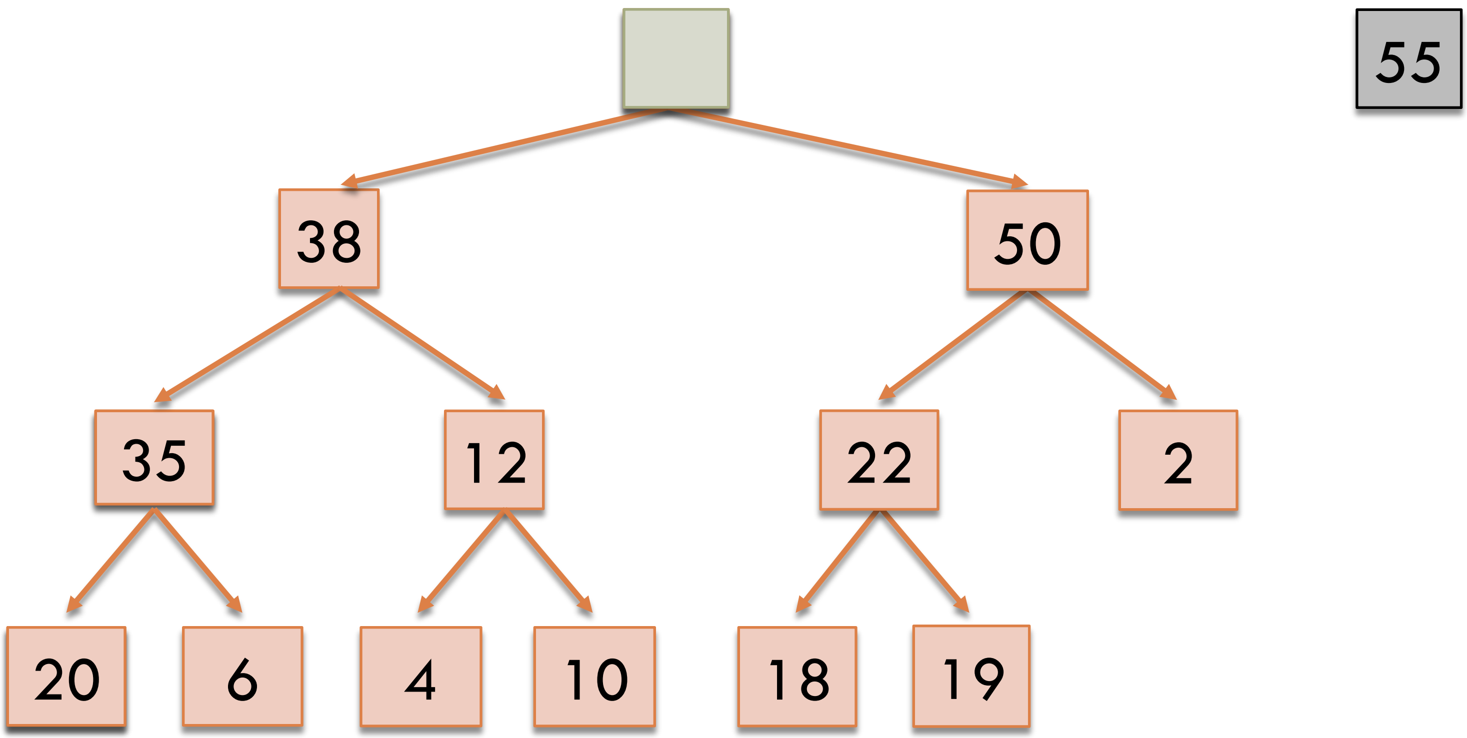
Heap: add(e)

Time is O(log n)



