HEAPS & PRIORITY QUEUES

Lecture 13

CS2110 Spring 2018

Announcements

- A4 goes out today!
- Prelim 1:
  - regrades are open
  - a few rubrics have changed
- No Recitations next week (Fall Break Mon & Tue)
- We'*ll* spend Fall Break taking care of loose ends

Abstract vs **concrete** data structures

- Abstract data structures are **interfaces**
  - specify only **interface** (method names and specs)
  - not **implementation** (method bodies, fields, …)
  - Have multiple possible implementations

- Concrete data structures are **classes**
  - These **are** the multiple possible implementations

Abstract data structures (the interfaces)

Interface	**definition**
List	an ordered collection (aka sequence)
Set	collection that contains no duplicate elements
Map	maps keys to values, no duplicate keys
Stack	a last-in-first-out (LIFO) stack of objects
Queue	collection for holding elements prior to processing
Priority Queue	*later this lecture!*

These definitions specify an interface for the user.
How you implement them is up to you!

Abstract data structures made concrete

Interface	Class (implementation)
List	ArrayList, LinkedList
Set	HashSet, TreeSet
Map	HashMap, TreeMap
Stack	can be done with a LinkedList
Queue	can be done with a LinkedList

2 classes that both implement List

- **List** is the **interface** (“abstract data type”)
  - has methods: add, get, remove, …
- These **2 classes** implement List ("concrete data types"):

Class:	ArrayList	LinkedList
Backing storage:	array	chained nodes
add(i, val)	O(n)	O(n)
add(0, val)	O(n)	O(1)
add(n, val)	O(1)	O(1)
get(i)	O(1)	O(n)
get(0)	O(1)	O(1)
get(n)	O(1)	O(1)

Priority Queue

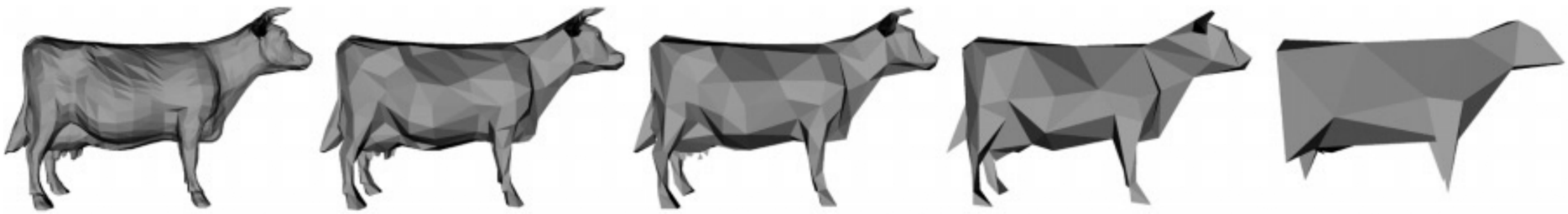
Unbounded queue with ordered elements

→ data items are **`Comparable`** (ties broken arbitrarily)

Priority order: **smaller** (determined by **`compareTo()`**) have **higher priority**

`remove()` : remove and return element with highest priority

Many uses of priority queues



Surface simplification [Garland and Heckbert 1997]

- Event-driven simulation: customers in a line
- Collision detection: "next time of contact" for colliding bodies
- Graph searching: Dijkstra's algorithm, Prim's algorithm
- AI Path Planning: A* search
- Statistics: maintain largest M values in a sequence
- Operating systems: load balancing, interrupt handling
- Discrete optimization: bin packing, scheduling
- College: prioritizing assignments for multiple classes.

java.util.PriorityQueue<E>


```
interface PriorityQueue<E> {
 boolean add(E e); //insert e.
 E poll(); //remove/return min elem.
 E peek() //return min elem.
 void clear() //remove all elems.
 boolean contains(E e);
 boolean remove(E e);
 int size();
 Iterator<E> iterator();
}
```

Priority queues can be maintained as:

A list

- `add()` put new element at front – O(1)
- `poll()` must search the list – O(n)
- `peek()` must search the list – O(n)

An ordered list

- `add()` must search the list – O(n)
- `poll()` min element at front – O(1)
- `peek()` O(1)

A red-black tree (*we'll cover later!*)

- `add()` must search the tree & rebalance – O(log n)
- `poll()` must search the tree & rebalance – O(log n)
- `peek()` O(log n)

Can we do better?

A Heap..

Is a binary tree satisfying 2 properties

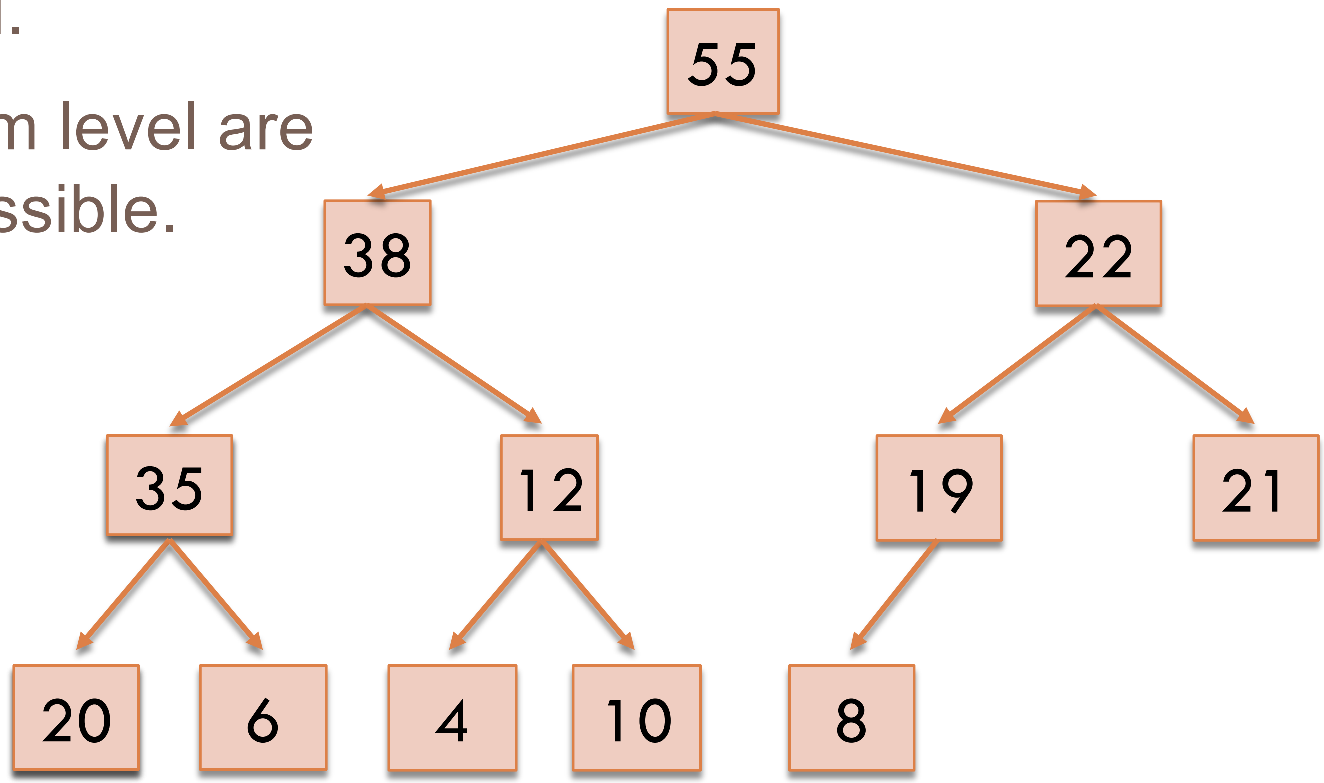
1) **Completeness.** Every level of the tree (except last) is completely filled, and on last level nodes are as far left as possible.

