CS211, LECTURE 25
MORE GRAPHS

ANNOUNCEMENTS:

- office hours today (Tues): 1-3pm
- A6 due Weds 4/30
- makeup assignment ("A7") info
- bonus points on prelim?

OVERVIEW:

- implicit graph reminder
- explicit graphs
- adjacency matrix and list representations
- design, algorithms and implentation for basic classes
- building graphs

1. Motivation

1.1 Up To Today:

- graph theory, which leads to …
- implicit graphs and help with homework, then…
- explicit graphs to help build generic graph classes…
- build graphs

1.2 What To Do With Graphs?

- next two lectures…
- generalize traversal: BFS, DFS
- use traversal for searching
- sorting
- shortest path to something
- more…?

2. Representations

2.1 Implicit

- rules/model creates a network of nodes/edges
- ex) puzzle moves
  - each move makes a new puzzle
  - treat each state as a node
  - so, rules implicit define a graph
- common for games!

2.2 Explicit

- define all nodes V and edges E ahead of time
- want system to represent edges
- why? it's the "biggest problem":
  - $G = (V,E)$ and each edge e in E is a pair (v1,v2)
  - most edges possible? $|V|$^2
    (form pairs from all nodes)
  - most sets of edges possible? 2^($|V|$^2)
- so, use container to represent edges
  - adjacency matrix
  - adjacency list

2.3 Adjacency Matrix

- ***adjacency matrix***

$$A_{ij} = \begin{cases} w_{ij} & \{v_i, v_j\} \in E \\ 0 & otherwise \end{cases}$$

- terms

  v_i : node i; v_j node j

  $\{v_i, v_j\} \in E$: edge between nodes i (v_i) and j (v_j) belongs to set of edges E

  w_{ij} : weight of edge between nodes i and j

- A_{ij} : the matrix (rectangular 2x2 array) as rows (i) and cols (j); coords correspond to nodes i and j

2.4 Adjacency List

- adjacency list: linked list of nodes adjacent to a node
- need $|V|$ lists

2.5 graph types to develop:

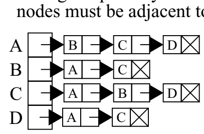
- undirected
- directed
- weighted

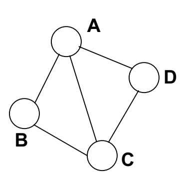
2.6 Undirected

$$A_{ij} = \begin{cases} 1 & \{v_i, v_j\} \in E \\ 0 & otherwise \end{cases}$$

i \ j	A	B	C	D
A		1	1	1
B	1		1	
C	1	1		1
D	1		1	

Use array A of lists:
A_i stores a linked list of nodes
no edge implied by order *in* list
nodes must be adjacent to A_i



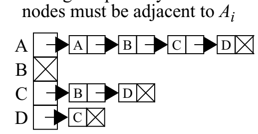


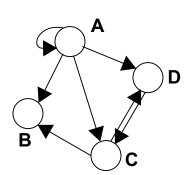
2.7 Directed

$$A_{ij} = \begin{cases} 1 & (v_i, v_j) \in E \\ 0 & otherwise \end{cases}$$

i \ j	A	B	C	D
A	1	1	1	1
B				
C		1		1
D			1	

Use array A of lists:
A_i stores a linked list of nodes
no edge implied by order *in* list
nodes must be adjacent to A_i





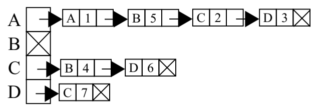
2.8 Weighted

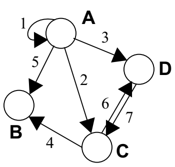
- assuming also weighted
- w_{ij} : cost or weight of edge from node i to node j
- sometimes use ***sentinel*** ∞ to represent "no edge" between i and j

$$A_{ij} = \begin{cases} w_{ij} & (v_i, v_j) \in E \\ 0 & otherwise \end{cases}$$

Use array *A* of lists: include weights
List for i contains j,w for edge (i,j)

i \ j	A	B	C	D
A	1	5	2	3
B				
C		4		6
D			7	





2.9 Choice of AM or AL?

- Adjacency Matrix
  - uses $O(|V^2|)$ space
  - can answer "is there an edge from i to j?" in $O(1)$ time
  - enumerating all nodes adjacent to i: $O(|V|)$ (find all nodes j in row for i)
  - could be sparse because of wasted space (0s)
  - better for dense graphs (lots of edges)!
- Adjacency List
  - uses $O(|V|+|E|)$ space ($|V|$ for i nodes, $|E|$ for j nodes emanating from each i node)
  - can answer question "is there an edge from i to j?" in $O(|E|)$ time
  - enumerating all nodes adjacent to i: $O(1)$ per adjacent node in linked list
  - better for sparse graphs (few edges)!