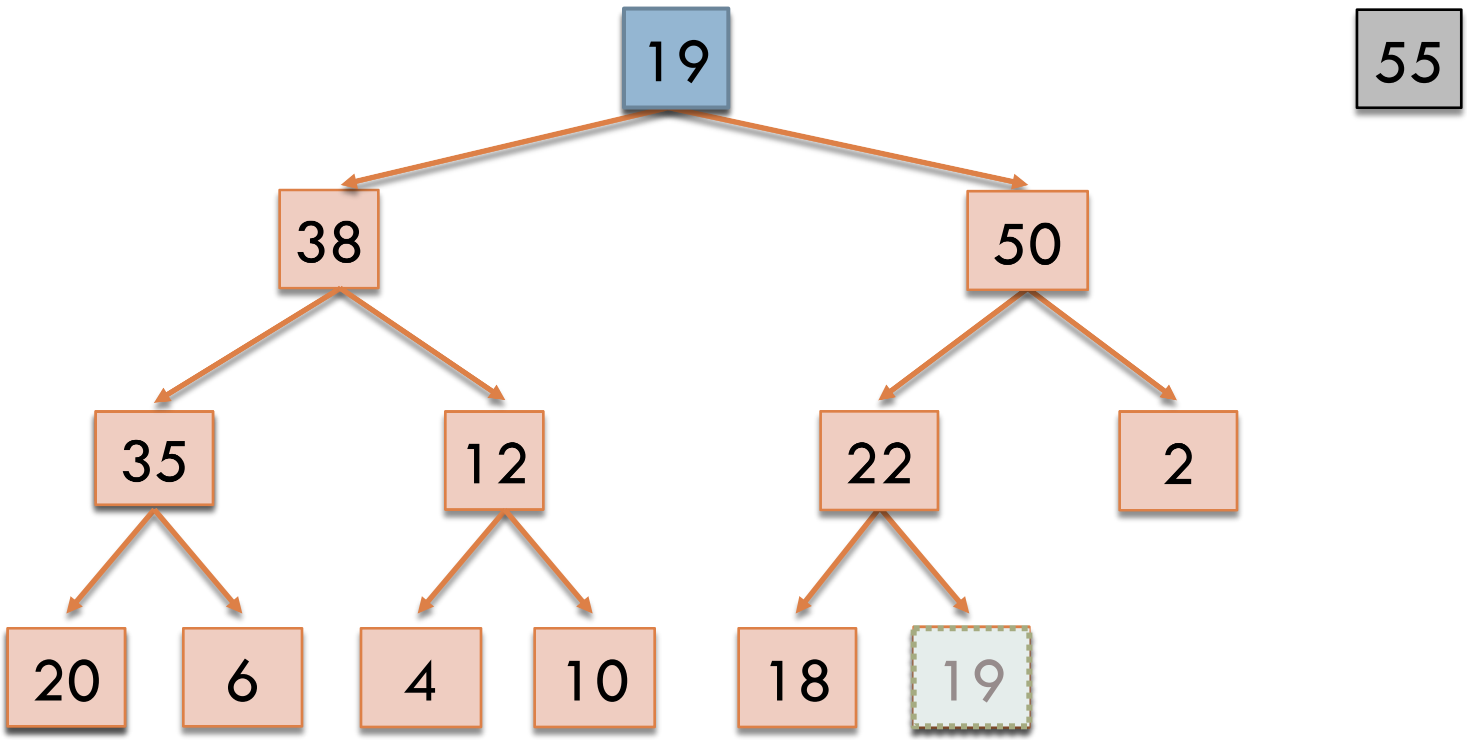
1. Put in the new element in a new node (leftmost empty leaf)
2. Bubble new element up while greater than parent

Heap: poll()