Do not confuse with heap memory, where a process dynamically allocates space–different usage of the word heap.

Completeness Property

Every level (except last) completely filled.

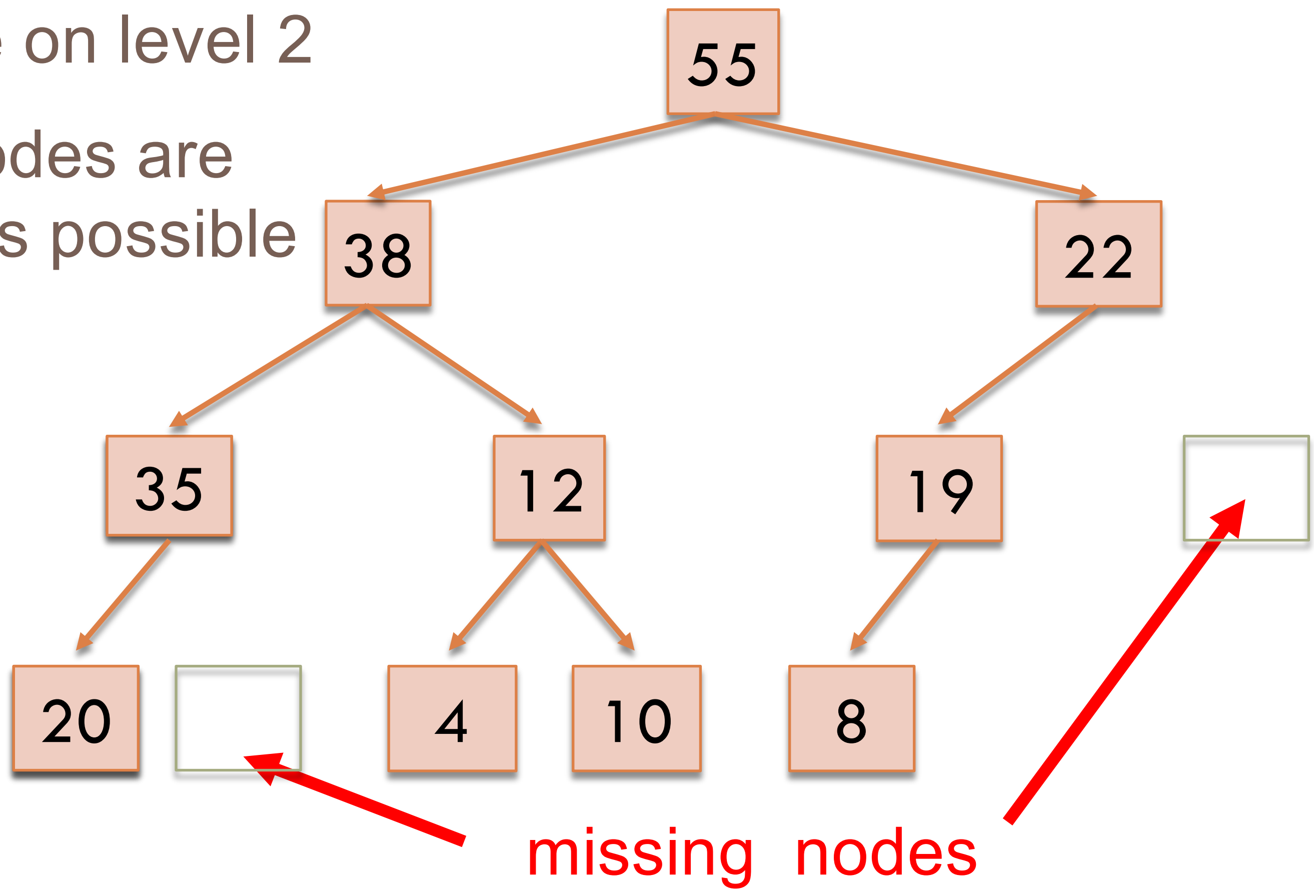
Nodes on bottom level are as far left as possible.



Completeness Property

Not a heap because:

- missing a node on level 2
- bottom level nodes are not as far left as possible



A Heap..

Is a binary tree satisfying 2 properties

1) **Completeness.** Every level of the tree (except last) is completely filled, and on last level nodes are as far left as possible.

2) **Heap Order Invariant.** "max on top"

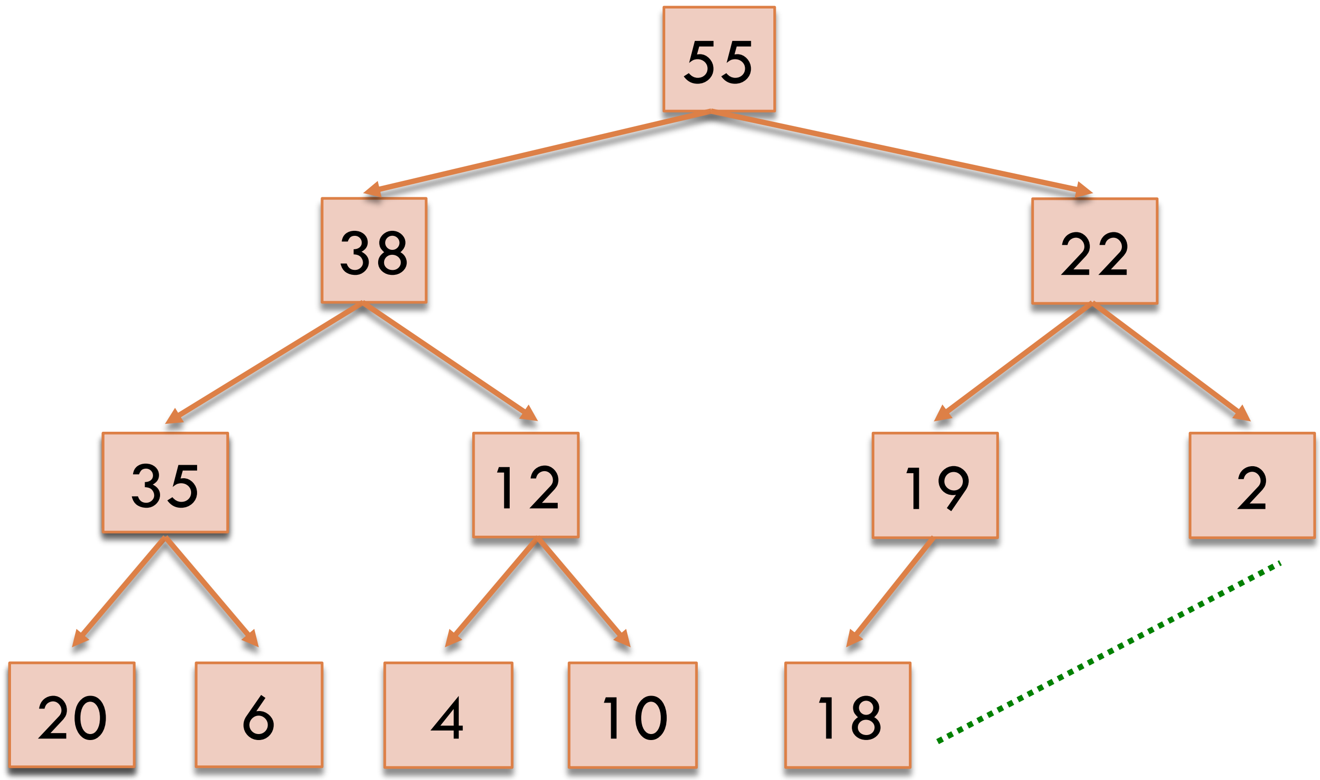
Max-Heap: every element in tree is <= its parent

Min-Heap: every element in tree is >= its parent

"min on top"

Order Property (max-heap)

Every element is <= its parent



Note: Bigger elements **can** be deeper in the tree!

