# PRIORITY QUEUES & HEAPS

Lecture 14
CS2110 Spring 2019

## JavaHyperText Topics

- Interface, implements
- Stack, queue
- Priority queue
- Heaps, heapsort

## Interface vs. Implementation

**Interface:** the operations of an ADT

- What you see on documentation web pages
- Method names and specifications
- Abstract from details: what to do, not how to do it
- Java syntax: `interface`

**Implementation:** the code for a data structure

- What you see in source files
- Fields and method bodies
- Provide the details: how to do operation
- Java syntax: `class`

Could be many implementations of an interface
e.g. List: ArrayList, LinkedList

## ADTs (interfaces)

| ADT | Description |
|---|---|
| List | Ordered collection (aka sequence) |
| Set | Unordered collection with no duplicates |
| Map | Collection of keys and values, like a dictionary |
| Stack | Last-in-first-out (LIFO) collection |
| Queue | First-in-first-out (FIFO) collection |
| Priority Queue | *Later this lecture!* |

## Implementations of ADTs

| Interface | Implementation (data structure) |
|---|---|
| List | ArrayList, LinkedList |
| Set | HashSet, TreeSet |
| Map | HashMap, TreeMap |
| Stack | *Can be done with a LinkedList* |
| Queue | *Can be done with a LinkedList* |
| Priority Queue | *Can be done with a heap — later this lecture!* |

## Efficiency Tradeoffs

| Class: | ArrayList | LinkedList |
|---|---|---|
| Backing storage: | array | chained nodes |
| prepend(val) | O(n) | O(1) |
| get(i) | O(1) | O(n) |

Which implementation to choose depends on expected workload for application

# Priority Queues

## Priority Queue

- Primary operation:
  - Stack: remove newest element
  - Queue: remove oldest element
  - Priority queue: remove highest priority element
- Priority:
  - Additional information for each element
  - Needs to be **`Comparable`**

## Priority Queue

| Priority | Task |
|---|---|
| | Practice for swim test |
| | Learn the Cornell *Alma Mater* |
| | Study for 2110 prelim |
| | Find Eric Andre ticket for sale |

## `java.util.PriorityQueue<E>`

```
class PriorityQueue<E> {
 boolean add(E e); //insert e.
 E poll(); //remove&return min elem.
 E peek(); //return min elem.
 boolean contains(E e);
 boolean remove(E e);
 int size();
 ...
}
```

## Implementations

LinkedList

| | |
|---|---|
| **`add()`** | put new element at front – O(1) |
| **`poll()`** | must search the list – O(n) |
| **`peek()`** | must search the list – O(n) |

LinkedList that is always sorted

| | |
|---|---|
| **`add()`** | must search the list – O(n) |
| **`poll()`** | highest priority element at front – O(1) |
| **`peek()`** | same – O(1) |

Balanced BST

| | |
|---|---|
| **`add()`** | must search the tree & rebalance – O(log n) |
| **`poll()`** | same – O(log n) |
| **`peek()`** | same – O(log n) |

Can we do better?

# Heaps

## A Heap..

Is a binary tree satisfying 2 properties:

1) **Completeness.** Every level of the tree (except last) is completely filled, and on last level nodes are as far left as possible.

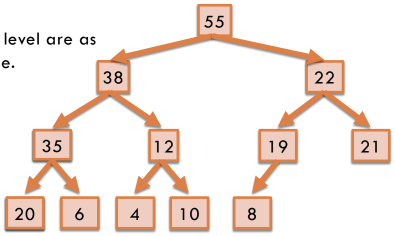
Do not confuse with heap memory – different use of the word heap.

---

## Completeness

Every level (except last) completely filled.

Nodes on bottom level are as far left as possible.

