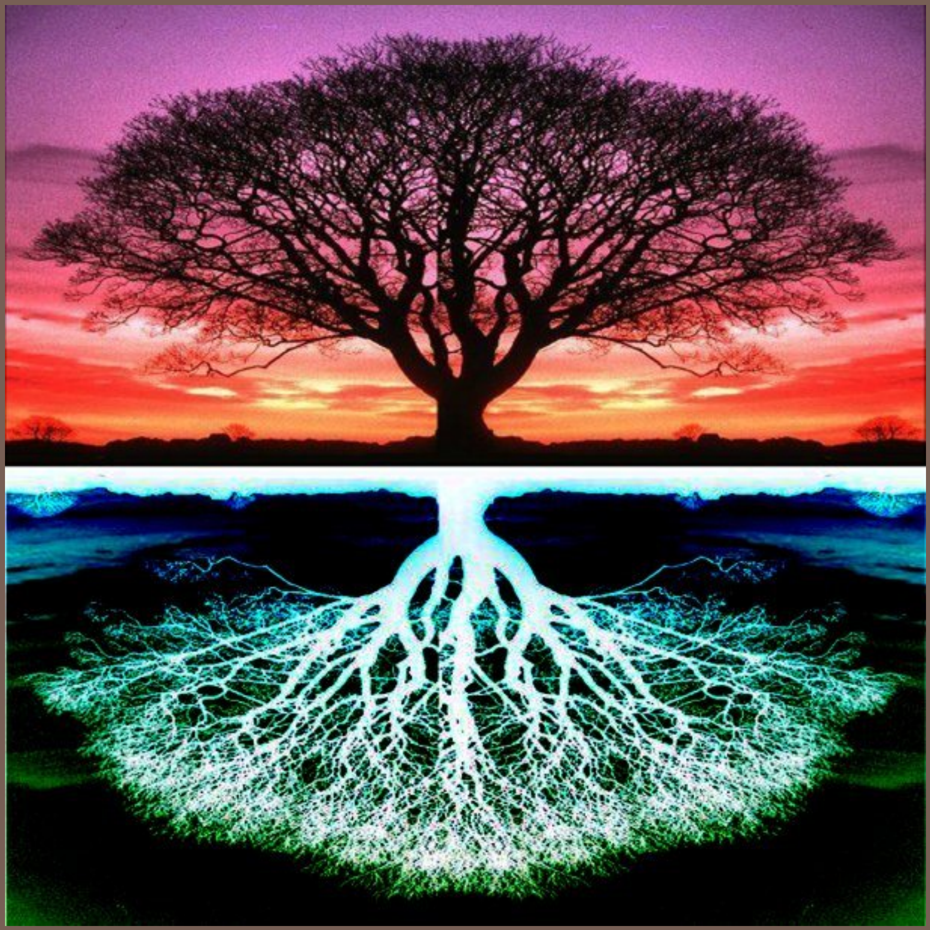


TREES

Lecture 12

CS2110 – Fall 2018

Prelim Updates

- Regrades are live until next Thursday @ 11:59PM
- A few rubric changes are happening
  - Recursion question: -0pts if you continued to print
  - Exception handling "write the output of execution of that statement" – rubrics change in place

Data Structures

- There are different ways of storing data, called **data structures**
- Each data structure has operations that it is good at and operations that it is bad at
- For any application, you want to choose a data structure that is good at the things you do often

Example Data Structures

Data Structure	add(val v)	get(int i)	contains(val v)
Array 2 | 1 | 3 | 0	$O(n)$	$O(1)$	$O(n)$
Linked List 2 → 1 → 3 → 0	$O(1)$	$O(n)$	$O(n)$

`add(v)`: append v to this list

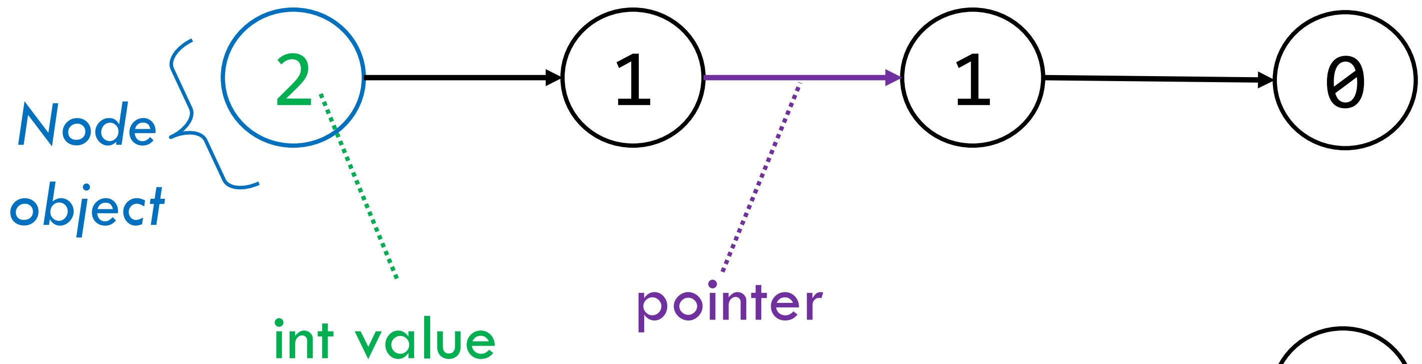
`get(i)`: return element at position i in this list

`contains(v)`: return true if this list contains v

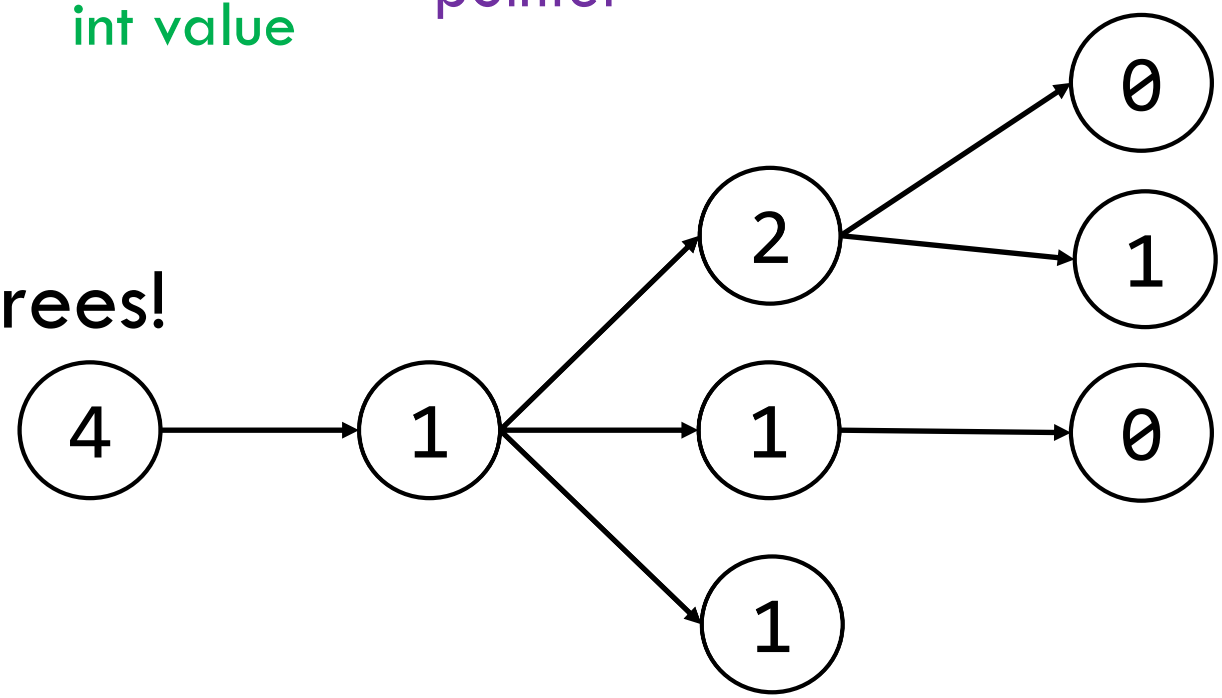
AKA add, lookup, search

Tree

Singly linked list:



Today: trees!