3. Implementation

3.1 Implicit

- can use containers to store node and edge info
- a bit too problem specific, though effective

3.2 Explicit

- Adjacency Matrix - left as exercise
- Adjacency List
  - using linked list to allow for flexible building
  - kind of gives implicit building by allowing for node/ edge creation "on the fly"
- focus on digragh, but could be weighted
  - Sections 3, 4, 5, 6
  - many methods left out – will see for graph problems

4. Verticies

4.1 Fields

- **`label`**: we like to have names, numbers, …
- **`edges`**: collection of all emanating edges from the current vertex
- **`visited`**: need later to tag vertex for searching…
- sometimes includes **`cost`** (cost to get *here* from somewhere)

4.2 Constructor

- set **`label`**
- create **`edges`** adjacency list (AL)

4.3 Methods

- **`addEdge`**: add to AL
- **`equals`**: need for path checking
- more?

```java
import java.util.*;

public class Vertex {

    private Object label;
    private LinkedList edges; // adjacent edges
    private boolean visited;  // tag

    public Vertex(Object o) {
        label = o;
        edges = new LinkedList();
    }

    public void addEdge(Edge e, int weight) {
        Vertex source = this;
        Vertex dest = e.getDest();
        edges.add(new Edge(source,dest,weight));
    }

    public void addEdge(Edge e) {
        addEdge(e,0);
    }

    public boolean equals(Vertex other) {
        return label.equals( ((Vertex)other).label );
    }

    public String toString() {
        return label.toString();
    }

    public Collection getEdges() { return edges; }

} // Class Vertex
```


5. Edges

5.1 Fields

- source: s->d, the node from which edge emanates
- dest: actually, all you need is this since Vertex keeps track of adjacent edges of source
- weight: could make double (sometimes called cost)

5.2 Constructors

- build edge from s->d
- can default to weight of 0 to handle unweighted graphs

5.3 Methods

- **`equals`** and **`compareTo`**:
  - many algorithms want to know shortest path
  - need to compare costs of going in different directions
- **`toString`**: "**`source-weight->dest`**"
- more?


```java
public class Edge implements Comparable {

    private Vertex source; // s (s->d)
    private Vertex dest;   // d
    private int weight;    // also called cost

    public Edge(Vertex source, Vertex dest, int weight) {
        this.source=source;
        this.dest=dest;
        this.weight=weight;
    }

    public Edge(Vertex source, Vertex dest) {
        this(source,dest,0);
    }

    // getters and setters not shown

    public boolean equals(Object other) {
        Edge e = (Edge) other;
        return weight == e.weight;
    }

    public int compareTo(Object other) {
        Edge e = (Edge) other;
        return (int) (weight-e.weight);
    }

    // Stringify as (d,--w->,s):
    public String toString() {
        return "("+source+"-"+weight+"->"+dest+")";
    }

} // Class Edge
```


6. Directed Graphs

6.1 Fields

- **`verticies`** dictionary:
  - key-val pairs of (VertexName,Vertex)
  - each Vertex points to its adjacency list!
- **`edgeCount`**

6.2 Constructors

- set **`verticies`** to LinkedHashMap
- maintains order of nodes in order created
- nodes *must* be created before edges this way!

6.3 Methods

- use vertex names/labels!
- **`addVertex`**: put **`Vertex`** in **`Map`**: (**`name`**, **`Vertex`**)
- **`addEdge`**: connect s and d nodes (they must exist!)

```
import java.util.*;
public class Digraph {

    private Map verticies; // dictionary of nodes
    private int edgeCount; // number of edges

    public Digraph( ) {
        verticies = new LinkedHashMap();
    }

    // Add vertex to map
    public void addVertex(Object name) {
        verticies.put(name, new Vertex(name));
    }

    // Adds edge (source and dest node must exist!):
    public void addEdge(Object s, Object d, int weight) {

        // Key is NAME of Vertex
        // Val is THE Vertex
        // So, get keys of s and d and use them to
        //      retrieve their vals (their Verticies):
        Vertex source = (Vertex)verticies.get(s);
        Vertex dest = (Vertex)verticies.get(d);

        // Create edge between source and dest:
        s.addEdge(new Edge(source,dest,weight));
        edgeCount++;
    }

    public void addEdge(Object source, Object dest) {
        addEdge(source,dest,0);
    }
```



```
    // Stringify: return edges with
    //            their adjacency lists:
    public String toString() {

        String s = "";

        Iterator it=verticies.keySet().iterator();

        while(it.hasNext()) {

            //  current node label:
            Object key = it.next();

            // current Vertex:
            Vertex val = (Vertex) verticies.get(key);

            // build string for current vertex in Map:
            s += "[" + val + "]" + "-->";
            s += val.getEdges();
            s += "\n";

        }

        return s;

    } // Method toString

} // Class Digraph
```


7. Demonstration

7.1 Code

```
public class TestDigraph {

    public static void main(String[] args) {

        Digraph g = new Digraph();
        g.addVertex("A");
        g.addVertex("B");
        g.addVertex("C");
        g.addEdge("A","B");
        g.addEdge("A","C");
        g.addEdge("B","C");
        System.out.println(g);

    }
}
```

7.2 Output

```
[A]-->[(A-0->B), (A-0->C)]
[B]-->[(B-0->C)]
[C]-->[]
```


8. Exercises

- Demonstrate why we use edges for explicit representations of graphs.
- Develop **`Vertex`**, **`Edge`**, **`Digraph`**, and **`TestDigraph`** classes for the adjacency matrix approach. You should develop methods to handle I/O in reading in a grid of adjacencies to help build a graph.
- Remove the **`source`** node field from class **`Edge`** and modify the remaining classes as necessary. This design is a bit more common than the examples given to you.
- Rewrite **`Digraph`**'s **`addEdge`** such that it does not assume that the nodes exist. You may either throw an exception or perhaps create more nodes….
- Graphical graph: This was once a final project long ago…develop a GUI tool that draws a graph that a user creates, either via the GUI or as a translation from the collection that contains the verticies and edges. A rudimentary application would naively draw each vertex according to a pre-determined grid and then draw the edges using the given vertex geometry.