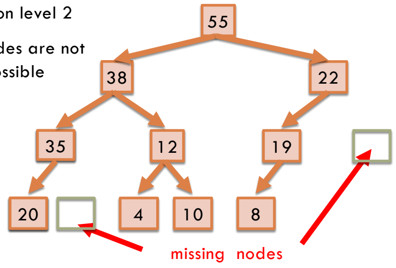


---

## Completeness

Not a heap because:

- missing a node on level 2
- bottom level nodes are not as far left as possible



---

## A Heap..

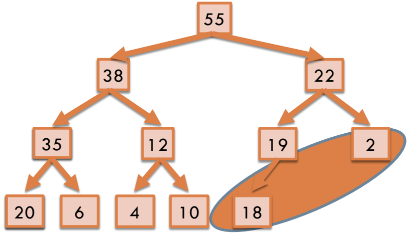
Is a binary tree satisfying 2 properties:

1) **Completeness.** Every level of the tree (except last) is completely filled, and on last level nodes are as far left as possible.
2) **Heap-order.** "max on top"
   Max-Heap: every element in tree is <= its parent
   Min-Heap: every element in tree is >= its parent
   "min on top"

---

## Heap-order (max-heap)

Every element is <= its parent



Note: Bigger elements can be deeper in the tree!

---

## Piazza Poll #1

## A Heap..

Is a binary tree satisfying 2 properties

1) **Completeness.** Every level of the tree (except last) is completely filled. All holes in last level are all the way to the right.
2) **Heap-order.**
   Max-Heap: every element in tree is <= its parent

Primary operations:

1) add(e): add a new element to the heap
2) poll(): delete the max element and return it
3) peek(): return the max element

## Priority queues

🎉🎉🎉

**Heaps can implement priority queues**

🎉🎉🎉

- Each heap node contains priority of a queue item
- (For values+priorities, see JavaHyperText)

## Priority queues

🎉🎉🎉

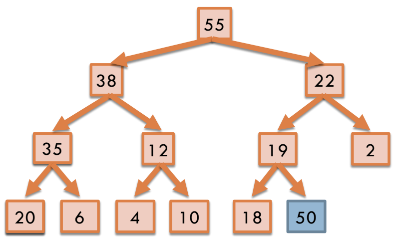
**Heaps can implement priority queues**

🎉🎉🎉

- Efficiency we will achieve:
  - add(): O(log n)
  - poll(): O(log n)
  - peek(): O(1)
- No linear time operations: better than lists
- peek() is constant time: better than balanced trees

## Heap Algorithms

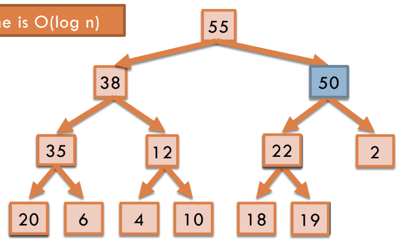
## Heap: add(e)



1. Put in the new element in a new node (leftmost empty leaf)

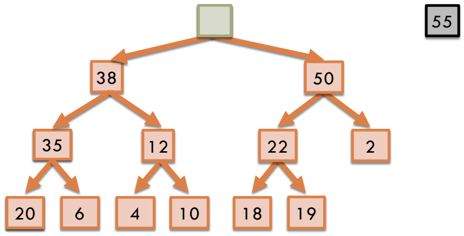
## Heap: add(e)

Time is O(log n)



1. Put in the new element in a new node (leftmost empty leaf)
2. Bubble new element up while greater than parent

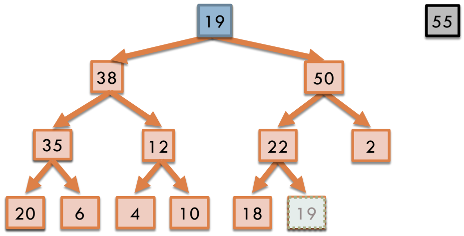
## Heap: poll()



1. Save root element in a local variable

---

## Heap: poll()

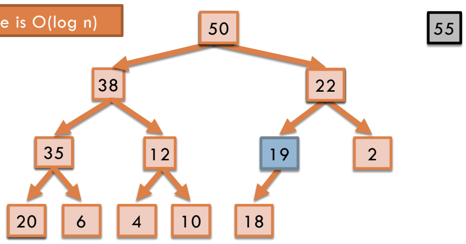


1. Save root element in a local variable
2. Assign last value to root, delete last node.

---

## Heap: poll()

Time is O(log n)