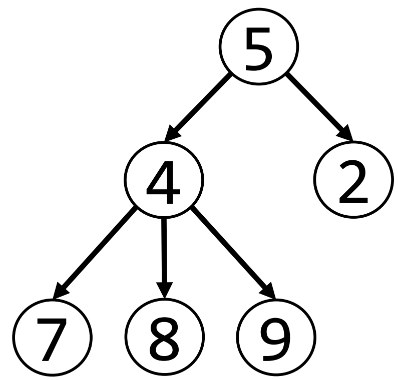


Tree Overview

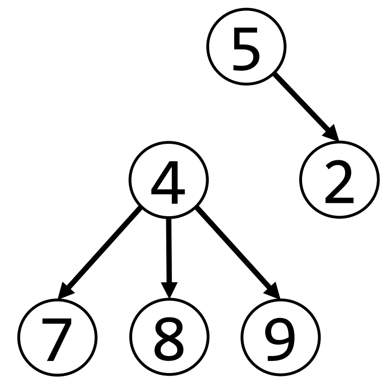
Tree: data structure with nodes, similar to linked list

- Each node may have zero or more *successors* (children)
- Each node has exactly one *predecessor* (parent) except the *root*, which has none
- All nodes are reachable from *root*

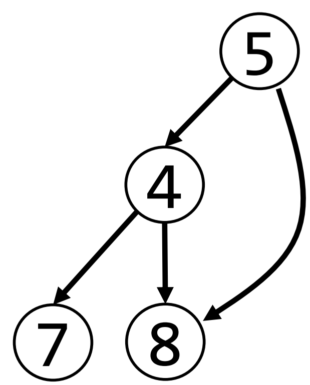
A tree or not a tree?



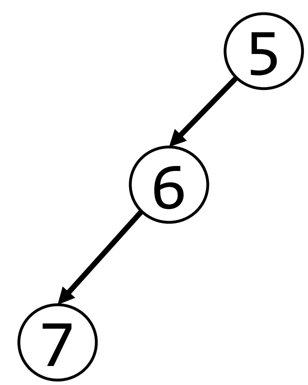
A tree



Not a tree

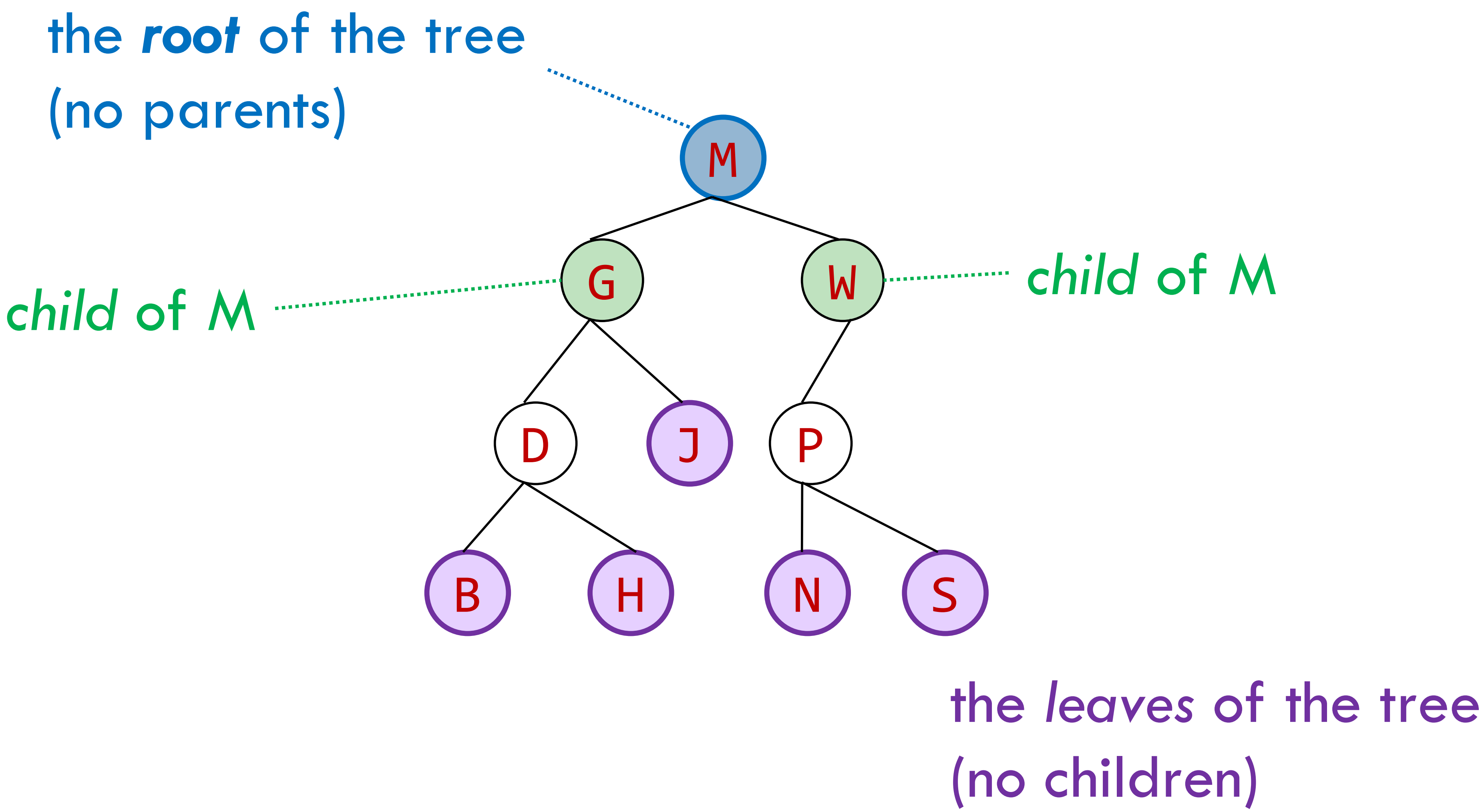


Not a tree

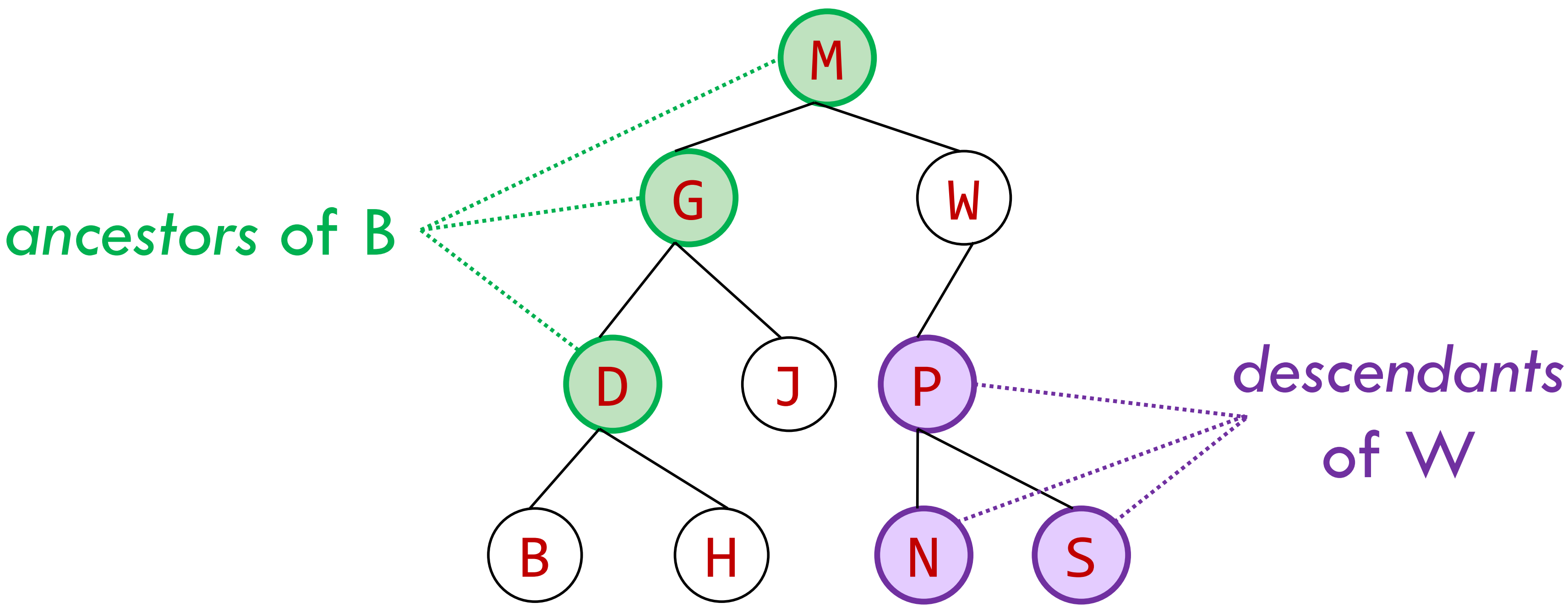


A tree

Tree Terminology (1)