1. Save root element in a local variable

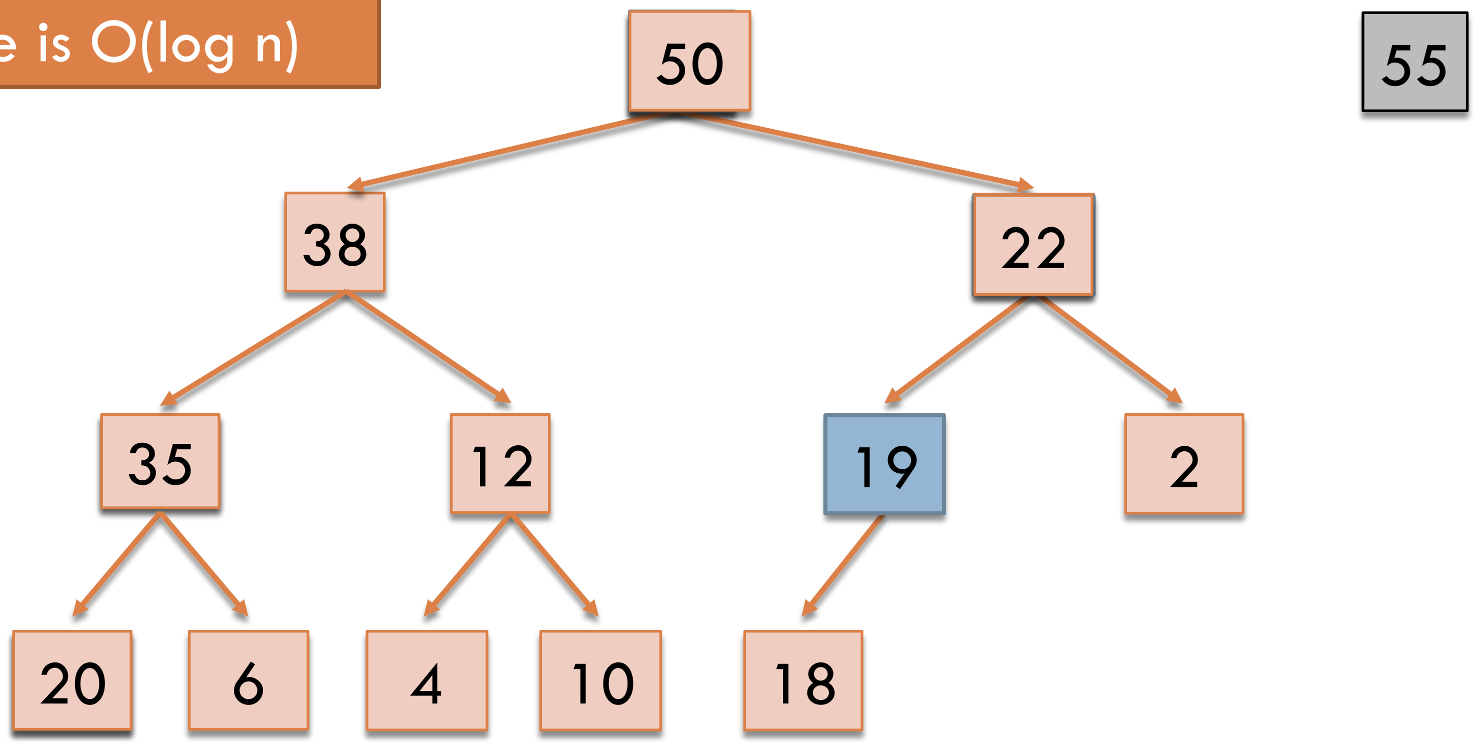
Heap: poll()



1. Save root element in a local variable
2. Assign last value to root, delete last node.

Heap: poll()

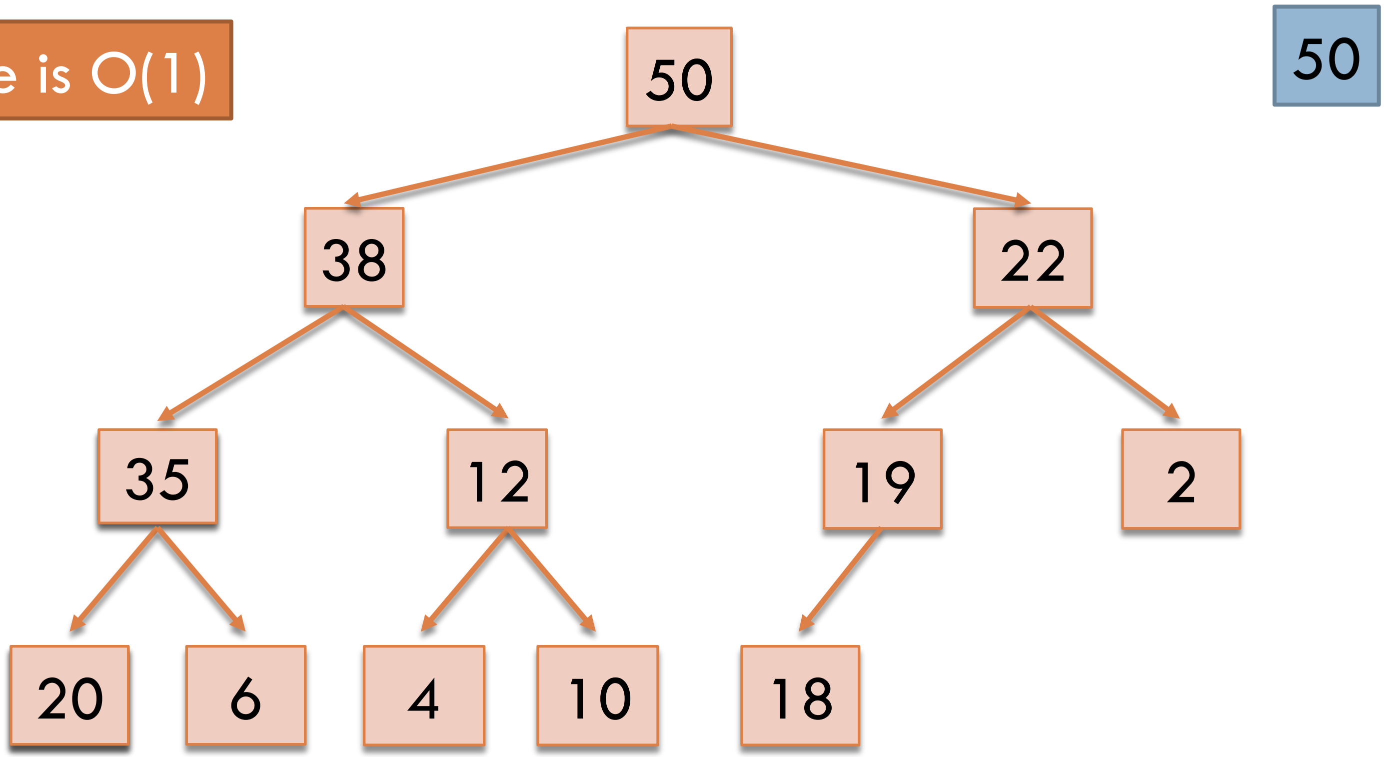
Time is O(log n)