Heap Quiz #1

A Heap..

Is a binary tree satisfying 2 properties

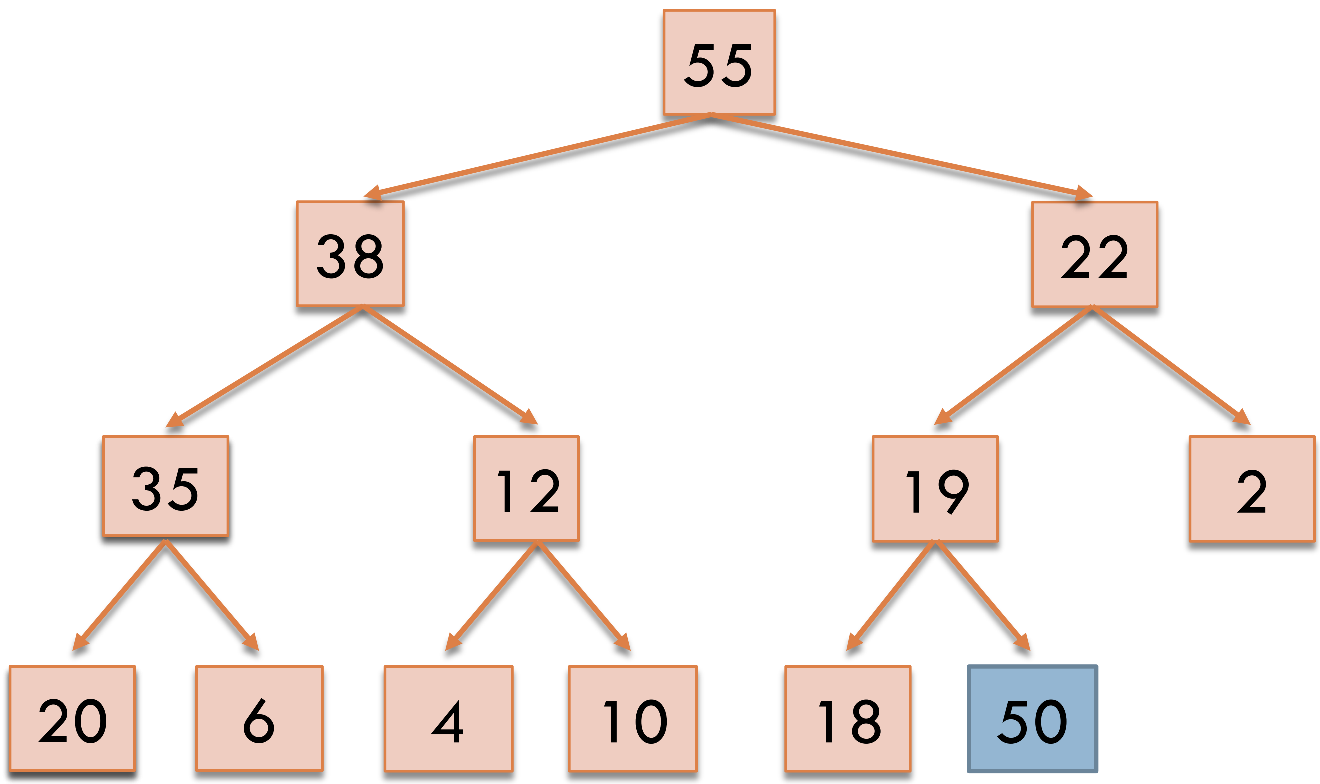
1) **Completeness.** Every level of the tree (except last) is completely filled. All holes in last level are all the way to the right.
2) **Heap Order Invariant.**

   Max-Heap: every element in tree is <= its parent

Implements 3 key methods:

1) **add**(e): add a new element to the heap
2) **poll**(): delete the max element and returns it
3) **peek**(): return the max element

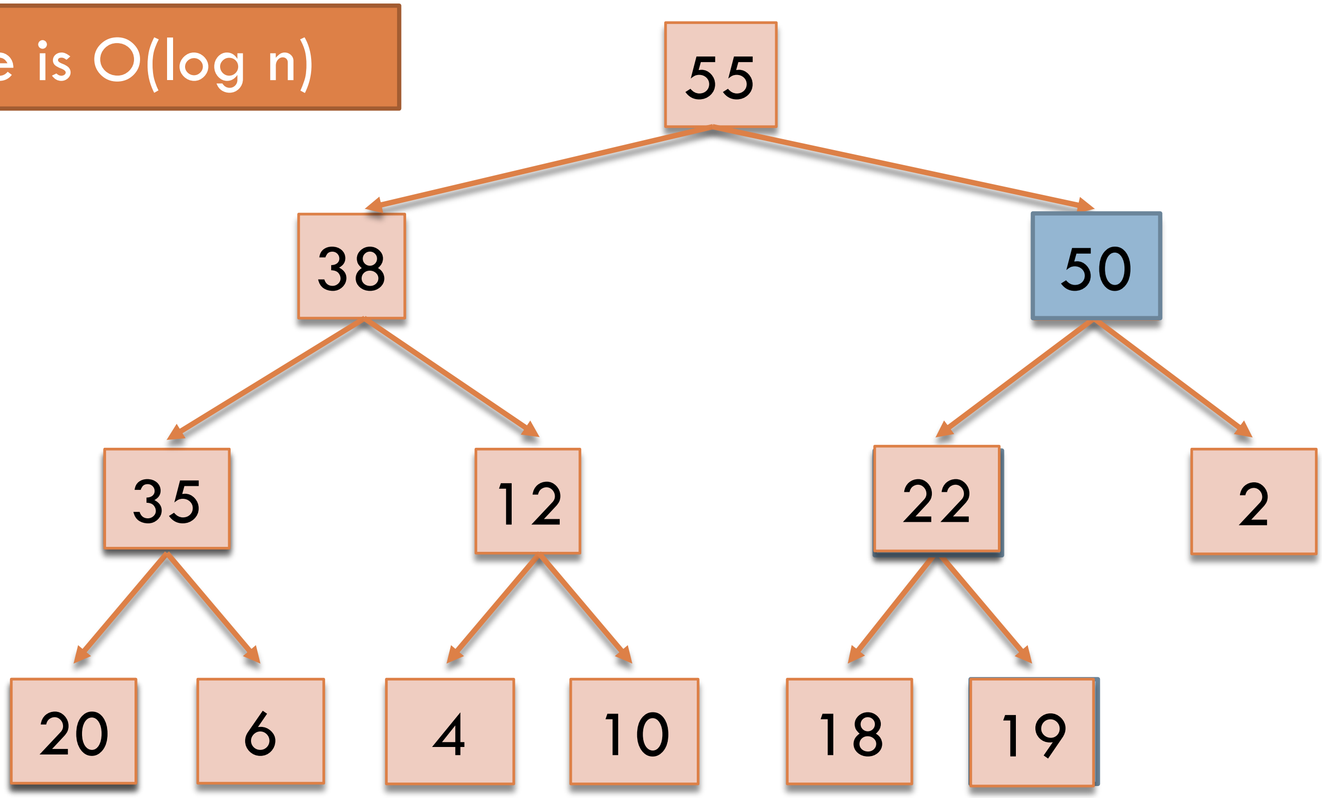
Heap: add(e)