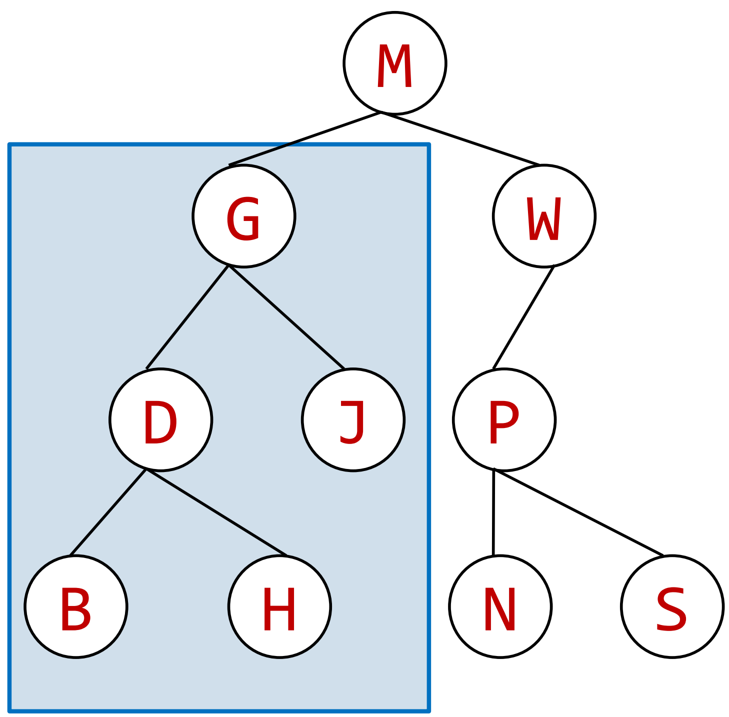


Tree Terminology (2)



Tree Terminology (3)

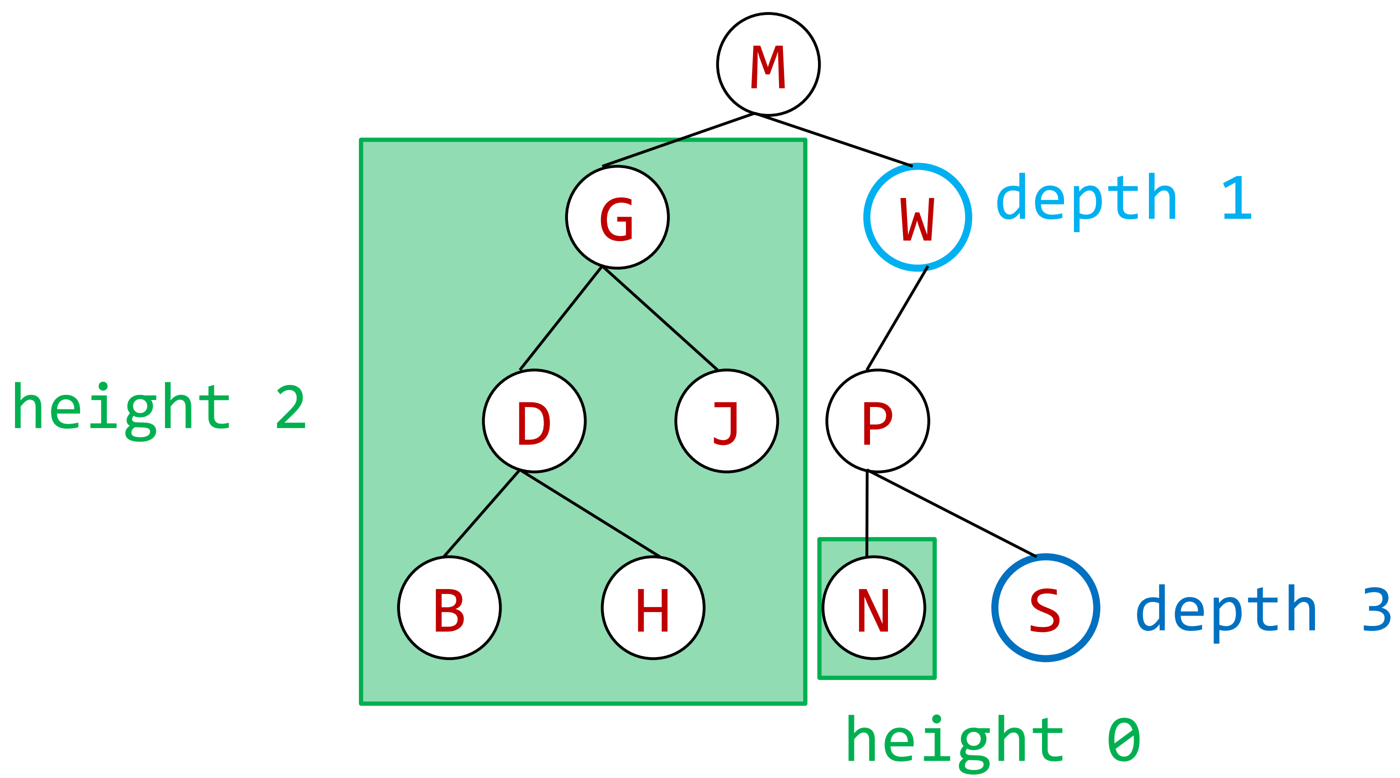
subtree of M



Tree Terminology (4)

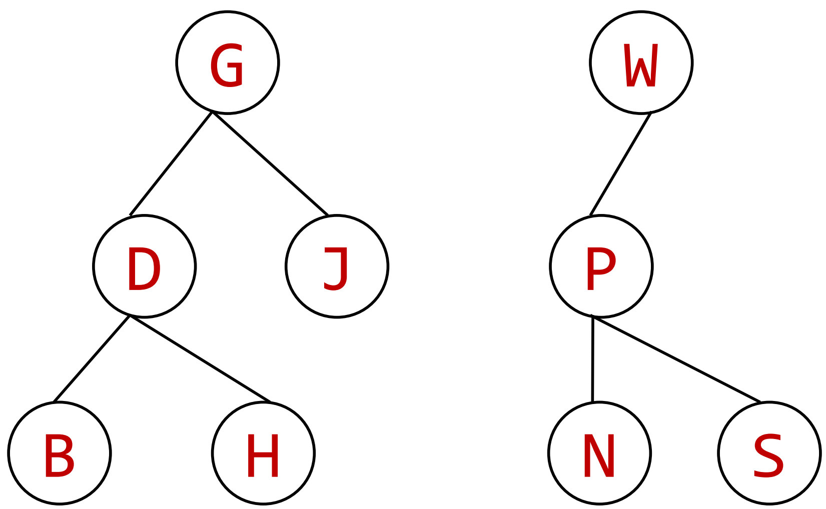
A node's ***depth*** is the length of the path to the root.

A tree's (or subtree's) ***height*** is the length of the longest path from the root to a leaf.