1. Save root element in a local variable
2. Assign last value to root, delete last node.
3. While less than a child, switch with bigger child (bubble down)

Heap: peek()

Time is O(1)



1. Return root value

Heap Implementation

(max heap)

Tree implementation


```
public class HeapNode<E> {
  private E value;
  private HeapNode left;
  private HeapNode right;
  ...
}
```

But since tree is complete, even more space-efficient implementation is possible…

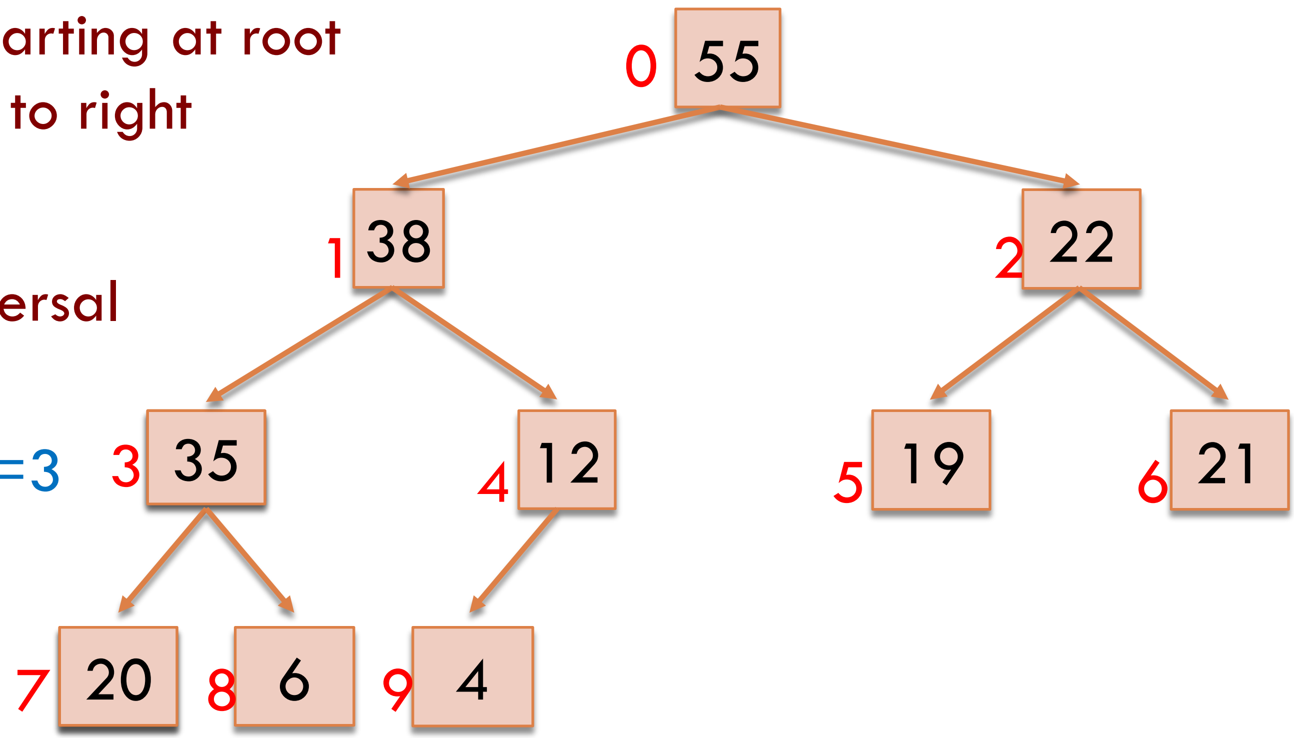
Array implementation


```
public class Heap<E> {
   (* represent tree as array *)
   private E[] heap;
   ...
}
```

