"Organizing is what you do before you do something, so that when you do it, it is not all mixed up."
~ A. A. Milne

SORTING

Lecture 11
CS2110 – Fall 2018

Prelim 1

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- Tonight!!!!
- Two Sessions:
  - You should know by now what room to take the final. Jenna emailed you.
- Bring your Cornell ID!!!
- We will grade this evening, and if everything works out well, you will receive an email in early morning from Gradescope telling you to look at your grade.

Prelim 1

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- Recitation 5. next week:
  Enums and Java Collections classes.
  Nothing to prepare for it!

But get A3 done.

Why Sorting?

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- Sorting is useful
  - Database indexing
  - Operations research
  - Compression
- There are lots of ways to sort
  - There isn't one right answer
  - You need to be able to figure out the options and decide which one is right for your application.
  - Today, we'll learn several different algorithms (and how to develop them)

Some Sorting Algorithms

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- Insertion sort
- Selection sort
- Quick sort
- Merge sort

InsertionSort

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pre: b [0 ? b.length]   post: b [0 sorted b.length]

inv: b [0 sorted i ? b.length]
or: b[0..i-1] is sorted

inv: b [0 processed i ? b.length]
or: b[0..i-1] is processed

A loop that processes elements of an array in increasing order has this invariant --- just replace "sorted" by "processed".

Each iteration, i= i+1; How to keep inv true?

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inv: b [0 … sorted … i … ? … b.length]

e.g. b [0: 2 5 5 5 7 | i: 3 ? … b.length]

b [0: 2 3 5 5 5 7 | i: ? … b.length]

What to do in each iteration?

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inv: b [0 … sorted … i … ? … b.length]

e.g. b [0: 2 5 5 5 7 | i: 3 ? … b.length]

Loop body (inv true before and after):

2 5 5 5 3 | 7 ?

2 5 5 3 5 | 7 ?

2 5 3 5 5 | 7 ?

2 3 5 5 5 | 7 ?

Push b[i] to its sorted position in b[0..i], then increase i

This will take time proportional to the number of swaps needed

b 2 3 5 5 5 7 | ?

Insertion Sort

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```
// sort b[], an array of int
// inv: b[0..i-1] is sorted
for (int i= 0; i < b.length; i= i+1) {
   // Push b[i] down to its sorted
   // position in b[0..i]
}
```

Present algorithm like this

Note English statement in body. **Abstraction**. Says **what** to do, not **how.**

This is the best way to present it. We expect you to present it this way when asked.

Later, can show how to implement that with an inner loop

Insertion Sort

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```
// sort b[], an array of int
// inv: b[0..i-1] is sorted
for (int i= 0; i < b.length; i= i+1) {
   // Push b[i] down to its sorted
   // position in b[0..i]
   int k= i;
   while (k > 0  &&  b[k] < b[k-1]) {
      Swap b[k] and b[k-1];
      k= k–1;
   }
}
```

invariant P: b[0..i] is sorted **except** that b[k] may be < b[k-1]

example: 2 5 3 (k) 5 5 | 7 (i) ?

start?

stop?

progress?

maintain invariant?

Insertion Sort

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```
// sort b[], an array of int
// inv: b[0..i-1] is sorted
for (int i= 0; i < b.length; i= i+1) {
   // Push b[i] down to its sorted
   // position in b[0..i]}
```

Pushing b[i] down can take i swaps. Worst case takes

1 + 2 + 3 + … n-1 = (n-1)*n/2

swaps.

Let n = b.length

- Worst-case: $O(n^2)$ (reverse-sorted input)
- Best-case: O(n) (sorted input)
- Expected case: $O(n^2)$

Performance

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Algorithm	Ave time.	Worst-case time	Space	Stable?
Insertion Sort	$O(n^2)$.	$O(n^2)$	$O(1)$	Yes
Merge Sort				
Quick Sort				

We'll talk about stability later

SelectionSort

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pre: b [0 … b.length: ?]   post: b [0 … b.length: sorted]

inv: b [0 … i: sorted, <= b[i..] | i … b.length: >= b[0..i-1]]

Additional term in invariant

Keep invariant true while making progress?

e.g.: b [0: 1 2 3 4 5 6 | i: 9 9 9 7 8 6 9 … b.length]

Increasing i by 1 keeps inv true only if b[i] is min of b[i..]

SelectionSort

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```
//sort b[], an array of int
// inv: b[0..i-1] sorted  AND
//        b[0..i-1]  <=  b[i..]
for (int i= 0; i < b.length; i= i+1) {
   int m= index of min of b[i..];
   Swap b[i] and b[m];
}
```