Tree Terminology (5)

Multiple trees: a ***forest***

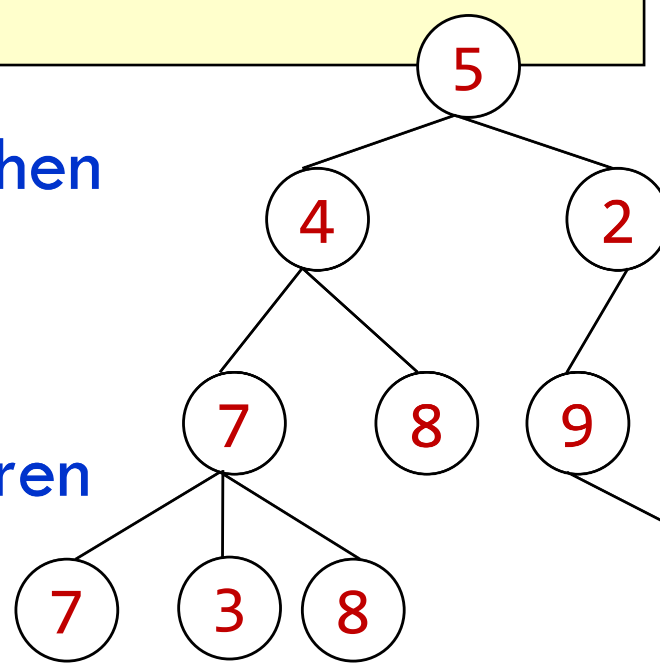


Class for general tree nodes

```
class GTreeNode<T> {
    private T value;
    private List<GTreeNode<T>> children;
    //appropriate constructors, getters,
    //setters, etc.
}
```

<T> means user picks a type when they create one (later lecture)

Parent contains a list of its children



General tree

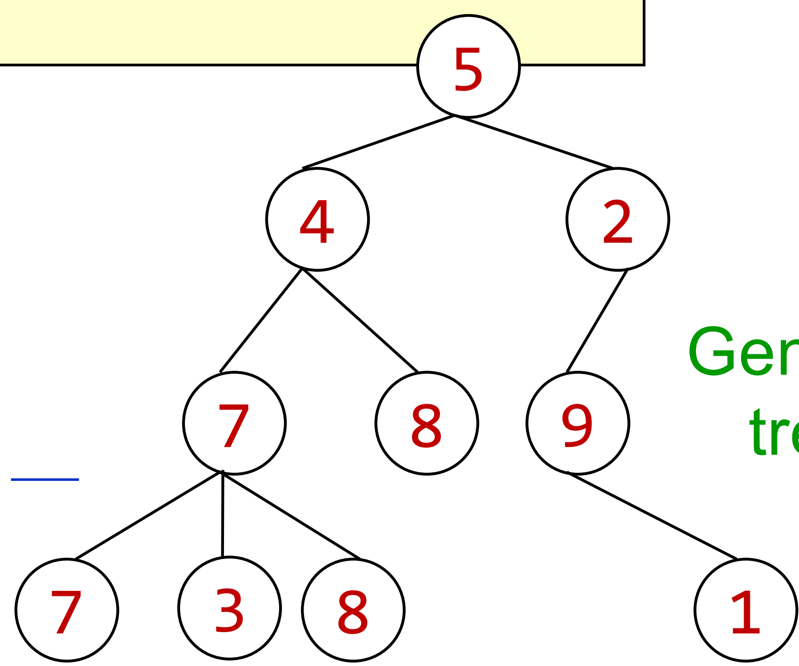
Class for general tree nodes

```
class GTreeNode<T> {
    private T value;
    private List<GTreeNode<T>> children;
    //appropriate constructors, getters,
    //setters, etc.
}
```

Java.util.List is an interface!

It defines the methods that all implementations must implement.

Whoever writes this class gets to decide what implementation to use — ArrayList? LinkedList? Etc.?

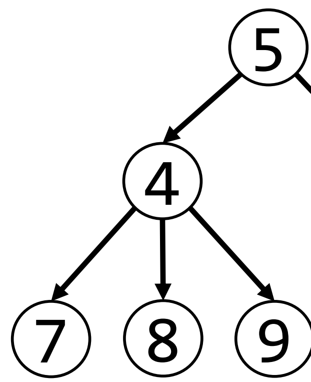


General tree

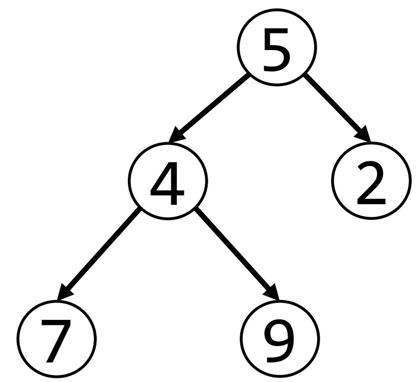
Binary Trees

A *binary tree* is a particularly important kind of tree in which every node as at most two children.

In a binary tree, the two children are called the *left* and *right* children.



Not a binary tree (a *general* tree)



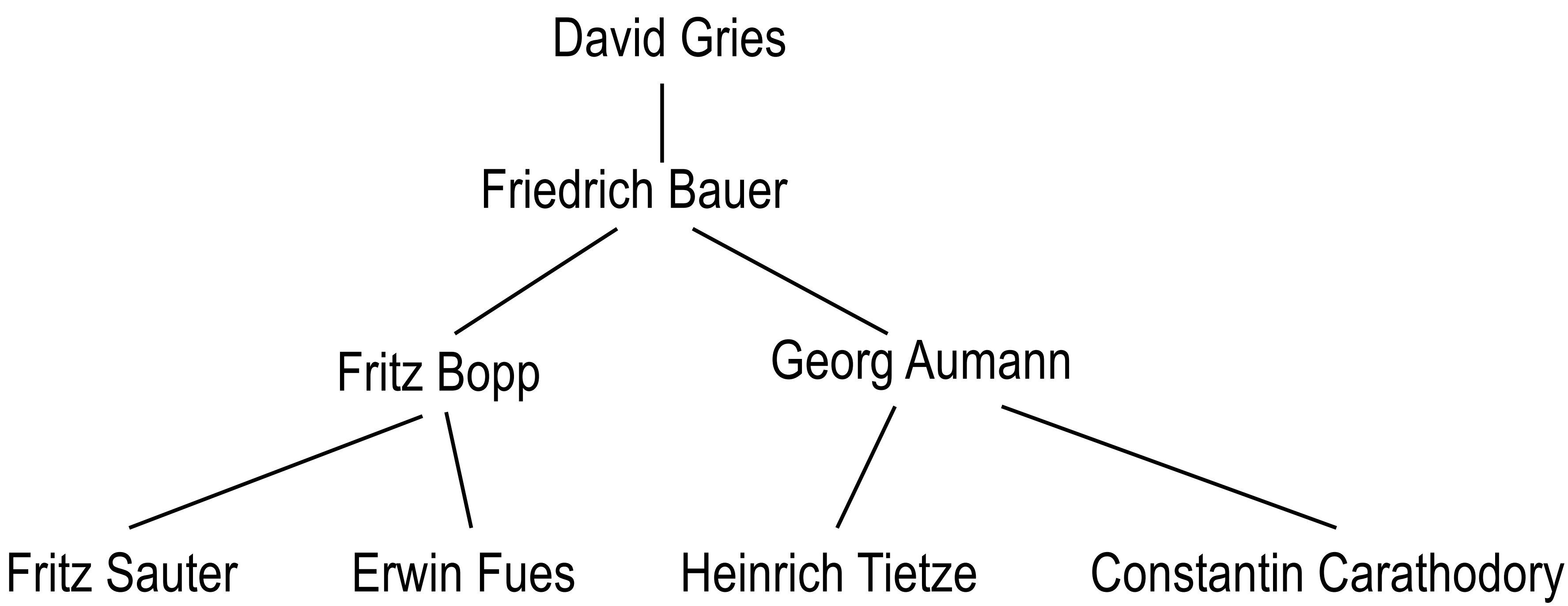
Binary tree

Binary trees were in A1!

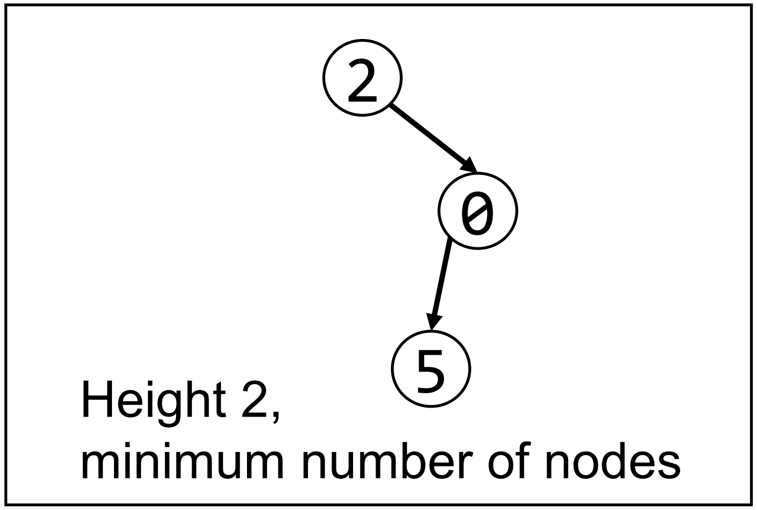
You have seen a binary tree in A1.