Numbering tree nodes

Number node starting at root row by row, left to right

Same order as level-order traversal



k=3

2(3)+1 = 7

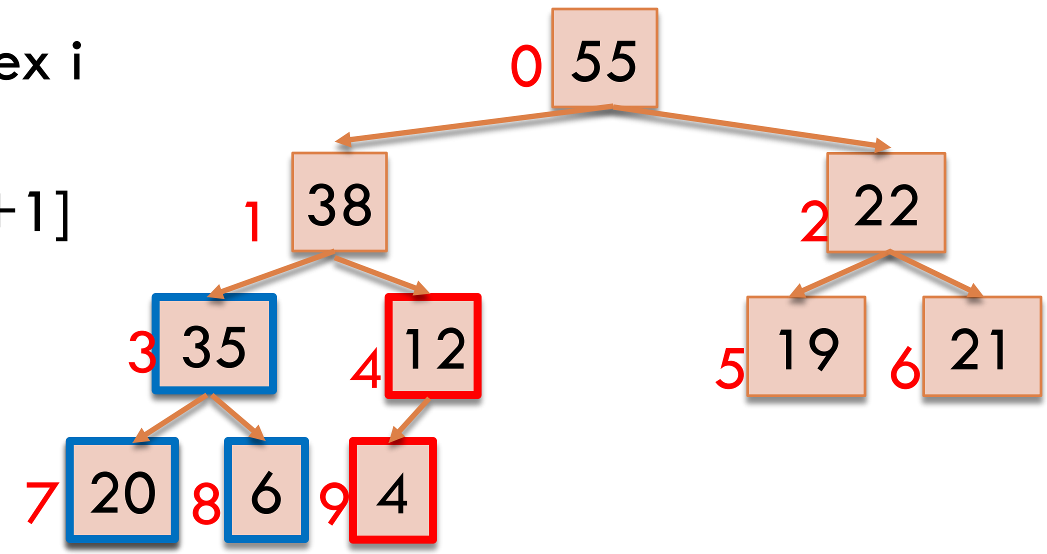
2(3)+2 = 8

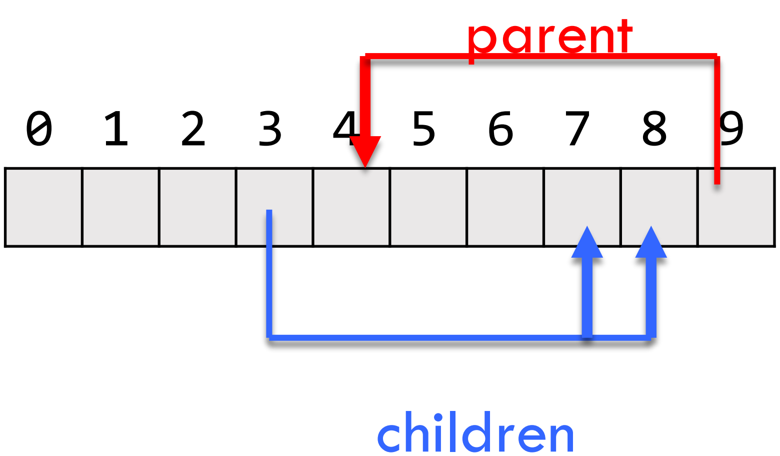
Children of node **k** are nodes **2k+1** and **2k+2**