1. Put in the new element in a new node (leftmost empty leaf)

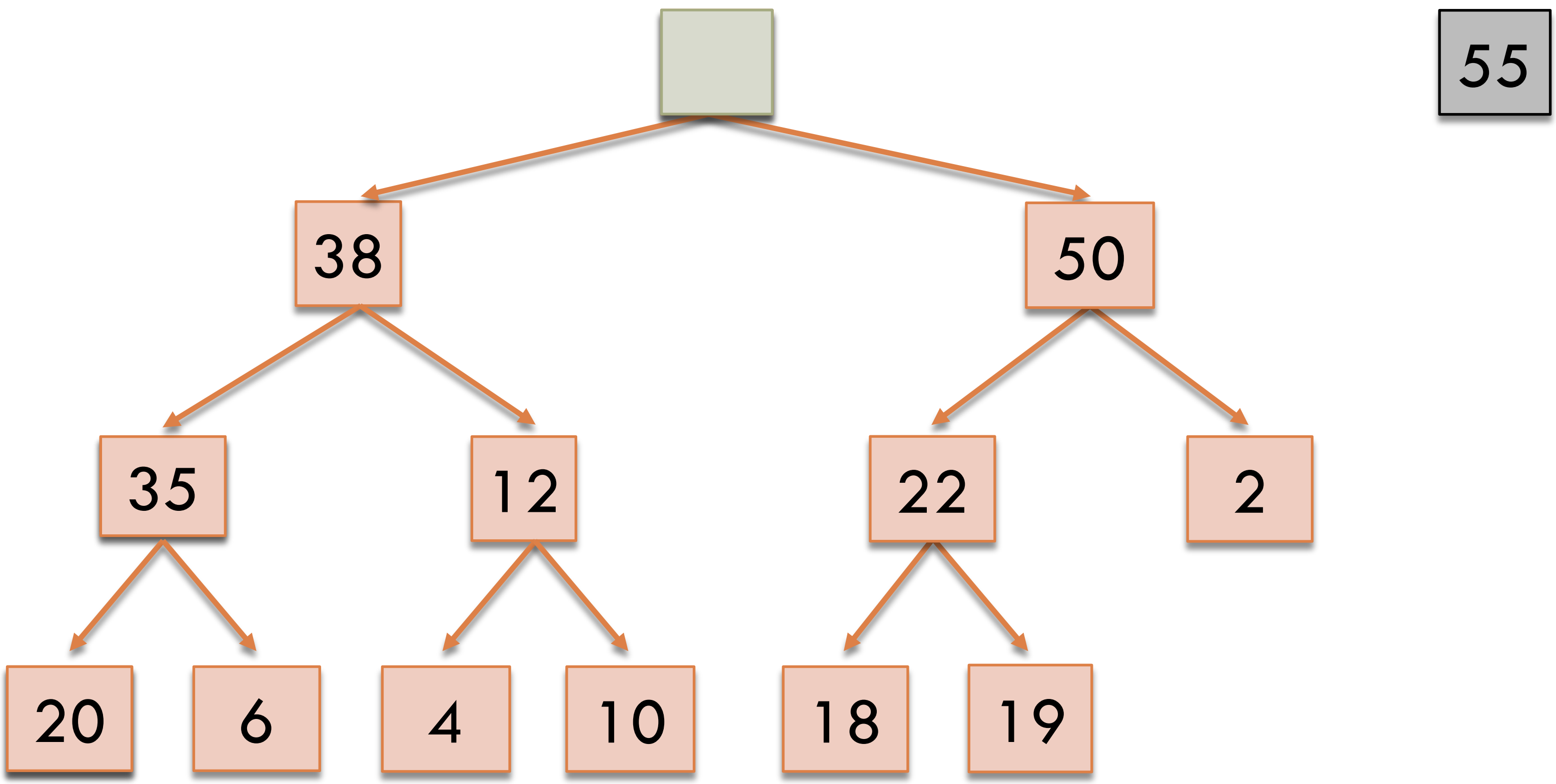
Heap: add(e)

Time is O(log n)



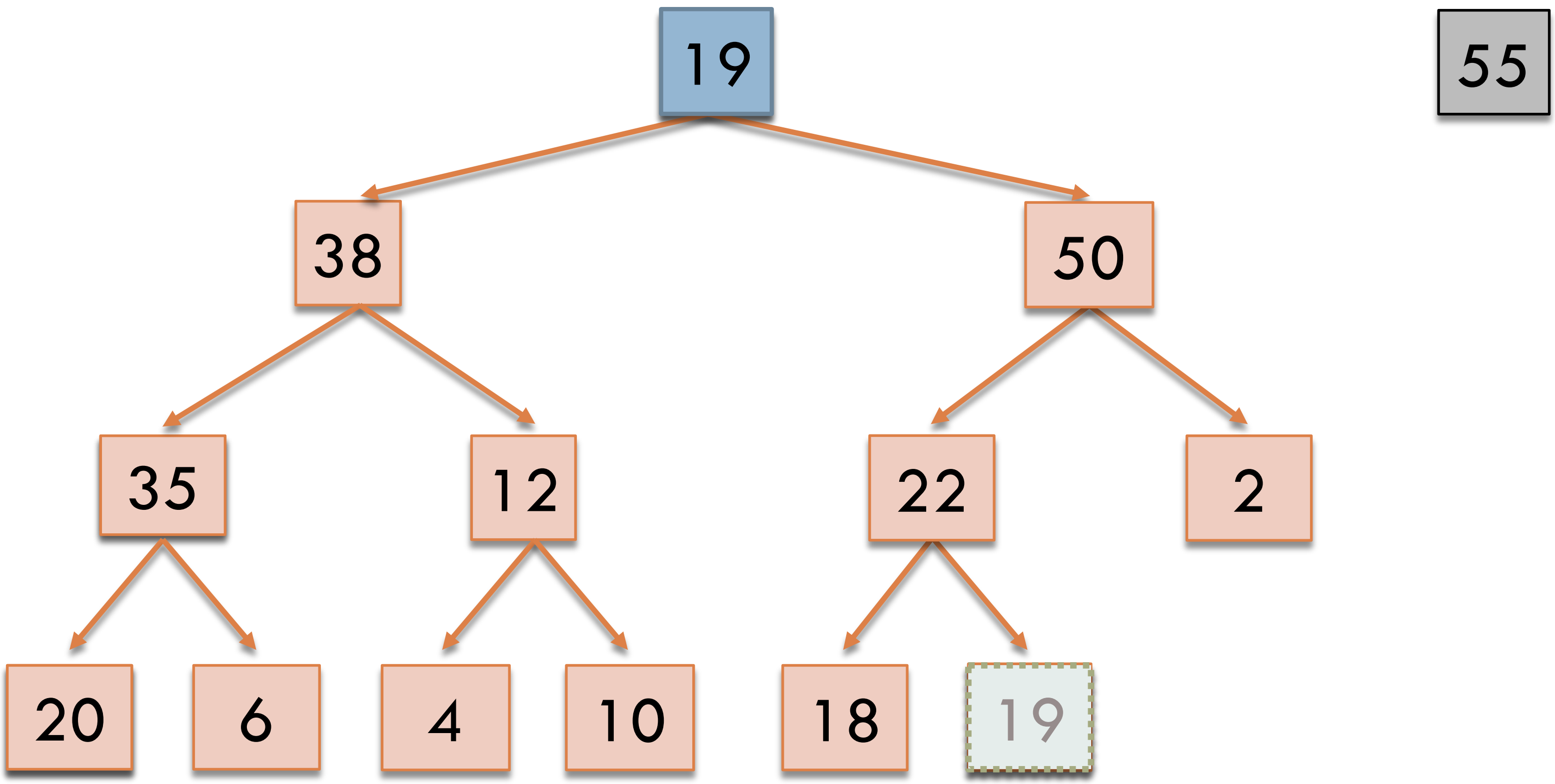
1. Put in the new element in a new node (leftmost empty leaf)
2. Bubble new element up while greater than parent