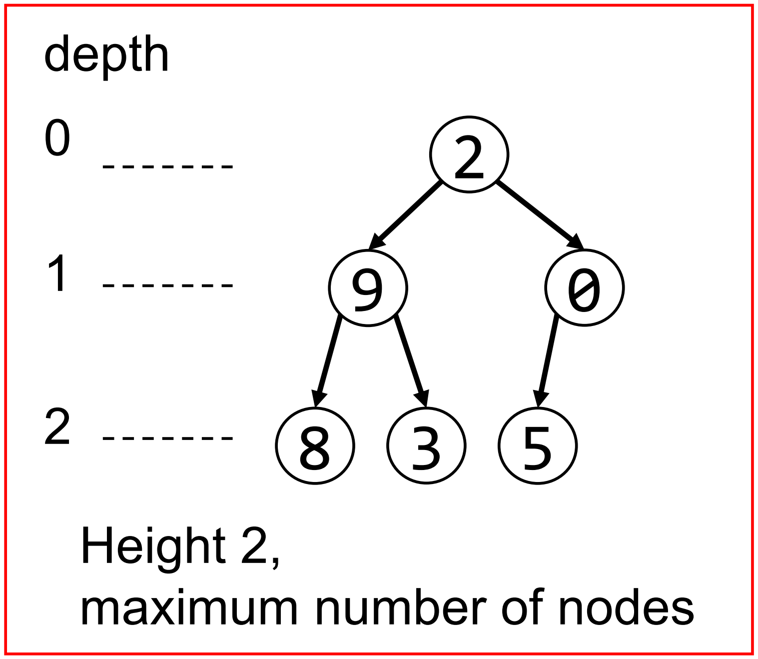
A PhD object has one or two advisors.

(Note: the advisors are the “children”.)



Useful facts about binary trees





Max # of nodes at depth d: 2^d

If height of tree is h:

- min # of nodes: $h + 1$
- max #of nodes:
- $2^0 + \ldots + 2^h = 2^{h+1} - 1$

Complete binary tree

Every level, except last, is completely filled, nodes on bottom level as far left as possible. No holes.

Class for binary tree node


```
class TreeNode<T> {
   private T datum;
   private TreeNode<T> left, right;

   /** Constructor: one-node tree with datum d */
   public TreeNode (T d) {datum= d; left= null; right= null;}

   /** Constr: Tree with root datum d, left tree l, right tree r */
   public TreeNode (T d, TreeNode<T> l, TreeNode<T> r) {
       datum= d; left= l; right= r;
   }

   // more methods: getValue, setValue, getLeft, setLeft, etc.

}
```

Either might be null if the subtree is empty.

Binary versus general tree

In a binary tree, each node has up to two pointers: to the left subtree and to the right subtree:

- One or both could be **null**, meaning the subtree is empty (remember, a tree is a set of nodes)

In a general tree, a node can have any number of child nodes (and they need not be ordered)

- Very useful in some situations ...
- ... one of which may be in an assignment!

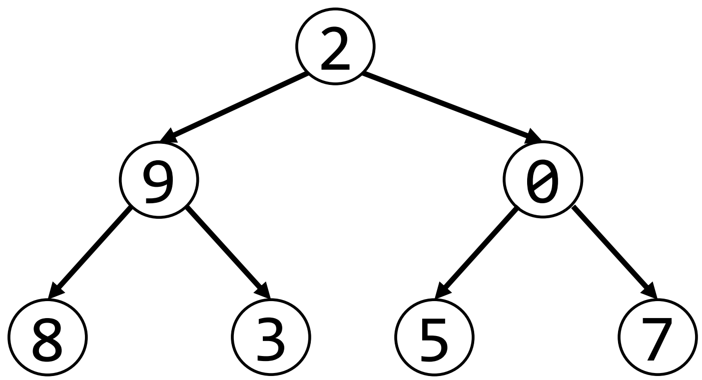
A Tree is a Recursive Thing

A binary tree is either `null` or an object consisting of a value, a left binary tree, and a right binary tree.