Another common way for people to sort cards

Runtime with n = b.length

- Worst-case $O(n^2)$
- Best-case $O(n^2)$
- Expected-case $O(n^2)$

b [0: sorted, smaller values | i: larger values … length]

Each iteration, swap min value of this section into b[i]

Performance

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Algorithm	Ave time.	Worst-case time	Space	Stable?
Insertion sort	$O(n^2)$.	$O(n^2)$	$O(1)$	Yes
Selection sort	$O(n^2)$.	$O(n^2)$	$O(1)$	No
Quick sort				
Merge sort				

QuickSort

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Quicksort developed by Sir Tony Hoare (he was knighted by the Queen of England for his contributions to education and CS).

84 years old.

Developed Quicksort in 1958. But he could not explain it to his colleague, so he gave up on it.

Later, he saw a draft of the new language Algol 58 (which became Algol 60). It had recursive procedures. First time in a procedural programming language. "Ah!," he said. "I know how to write it better now." 15 minutes later, his colleague also understood it.

Dijkstras, Hoares, Grieses, 1980s

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Partition algorithm of quicksort

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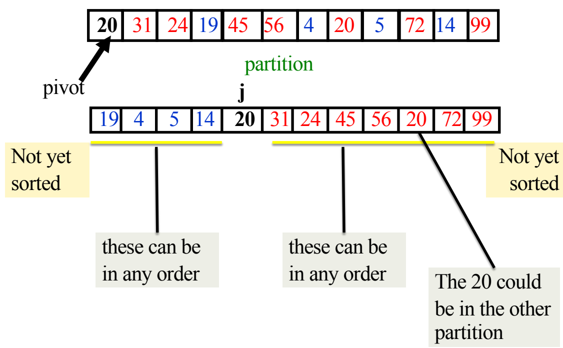
pre: [h: x | h+1 … k: ?]

x is called the pivot

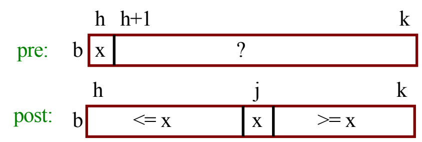
Swap array values around until b[h..k] looks like this:

post: [h: <= x | j: x | >= x … k]

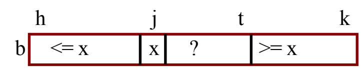
Partition algorithm of quicksort



Partition algorithm

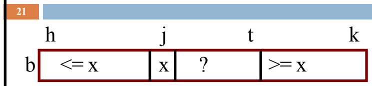


Combine pre and post to get an invariant



invariant needs at least 4 sections

Partition algorithm




```
j= h; t= k;
while (j < t) {
  if (b[j+1] <= b[j]) {
     Swap b[j+1] and b[j];   j= j+1;
  } else {
     Swap b[j+1] and b[t];   t= t-1;
  }
}
```

Takes linear time: O(k+1-h)

Initially, with j = h and t = k, this diagram looks like the start diagram

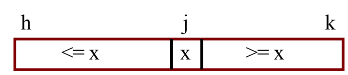
Terminate when j = t, so the "?" segment is empty, so diagram looks like result diagram