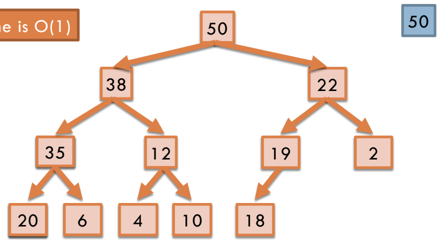


1. Save root element in a local variable
2. Assign last value to root, delete last node.
3. While less than a child, switch with bigger child (bubble down)

---

## Heap: peek()

Time is O(1)



1. Return root value

---

## Heap Implementation

(max heap)

---

## Tree implementation

```
public class HeapNode<E> {
  private E value;
  private HeapNode left;
  private HeapNode right;
  ...
}
```

But since tree is complete, even more space-efficient implementation is possible…

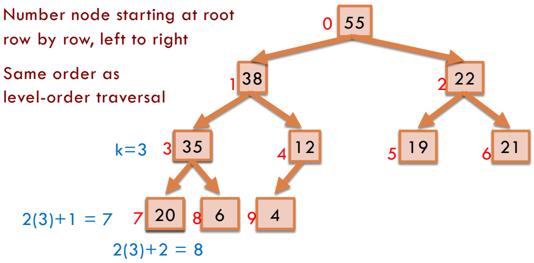
## Array implementation

```
public class Heap<E> {
  (* represent tree as array *)
  private E[] heap;
  ...
}
```

## Numbering tree nodes

Number node starting at root
row by row, left to right

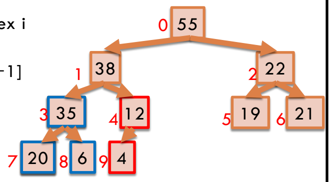
Same order as
level-order traversal

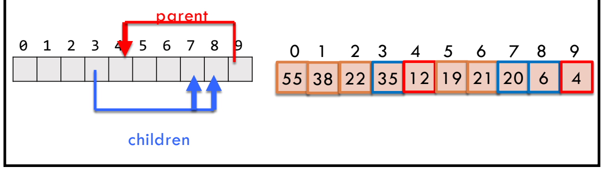


Children of node k are nodes **2k+1** and **2k+2**
Parent of node k is node (k-1)/2

## Represent tree with array

- Store node number i in index i of array b
- Children of b[k] are b[2k +1] and b[2k +2]
- Parent of b[k] is b[(k-1)/2]





## Constructor

```
class Heap<E> {
  E[] b; // heap is b[0..n-1]
  int n;

  /** Create heap with max size */
  public Heap(int max) {
    b= new E[max];
    // n == 0, so heap invariant holds
    // (completeness & heap-order)
  }
}
```

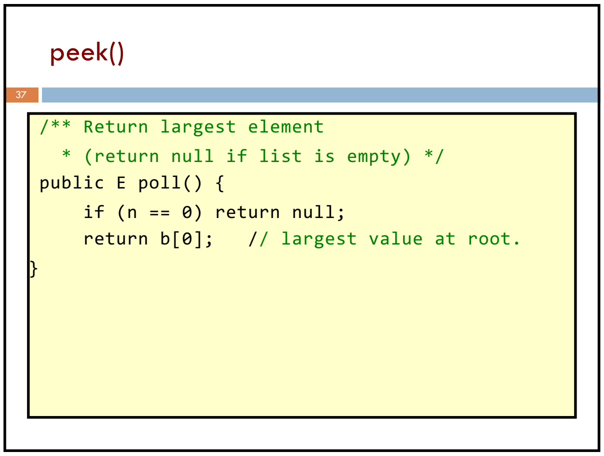
## add() (assuming enough room in array)

```
class Heap<E> {

  /** Add e to the heap */
  public void add(E e) {
    b[n]= e;
    n= n + 1;
    bubbleUp(n - 1); // on next slide
  }
}
```