Looking at trees recursively

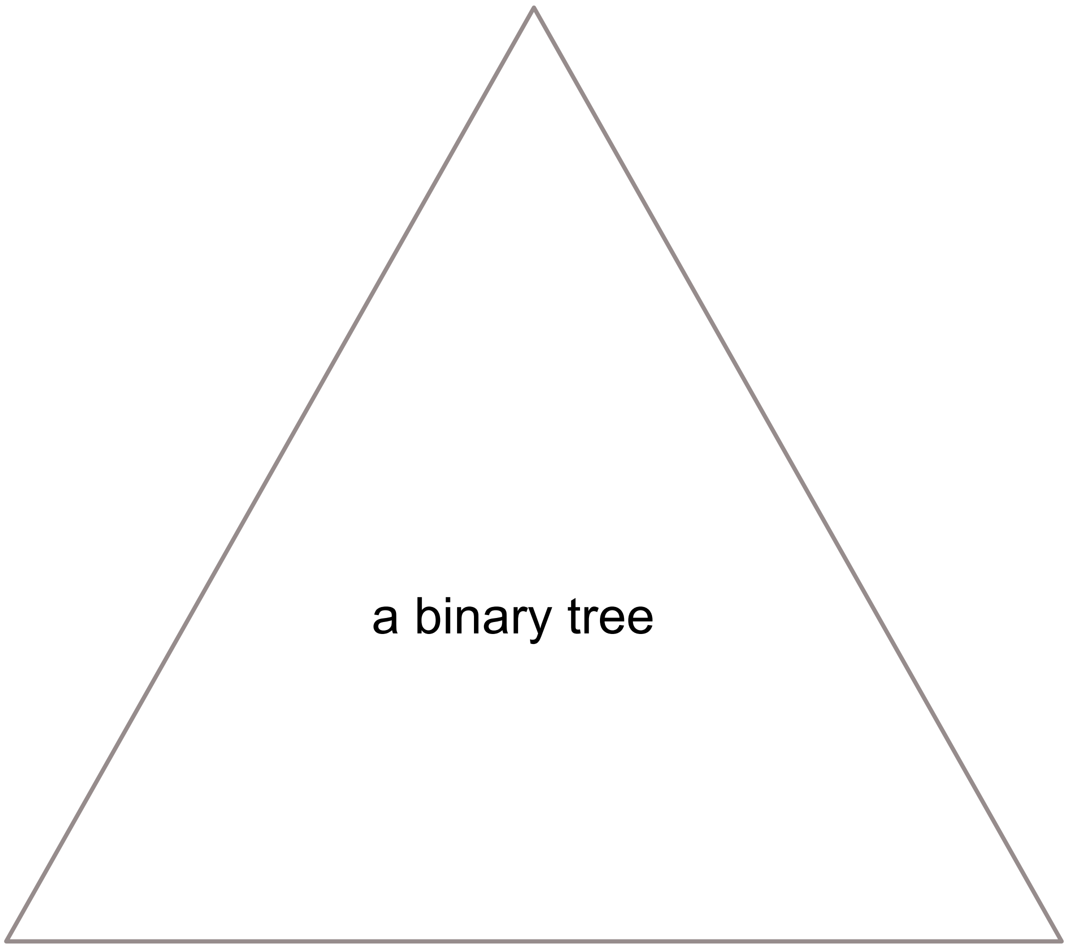
Binary Tree

Left subtree, which is also a binary tree

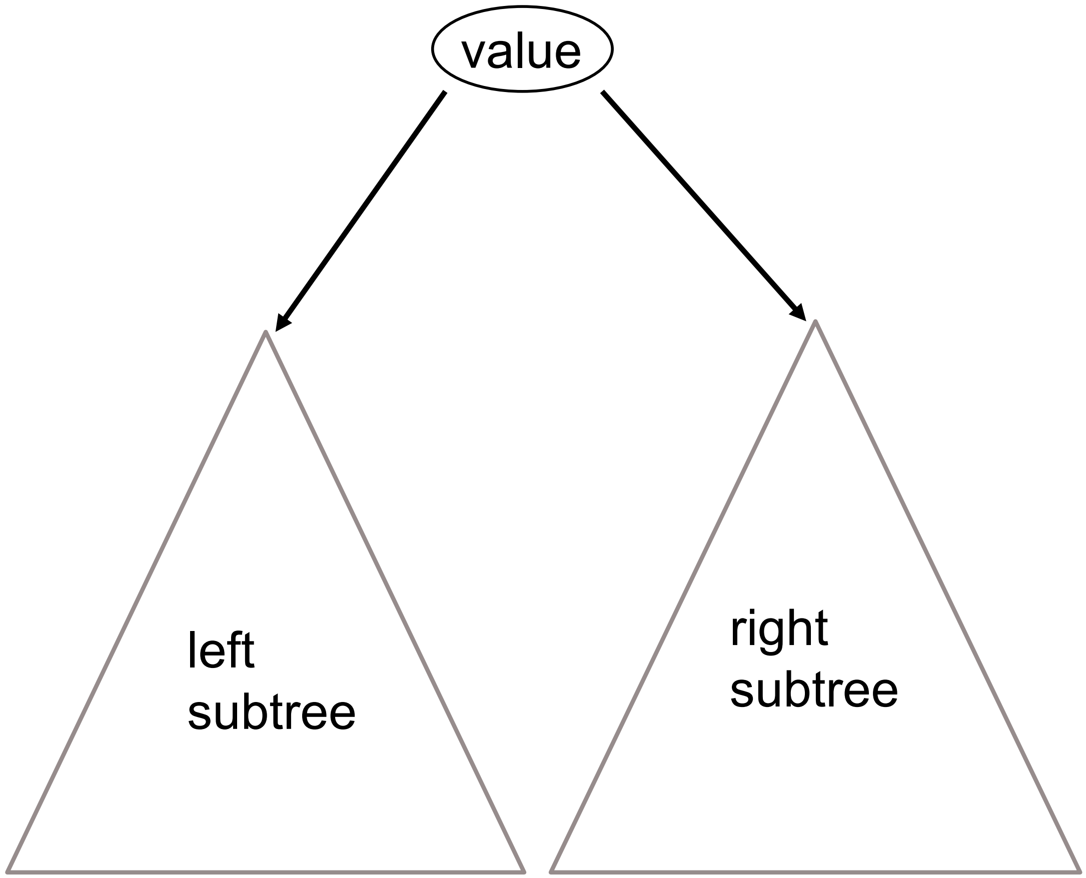


Right subtree (also a binary tree)

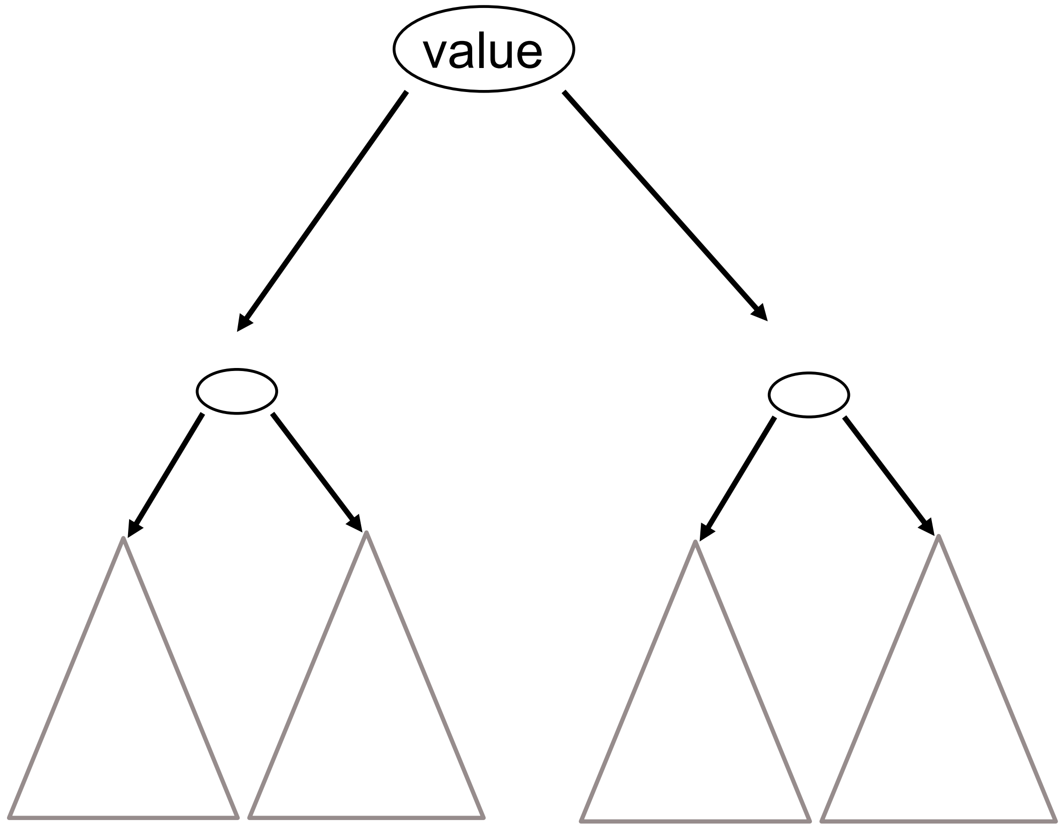
Looking at trees recursively



Looking at trees recursively



Looking at trees recursively



A Recipe for Recursive Functions

Base case:

If the input is "easy," just solve the problem directly.

Recursive case:

Get a smaller part of the input (or several parts).

Call the function on the smaller value(s).

Use the recursive result to build a solution for the full input.

A Recipe for Recursive Functions on Binary Trees

Base case:

an empty tree (null), or possibly a leaf

If the input is ~~"easy,"~~ just solve the problem directly.

Recursive case:

~~Get a smaller part of the input (or several parts).~~

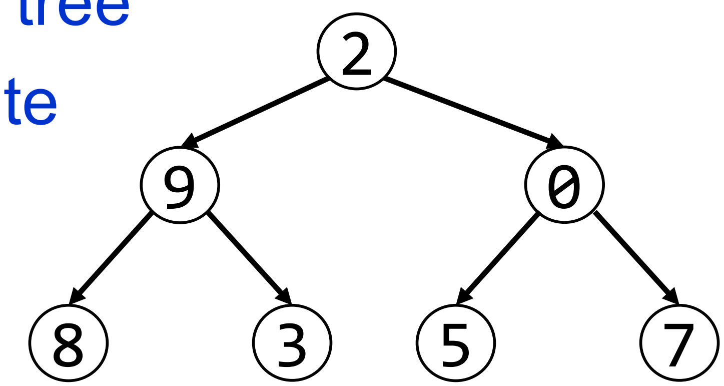
Call the function on ~~the smaller value(s).~~ each subtree

Use the recursive result to build a solution for the full input.

Searching in a Binary Tree


```
/** Return true iff x is the datum in a node of tree  t*/
public static boolean treeSearch(T x, TreeNode<T> t) {
   if (t == null) return false;
   if (x.equals(t.datum)) return true;
   return treeSearch(x, t.left) || treeSearch(x, t.right);
}
```

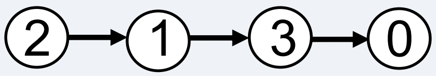
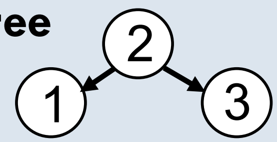
- Analog of linear search in lists: given tree and an object, find out if object is stored in tree
- Easy to write recursively, harder to write iteratively



We sometimes talk of the root of the tree, **t**.

But we also use **t** to denote the whole tree.