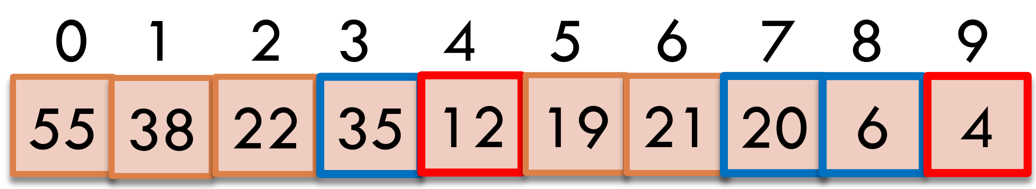
Parent of node **k** is node **(k-1)/2**

Represent tree with array

- Store node number i in index i of array b
- Children of b[k] are b[2k +1] and b[2k +2]
- Parent of b[k] is b[(k-1)/2]







Constructor


```
class Heap<E> {
  E[] b; // heap is b[0..n-1]
  int n;

  /** Create heap with max size */
  public Heap(int max) {
    b= new E[max];
    // n == 0, so heap invariant holds
    // (completeness & heap-order)
  }
}
```

add() (assuming enough room in array)


```
class Heap<E> {

  /** Add e to the heap */
  public void add(E e) {
    b[n]= e;
    n= n + 1;
    bubbleUp(n - 1); // on next slide
  }
}
```

add(). heap is in b[0..n-1]


```
class Heap<E> {
  /** Bubble element #k up to its position.
    * Pre: heap inv holds except maybe for k */
  private void bubbleUp(int k) {
     int p= (k-1)/2;
     // inv: p is parent of k and every element
     // except perhaps k is <= its parent
     while (k > 0  &&  b[k].compareTo(b[p]) > 0) {
         swap(b[k], b[p]);
         k= p;
         p= (k-1)/2;
    }
}
```

peek()


```
/** Return largest element
  * (return null if list is empty) */
public E poll() {
    if (n == 0) return null;
    return b[0];   // largest value at root.
}
```

poll(). heap is in b[0..n-1]


```
/** Remove and return the largest element
  * (return null if list is empty) */
public E poll() {
    if (n == 0) return null;
    E v= b[0];     // largest value at root
    n= n – 1;      // move last
    b[0]= b[n];    // element to root
    bubbleDown(); // on next slide
    return v;
}
```

poll()


```
/** Bubble root down to its heap position.
    Pre: b[0..n-1] is a heap except maybe b[0] */
private void bubbleDown() {
    int k= 0;
    int c= biggerChild(k); // on next slide
   // inv: b[0..n-1] is a heap except maybe b[k] AND
   //      b[c] is b[k]'s biggest child
   while ( c < n &&  b[k] < b[c]) {

      swap(b[k], b[c]);
      k= c;
      c= biggerChild(k);

  }
}
```

poll()


```
/** Return index of bigger child of node k */
public int biggerChild(int k) {
    int c= 2*k + 2;     // k’s right child
    if (c >= n || b[c-1] > b[c])
        c= c-1;
    return c;
}
```

Piazza Poll #2

Efficiency


```
class PriorityQueue<E> {                TIME*
 boolean add(E e); //insert e.          log
 E poll(); //remove&return min elem.    log
 E peek(); //return min elem.           constant
 boolean contains(E e);                 linear
 boolean remove(E e);                   linear
 int size();                            constant
}
```

*IF implemented with a heap!

Heapsort

(if time, in JavaHyperText if not)