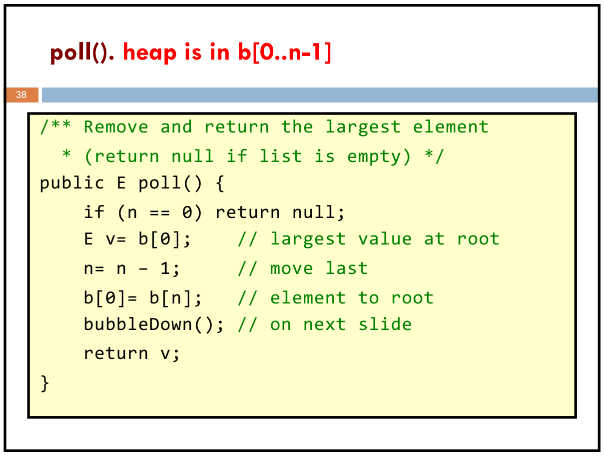
## add(). heap is in b[0..n-1]

```
class Heap<E> {
  /** Bubble element #k up to its position.
    * Pre: heap inv holds except maybe for k */
  private void bubbleUp(int k) {
    int p= (k-1)/2;
    // inv: p is parent of k and every element
    // except perhaps k is <= its parent
    while (k > 0  &&  b[k].compareTo(b[p]) > 0) {
       swap(b[k], b[p]);
       k= p;
       p= (k-1)/2;
    }
}
```



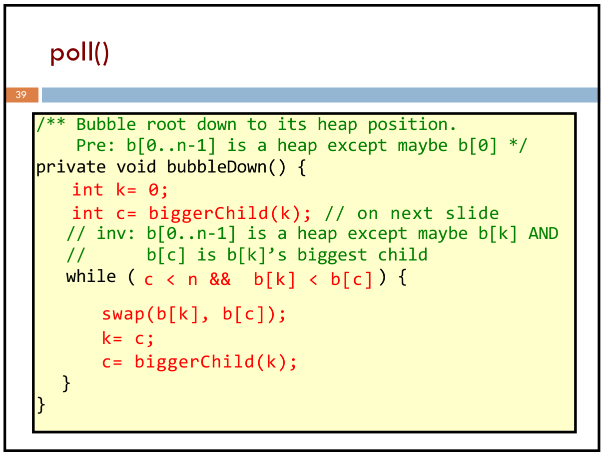
## peek()

```
/** Return largest element
  * (return null if list is empty) */
public E poll() {
    if (n == 0) return null;
    return b[0];    // largest value at root.
}
```



## poll(). heap is in b[0..n-1]

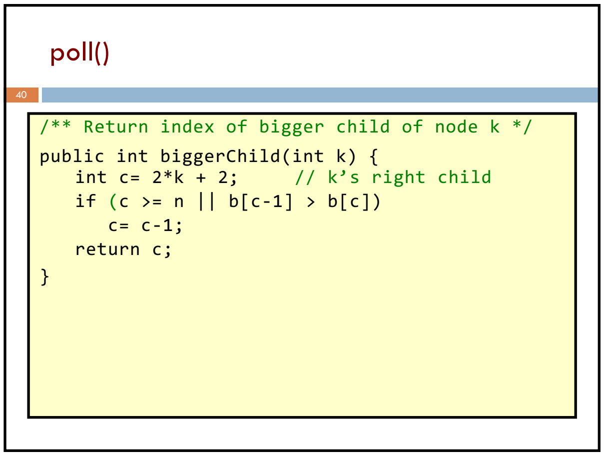
```
/** Remove and return the largest element
  * (return null if list is empty) */
public E poll() {
    if (n == 0) return null;
    E v= b[0];    // largest value at root
    n= n - 1;     // move last
    b[0]= b[n];   // element to root
    bubbleDown(); // on next slide
    return v;
}
```