Heap: poll()



1. Save root element in a local variable

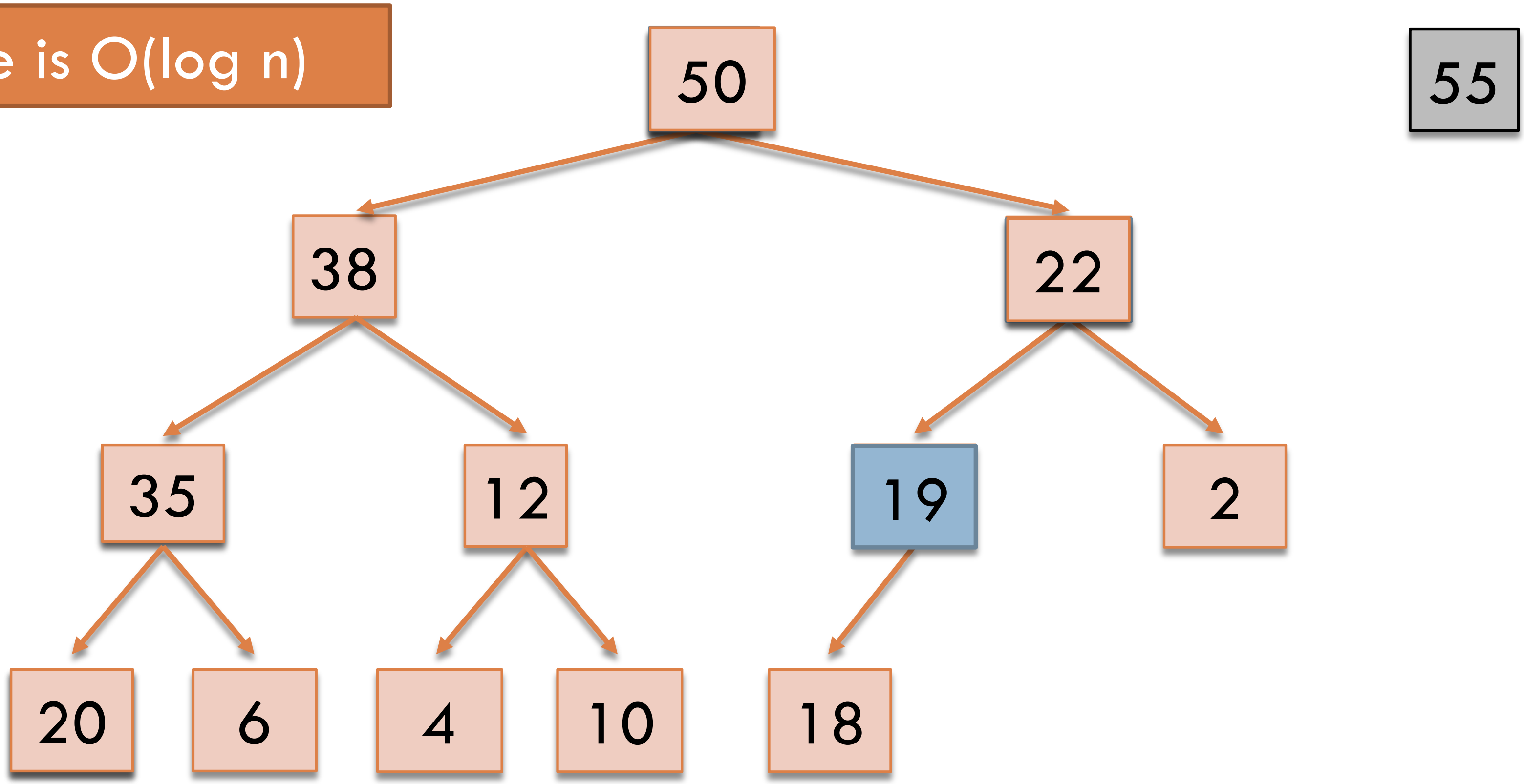
Heap: poll()



1. Save root element in a local variable
2. Assign last value to root, delete last node.

Heap: poll()

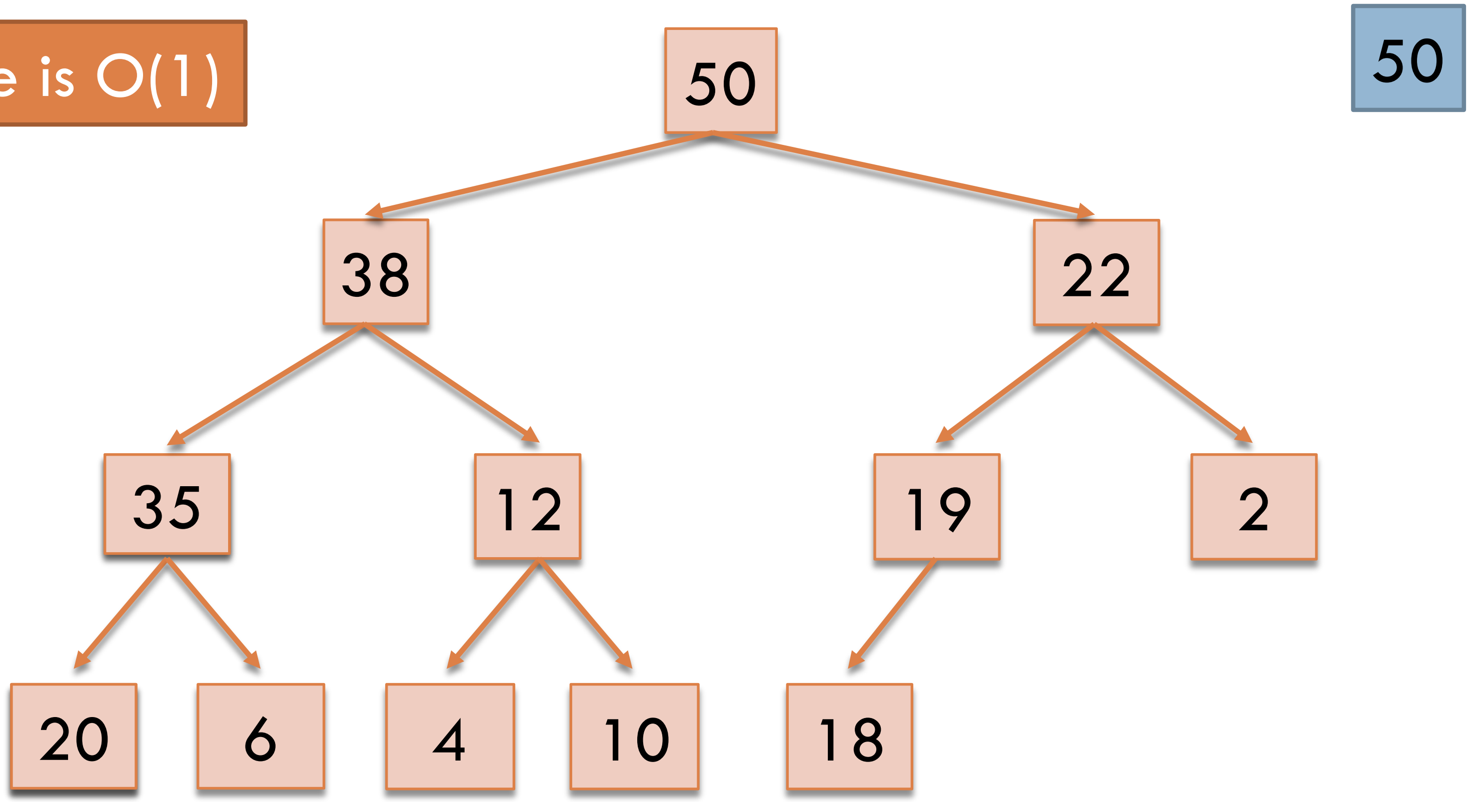
Time is O(log n)