Heapsort

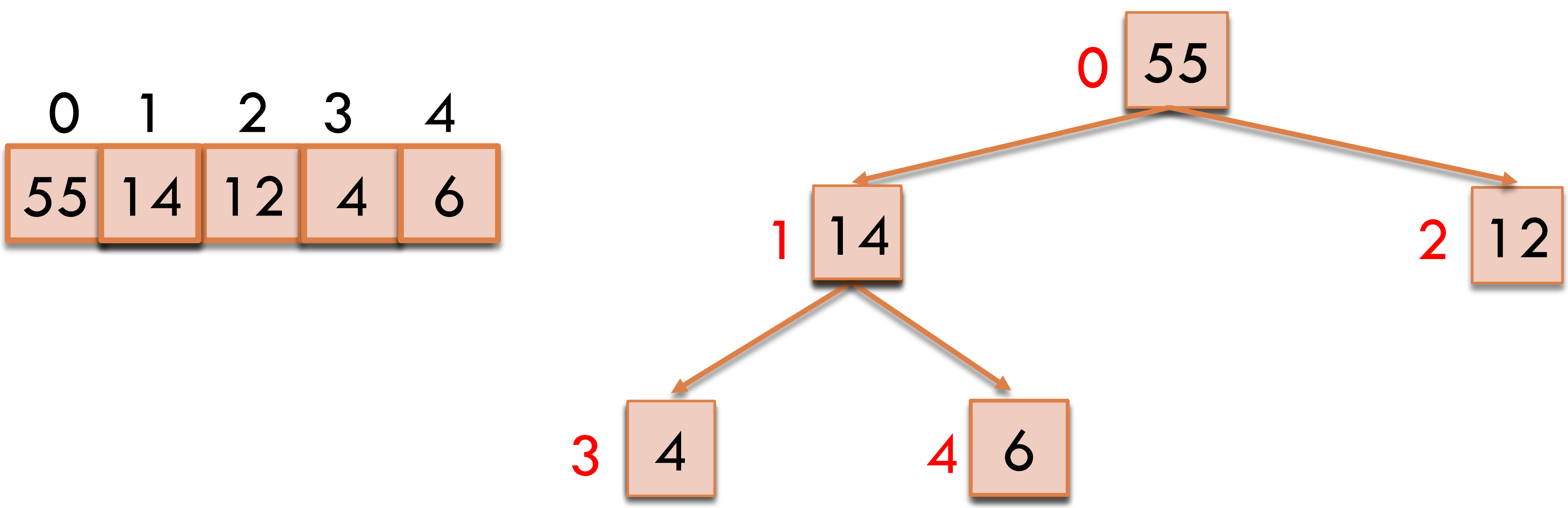
0	1	2	3	4
55	4	12	6	14

Goal: sort this array in place

Approach: turn the array into a heap and then poll repeatedly

Heapsort

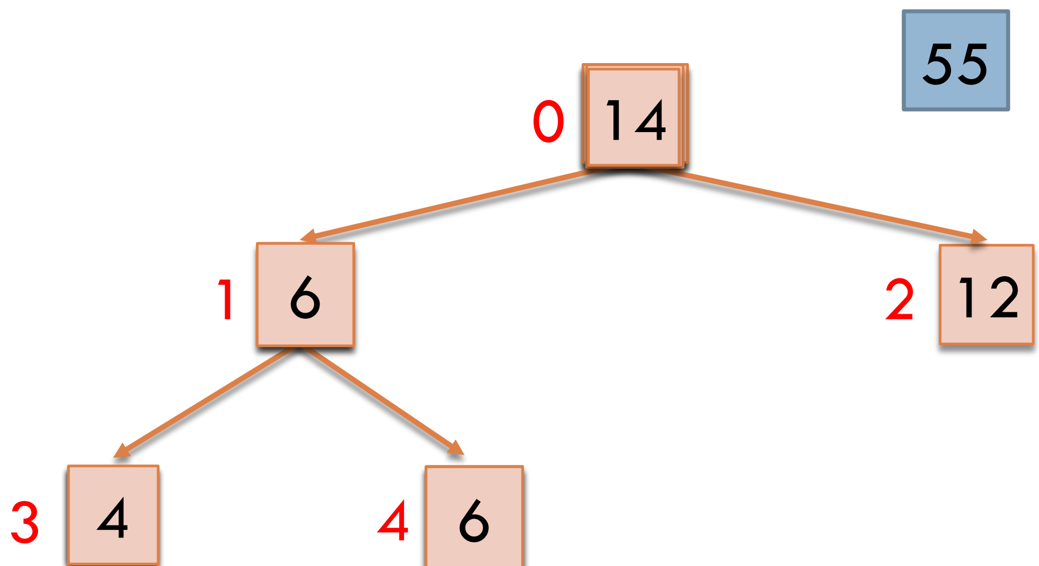
// Make b[0..n-1] into a max-heap (in place)



Heapsort


```
// Make b[0..n-1] into a max-heap (in place)
// inv:   b[0..k] is a heap, b[0..k] <= b[k+1..], b[k+1..] is sorted
    for (k= n-1; k > 0; k= k-1) {
            b[k]= poll  – i.e., take max element out of heap.
     }
```

0	1	2	3	4
14	6	12	4	55



Heapsort

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    for (k= n-1; k > 0; k= k-1) {
            b[k]= poll  – i.e., take max element out of heap.
    }
```

0	1	2	3	4
4	6	12	14	55