Comparing Data Structures

Data Structure	add(val v)	get(int i)	contains(val v)
Array 	$O(n)$	$O(1)$	$O(n)$
Linked List 	$O(1)$	$O(n)$	$O(n)$
Binary Tree 	$O(1)$	$O(n)$	$O(n)$

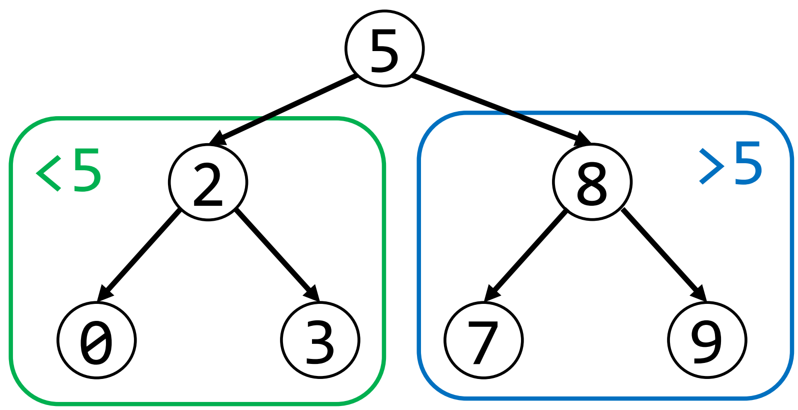
Index set by pre-determined traversal order (see slide 36); have to go through the whole tree (no short cut like array indexing)

Node you seek could be *anywhere* in the tree; have to search the whole thing.

Binary Search Tree (BST)

A *binary search tree* is a binary tree that is **ordered** and **has no duplicate values.** In other words, for *every* node:

- All nodes in the left subtree have values that are less than the value in that node, and
- All values in the right subtree are greater.



A BST is the key to making search way faster.

Building a BST

To insert a new item:

- Pretend to look for the item
- Put the new node in the place where you fall off the tree

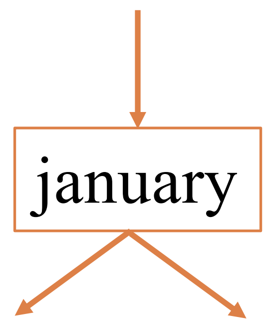
Building a BST

insert: January

Note: Inserting them *chronologically*, (January, then February…) but the BST places them alphabetically (Feb comes *before* Jan, etc.)

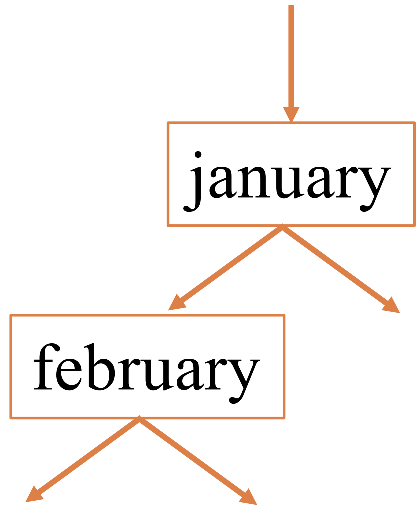
Building a BST

insert: February



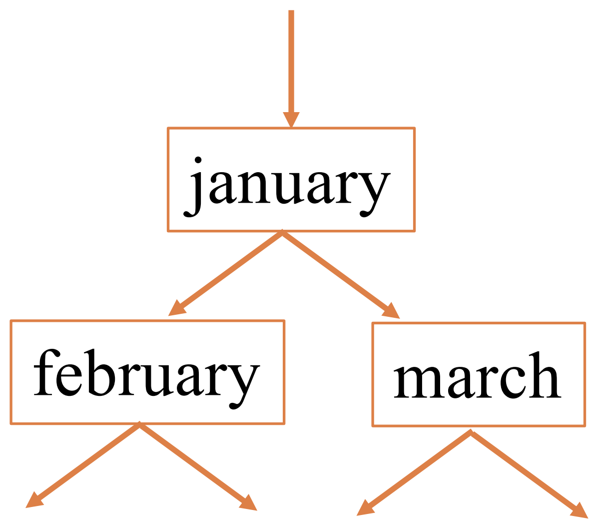
Building a BST

insert: March

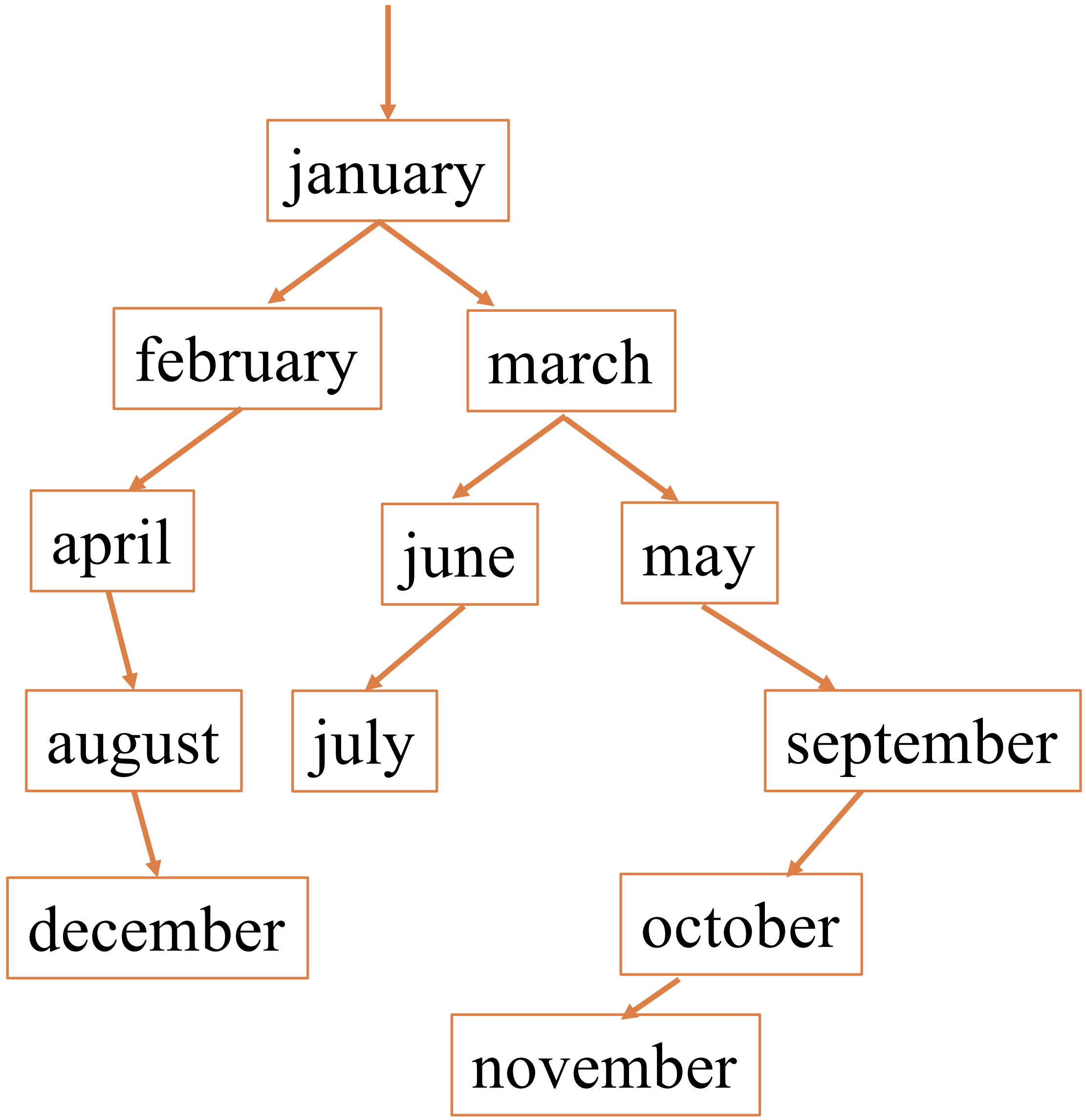


Building a BST

insert: April



Building a BST



Printing contents of BST


```
/** Print BST t in alpha order */
private static void
print(TreeNode<T> t) {
   if (t == null) return;
   print(t.left);
   System.out.print(t.value);
   print(t.right);
}
```

Because of ordering rules for BST, easy to print alphabetically

- Recursively print left subtree
- Print the root
- Recursively print right subtree