1. Save root element in a local variable
2. Assign last value to root, delete last node.
3. While less than a child, switch with bigger child (bubble down)

Heap: peek()

Time is O(1)



1. Return root value

Implementing Heaps


```
public class HeapNode<E> {
  private E value;
  private HeapNode left;
  private HeapNode right;
  ...
}
```

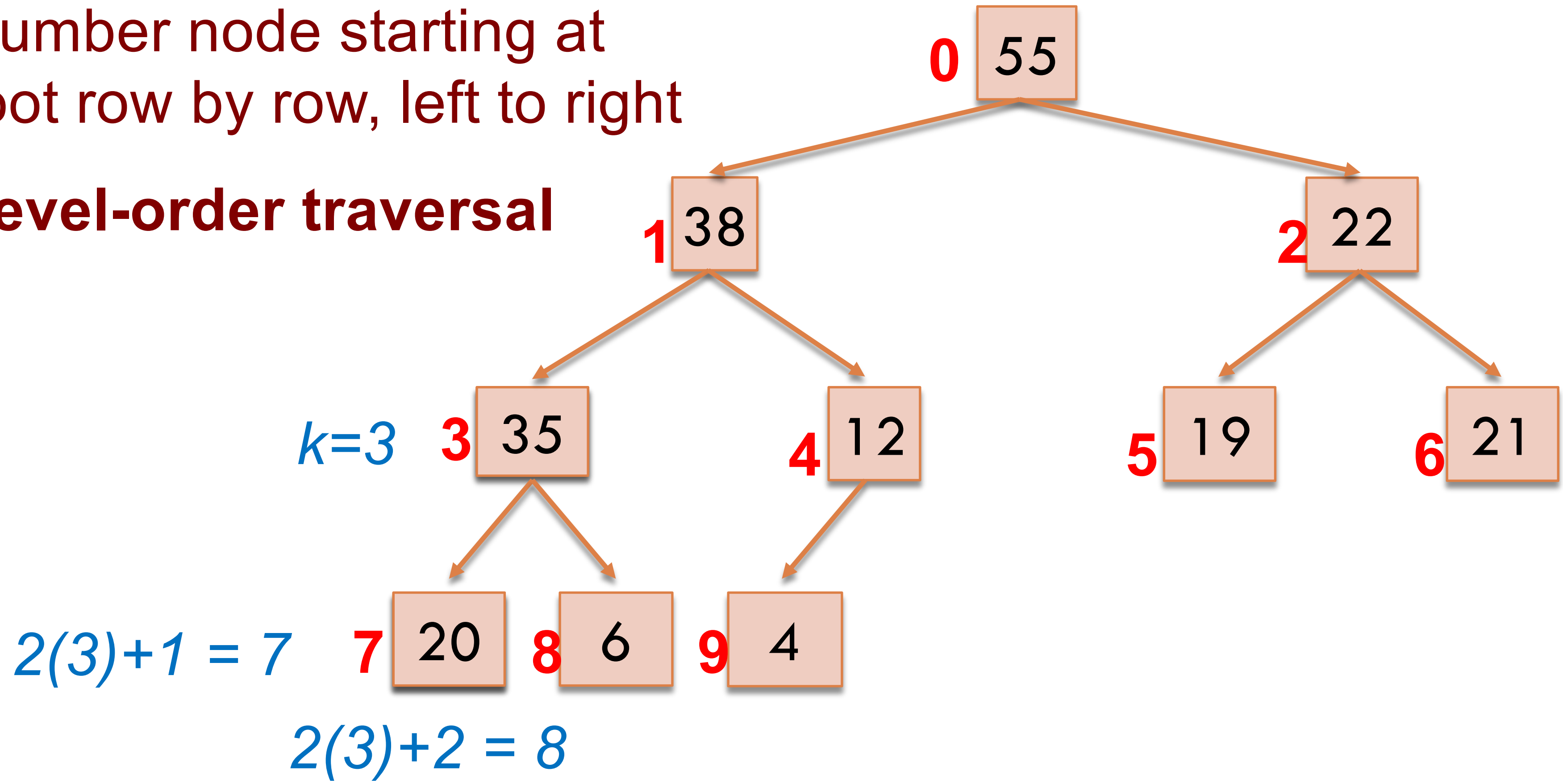
Implementing Heaps


```
public class Heap<E> {
  private E[] heap;
  ...
}
```