## poll()

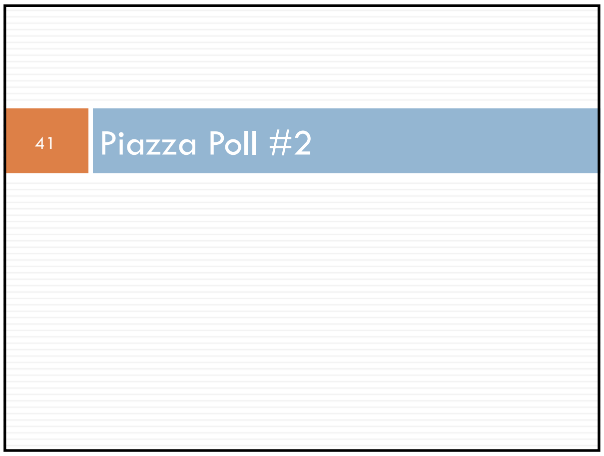
```
/** Bubble root down to its heap position.
    Pre: b[0..n-1] is a heap except maybe b[0] */
private void bubbleDown() {
    int k= 0;
    int c= biggerChild(k); // on next slide
   // inv: b[0..n-1] is a heap except maybe b[k] AND
   //      b[c] is b[k]'s biggest child
   while ( c < n &&  b[k] < b[c] ) {

       swap(b[k], b[c]);
       k= c;
       c= biggerChild(k);
   }
}
```

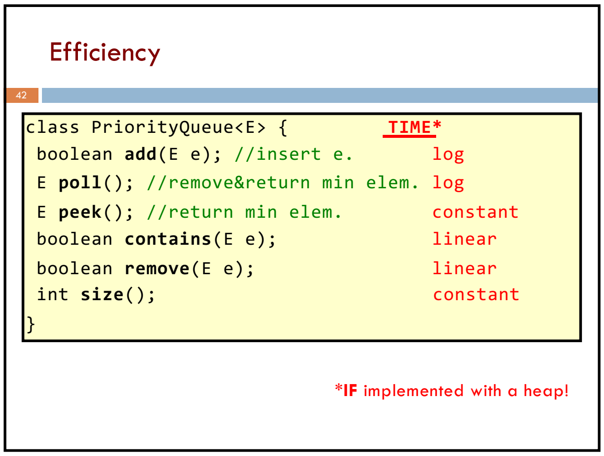


## poll()

```
/** Return index of bigger child of node k */
public int biggerChild(int k) {
   int c= 2*k + 2;      // k's right child
   if (c >= n || b[c-1] > b[c])
       c= c-1;
   return c;
}
```



## Piazza Poll #2



## Efficiency

```
class PriorityQueue<E> {             TIME*
 boolean add(E e); //insert e.        log
 E poll(); //remove&return min elem.  log
 E peek(); //return min elem.         constant
 boolean contains(E e);               linear
 boolean remove(E e);                 linear
 int size();                          constant
}
```

*IF implemented with a heap!

# Heapsort

(if time, in JavaHyperText if not)

---

## Heapsort

| 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 55 | 4 | 12 | 6 | 14 |

Goal: sort this array in place

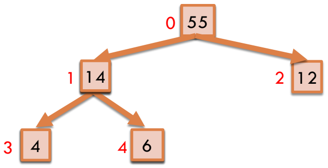
Approach: turn the array into a heap and then poll repeatedly

---

## Heapsort

```
// Make b[0..n-1] into a max-heap (in place)
```

| 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 55 | 14 | 12 | 4 | 6 |

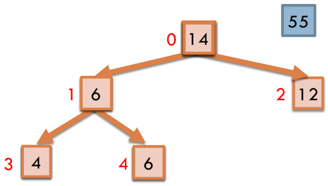


---

## Heapsort

```
// Make b[0..n-1] into a max-heap (in place)
// inv:   b[0..k] is a heap, b[0..k] <= b[k+1..], b[k+1..] is sorted
    for (k= n-1; k > 0; k= k-1) {
          b[k]= poll  – i.e., take max element out of heap.
    }
```

| 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 14 | 6 | 12 | 4 | 55 |



---

## Heapsort

```
// Make b[0..n-1] into a max-heap (in place)
// inv:   b[0..k] is a heap, b[0..k] <= b[k+1..], b[k+1..] is sorted
    for (k= n-1; k > 0; k= k-1) {
          b[k]= poll  – i.e., take max element out of heap.
    }
```

| 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 4 | 6 | 12 | 14 | 55 |