Tree traversals

"Walking" over the whole tree is a tree traversal

- Done often enough that there are standard names

Previous example:
in-order traversal

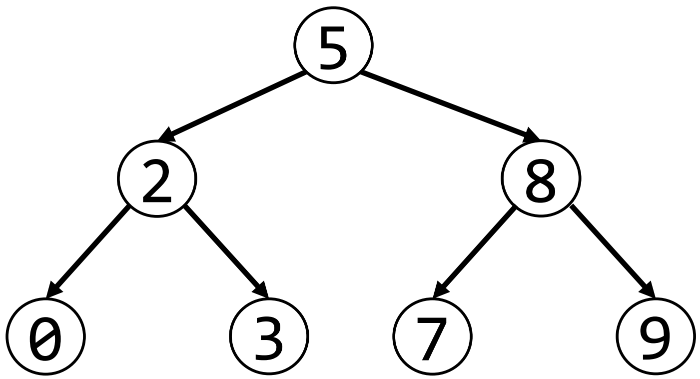
- Process left subtree
- Process root
- Process right subtree

Note: Can do other processing besides printing

Other standard kinds of traversals

- preorder traversal
  - Process root
  - Process left subtree
  - Process right subtree
- postorder traversal
  - Process left subtree
  - Process right subtree
  - Process root
- level-order traversal
  - Not recursive: uses a queue (we'll cover this later)

Binary Search Tree (BST)



Compare binary tree to binary search tree:

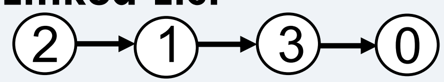
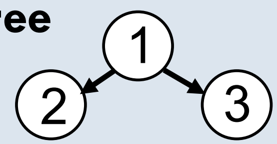
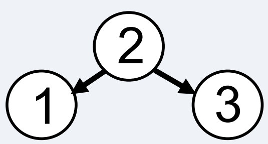
```
boolean searchBT(n, v):
  if n == null, return false
  if n.v == v, return true
  return searchBT(n.left, v)
      || searchBT(n.right, v)
```

2 recursive calls

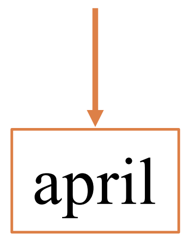
```
boolean searchBST(n, v):
  if n == null, return false
  if n.v == v, return true
  if v < n.v
    return searchBST(n.left, v)
  else
    return searchBST(n.right, v)
```

1 recursive call

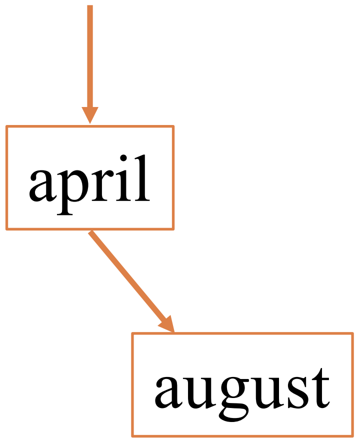
Comparing Data Structures

Data Structure	add(val x)	get(int i)	contains(val x)
Array 	$O(n)$	$O(1)$	$O(n)$
Linked List 	$O(1)$	$O(n)$	$O(n)$
Binary Tree 	$O(1)$	$O(n)$	$O(n)$
BST 	$O(depth)$	$O(depth)$	$O(depth)$

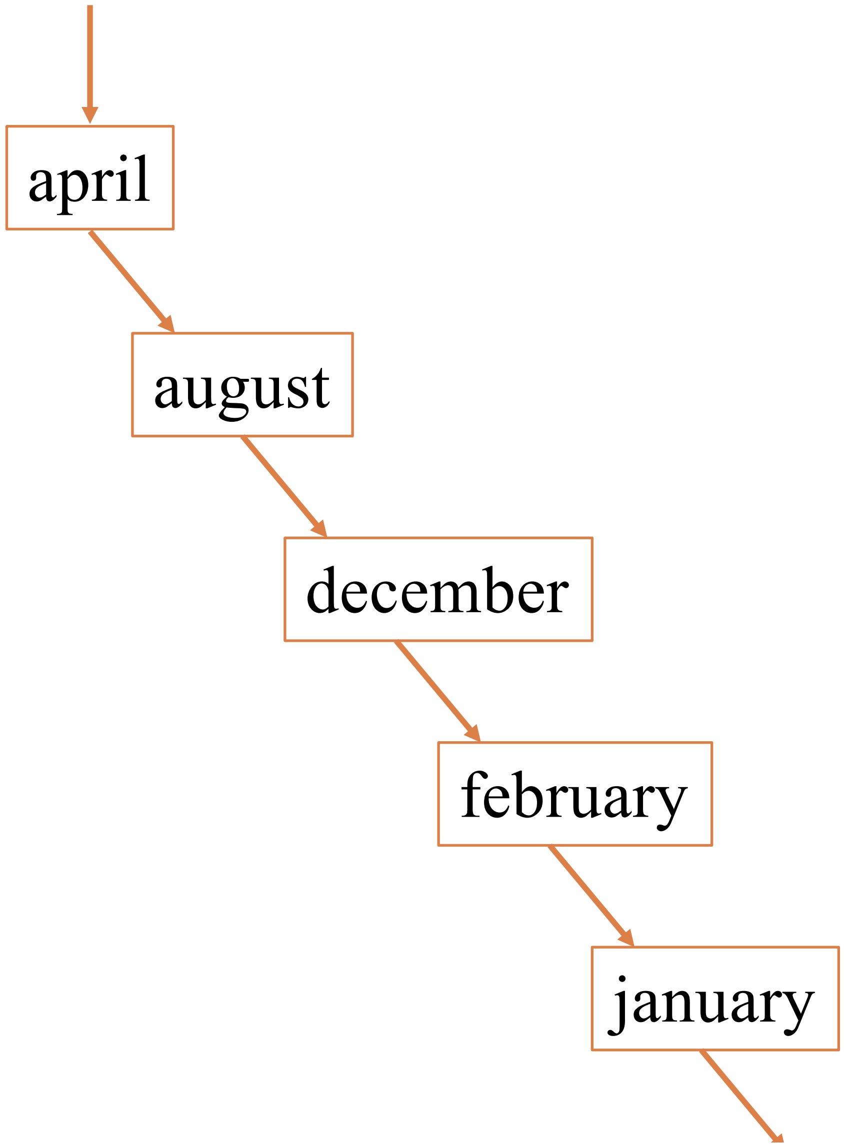
Inserting in Alphabetical Order



Inserting in Alphabetical Order



Inserting in Alphabetical Order

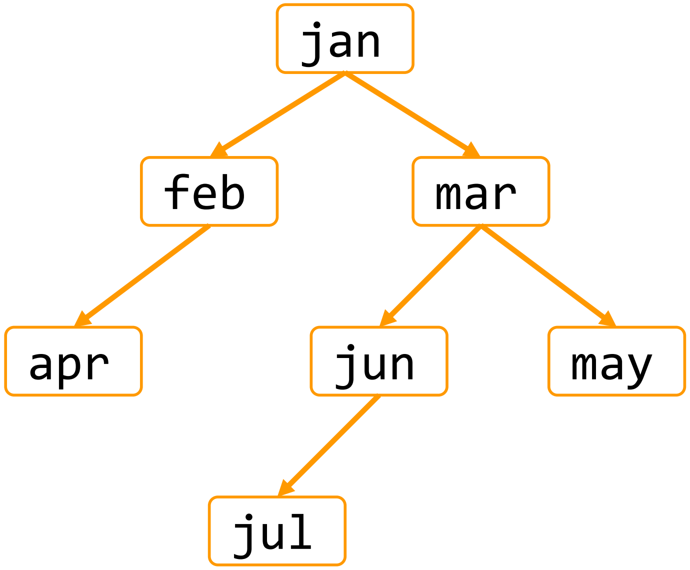


Insertion Order Matters

- A *balanced* binary tree is one where the two subtrees of any node are about the same size.
- Searching a binary search tree takes O(h) time, where h is the height of the tree.
- In a balanced binary search tree, this is O(log n).
- But if you insert data in sorted order, the tree becomes imbalanced, so searching is O(n).

Things to think about

What if we want to *delete* data from a BST?



A BST works great as long as it's *balanced*.

There are kinds of trees that can *automatically* keep themselves balanced as things are inserted!