Numbering the nodes

Number node starting at root row by row, left to right

Level-order traversal

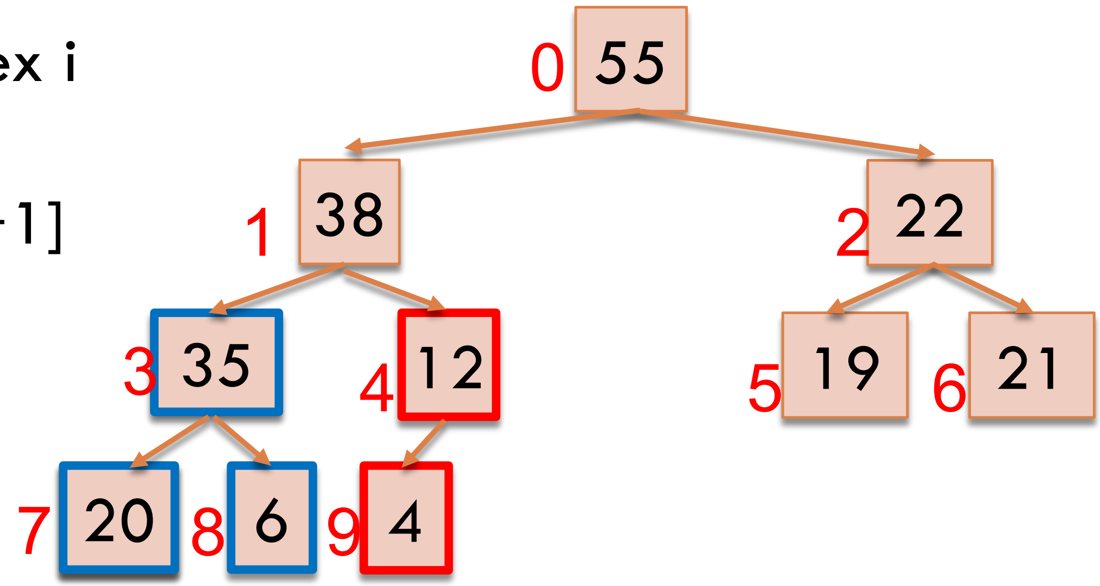


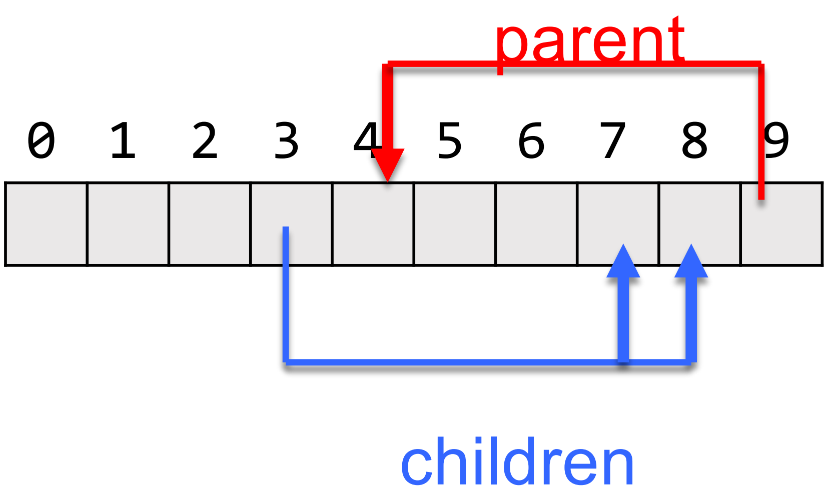
Children of node k are nodes 2k+1 and 2k+2

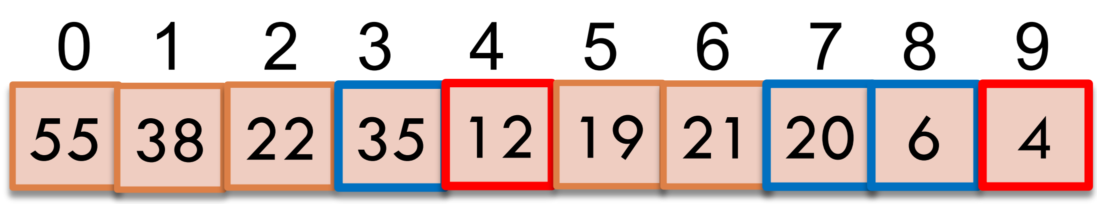
Parent of node k is node (k-1)/2

Storing a heap in an array

- Store node number i in index i of array b
- Children of b[k] are b[2k +1] and b[2k +2]
- Parent of b[k] is b[(k-1)/2]







add() (assuming there is space)


```
/** An instance of a heap */
class Heap<E> {
  E[] b= new E[50];  // heap is b[0..n-1]
  int n= 0;          // heap invariant is true

  /** Add e to the heap */
  public void add(E e) {
    b[n]= e;
    n= n + 1;
    bubbleUp(n - 1); // given on next slide
  }
}
```

add(). Remember, heap is in b[0..n-1]


```
class Heap<E> {
  /** Bubble element #k up to its position.
    * Pre: heap inv holds except maybe for k */
  private void bubbleUp(int k) {
    int p= (k-1)/2;
    // inv: p is parent of k and every elmnt
    // except perhaps k is <= its parent
    while (k > 0  &&  b[k].compareTo(b[p]) > 0) {
        swap(b[k], b[p]);
        k= p;
        p= (k-1)/2;
    }
}
```

poll(). Remember, heap is in b[0..n-1]


```
/** Remove and return the largest element
  * (return null if list is empty) */
public E poll() {
    if (n == 0) return null;
    E v=  b[0];   // largest value at root.
    n= n – 1;     // move last
    b[0]= b[n];   // element to root
    bubbleDown(0);
    return v;
}
```

poll()


```
/** Tree has n node.
 *  Return index of bigger child of node k
    (2k+2 if k >= n) */
public int biggerChild(int k, int n) {
   int c= 2*k + 2;     // k's right child
   if (c >= n || b[c-1] > b[c])
      c= c-1;
   return c;
}
```

poll()


```
/** Bubble root down to its heap position.
    Pre: b[0..n-1] is a heap except maybe b[0] */
private void bubbleDown() {
    int k= 0;
    int c= biggerChild(k, n);
   // inv: b[0..n-1] is a heap except maybe b[k] AND
   //      b[c] is b[k]'s biggest child
   while ( c < n &&  b[k] <) b{[c]

      swap(b[k], b[c]);
      k= c;
      c= biggerChild(k, n);
   }
}
```

peek(). Remember, heap is in b[0..n-1]


```
/** Return largest element
  * (return null if list is empty) */
public E poll() {
    if (n == 0) return null;
    return b[0];   // largest value at root.
}
```

Heap Quiz #2

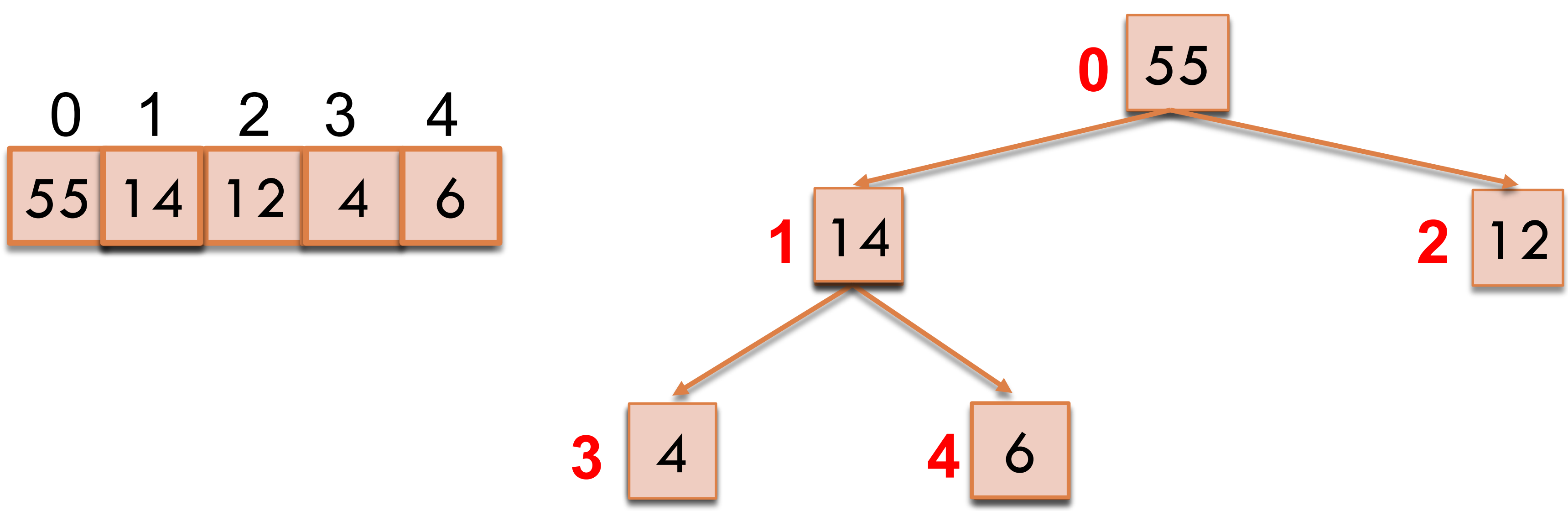
HeapSort

0	1	2	3	4
55	4	12	6	14

Goal: sort this array in place

HeapSort

// Make b[0..n-1] into a max-heap (in place)