QuickSort procedure

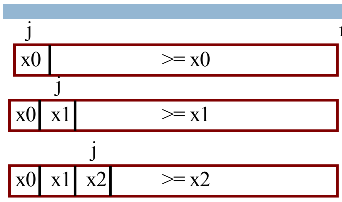
```
/** Sort b[h..k]. */
public static void QS(int[] b, int h, int k) {
  if (b[h..k] has < 2 elements) return;
  int j= partition(b, h, k);
    // We know b[h..j–1] <= b[j] <= b[j+1..k]
    // Sort b[h..j-1] and b[j+1..k]
    QS(b, h, j-1);
    QS(b, j+1, k);
}
```

Base case

Function does the partition algorithm and returns position j of pivot



Worst case quicksort: pivot always smallest value



partioning at depth 0

partioning at depth 1

partioning at depth 2

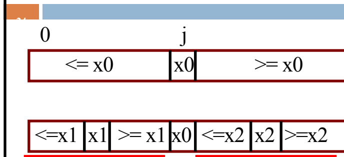
Depth of recursion: O(n)

Processing at depth i: O(n-i)

O(n*n)

```
/** Sort b[h..k]. */
public static void QS(int[] b, int h, int k) {
  if (b[h..k] has < 2 elements) return;
  int j= partition(b, h, k);
  QS(b, h, j-1);    QS(b, j+1, k);
```

Best case quicksort: pivot always middle value



depth 0. 1 segment of size ~n to partition.

Depth 2. 2 segments of size ~n/2 to partition.

Depth 3. 4 segments of size ~n/4 to partition.

Max depth: O(log n). Time to partition on each level: O(n)
Total time: O(n log n).

Average time for Quicksort: n log n. Difficult calculation

QuickSort complexity to sort array of length n

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Time complexity
Worst-case: O(n*n)
Average-case: O(n log n)

```
/** Sort b[h..k]. */
public static void QS(int[] b, int h, int k) {
   if (b[h..k] has < 2 elements) return;
   int j=  partition(b, h, k);
   // We know b[h..j–1] <= b[j] <= b[j+1..k]
   // Sort b[h..j-1] and b[j+1..k]
   QS(b, h, j-1);
   QS(b, j+1, k);
}
```

Worst-case space: ?
What's depth of recursion?

Worst-case space: O(n)!
--depth of recursion can be n
Can rewrite it to have space O(log n)
Show this at end of lecture if we have time

Partition. Key issue. How to choose pivot

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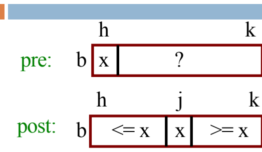

Choosing pivot
Ideal pivot: the median, since it splits array in half
But computing the median is O(n), quite complicated

Popular heuristics: Use
- first array value (not so good)
- middle array value (not so good)
- Choose a random element (not so good)
- median of first, middle, last, values (often used)!

Performance

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Algorithm	Ave time.	Worst-case time	Space	Stable?
Insertion sort	$O(n^2)$.	$O(n^2)$	$O(1)$	Yes
Selection sort	$O(n^2)$.	$O(n^2)$	$O(1)$	No
Quick sort	$O(n \log n)$.	$O(n^2)$	O(log n)*	No
Merge sort				

* The first algorithm we developed takes space O(n) in the worst case, but it can be reduced to O(log n)

Merge two adjacent sorted segments

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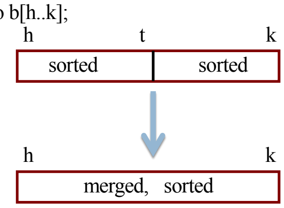
```
/* Sort b[h..k]. Precondition: b[h..t] and b[t+1..k] are sorted.  */
public static merge(int[] b, int h, int t, int k) {
    Copy b[h..t] into a new array c;
    Merge c and b[t+1..k] into b[h..k];
}
```