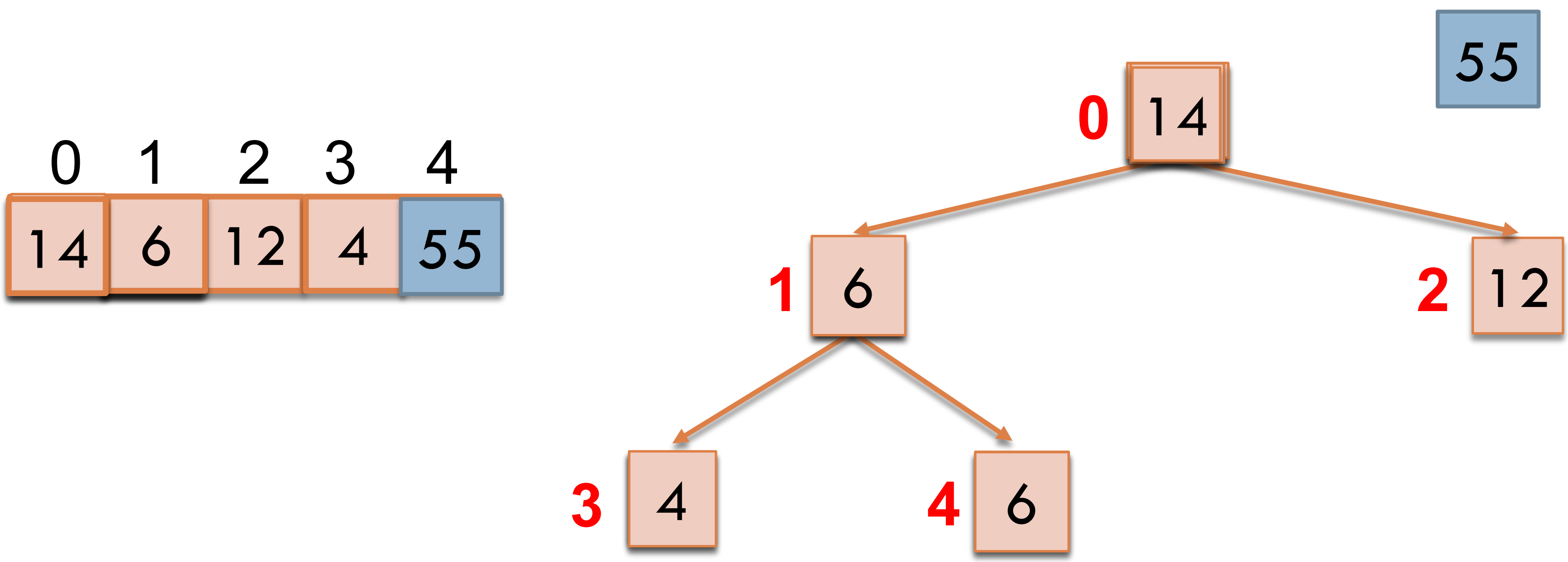


HeapSort

// Make b[0..n-1] into a max-heap (in place)

// inv: b[0..k] is a heap, b[0..k] <= b[k+1..], b[k+1..] is sorted

```
for (k= n-1; k > 0; k= k-1) {
        b[k]= poll – i.e., take max element out of heap.
}
```



HeapSort

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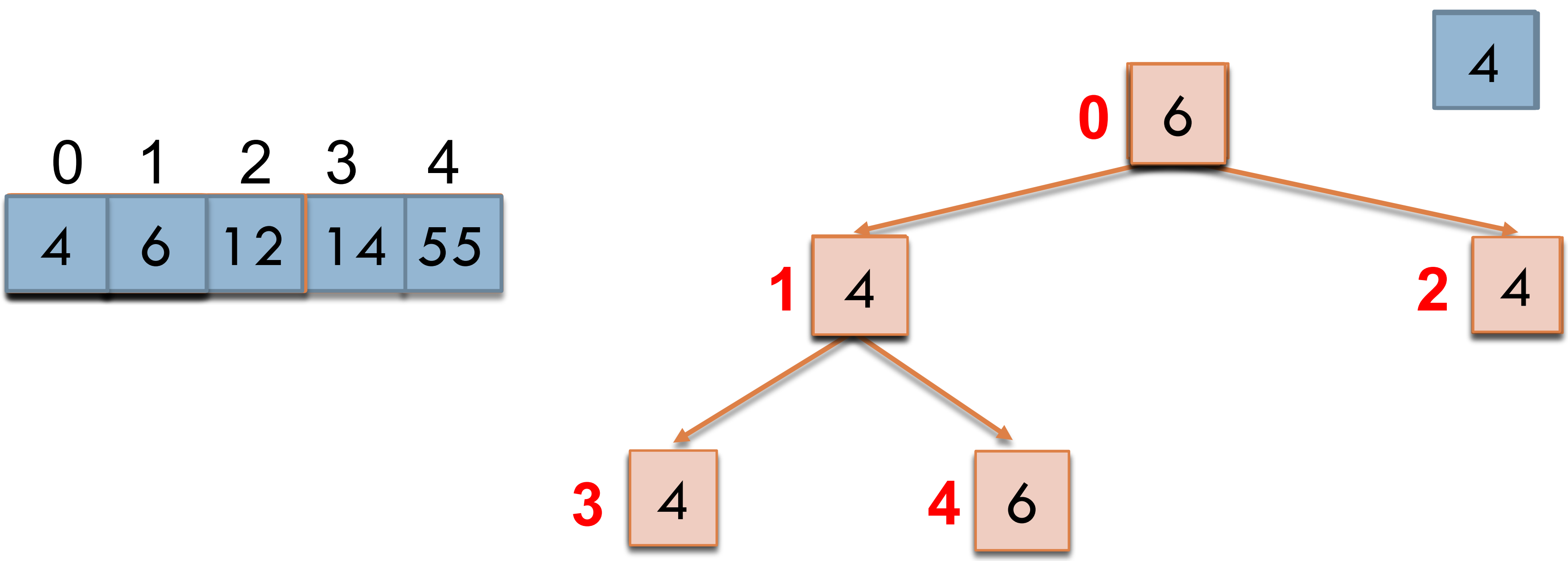
// inv: b[0..k] is a heap, b[0..k] <= b[k+1..], b[k+1..] is sorted

for (k= n-1; k > 0; k= k-1) {

b[k]= poll – *i.e.*, take max element out of heap.

}



Priority queues as heaps

- A *heap* can be used to implement priority queues
  - Note: need a min-heap instead of a max-heap
- Gives better complexity than either ordered or unordered list implementation:
  - **`add()`**: O(log n) (n is the size of the heap)
  - **`poll()`**: O(log n)
  - **`peek()`**: O(1)

java.util.PriorityQueue<E>


```
interface PriorityQueue<E> {                        TIME*
 boolean add(E e); //insert e.                      log
 void clear(); //remove all elems.
 E peek(); //return min elem.                       constant
 E poll(); //remove/return min elem.                log
 boolean contains(E e);                             linear
 boolean remove(E e);                               linear
 int size();                                        constant
 Iterator<E> iterator();
}
```

*IF implemented with a heap!

What if priority is independent from the value?

Separate priority from value and do this:

add(e, p); //add element e with priority p (a double)

THIS IS EASY!

Be able to change priority

change(e, p); //change priority of e to p

THIS IS HARD!

Big question: How do we find e in the heap?
Searching heap takes time proportional to its size! No good!
Once found, change priority and bubble up or down. OKAY

Assignment A4: implement this heap! Use a second data structure to make change-priority expected log n time