Merge two adjacent sorted segments

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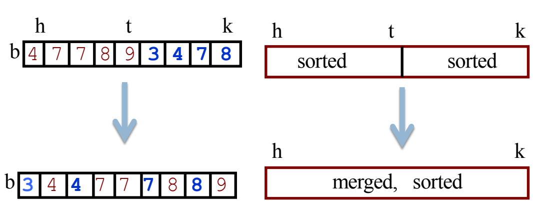
```
/* Sort b[h..k]. Precondition: b[h..t] and b[t+1..k] are sorted.  */
public static merge(int[] b, int h, int t, int k) {
}
```



Merge two adjacent sorted segments

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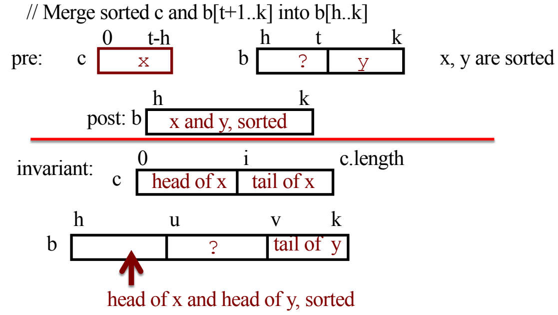
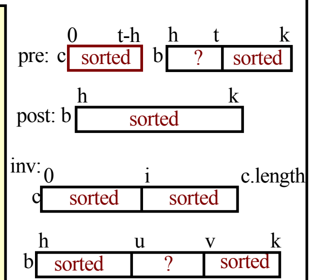
// Merge sorted c and b[t+1..k] into b[h..k]



Merge

```
int i = 0;
int u = h;
int v = t+1;
while( i <= t-h){
    if(v <= k && b[v] < c[i]) {
        b[u] = b[v];
        u++; v++;
    }else {
        b[u] = c[i];
        u++; i++;
    }
  }
}
```



Mergesort

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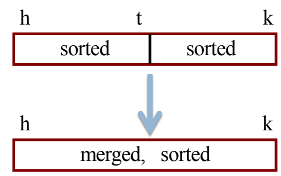
```
/** Sort b[h..k] */
public static void mergesort(int[] b, int h, int k]) {
    if (size b[h..k] < 2)
        return;
    int t= (h+k)/2;
    mergesort(b, h, t);
    mergesort(b, t+1, k);
    merge(b, h, t, k);
}
```



QuickSort versus MergeSort

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```
/** Sort b[h..k] */
public static void QS
      (int[] b, int h, int k) {
   if (k – h < 1) return;
   int j= partition(b, h, k);
   QS(b, h, j-1);
   QS(b, j+1, k);
}
```

```
/** Sort b[h..k] */
public static void MS
      (int[] b, int h, int k) {
   if (k – h < 1) return;
   MS(b, h, (h+k)/2);
   MS(b, (h+k)/2 + 1, k);
   merge(b, h, (h+k)/2, k);
}
```

One processes the array then recurses.
One recurses then processes the array.

Performance

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Algorithm	Ave time.	Worst-case time	Space	Stable?
Insertion sort	$O(n^2)$.	$O(n^2)$	$O(1)$	Yes
Selection sort	$O(n^2)$.	$O(n^2)$	$O(1)$	No
Quick sort	$O(n \log n)$.	$O(n^2)$	O(log n)*	No
Merge sort	$O(n \log n)$.	$O(n \log n)$	O(n)	Yes

* The first algorithm we developed takes space O(n) in the worst case, but it can be reduced to O(log n)

Sorting in Java

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- Java.util.Arrays has a method sort(array)
  - implemented as a collection of overloaded methods
  - for primitives, sort is implemented with a version of quicksort
  - for Objects that implement Comparable, sort is implemented with timSort, a modified mergesort developed in 1993 by Tim Peters
- Tradeoff between speed/space and stability/performance guarantees