CS211, LECTURE 22
HIERARCHICAL SEQUENCE STRUCTURE

ANNOUNCEMENTS:

OVERVIEW:

- Motivation
- PQ
- Linear PQ
- Heaps
- PQ as Heap
- Array representation

1. Motivation

1.1 Sequence Structure Reminder

- want ADT to store and retrieve items
- use put and get to create and manipulate a pile of things
- not interested in search

1.2 Queues

- line things up, take them out in FIFO order
- weakness: what if some things are more important than others?
- Analogies: patients needing emergency care, small and large print jobs, office hours…

1.3 Priority Queues

- insert items in any order:
  include a ***priority*** (numerical rating of importance)
- extract items according to priority

1.4 For later…

- some algorithms need to be broken into smaller tasks
- use PQs to prioritize which tasks occur early/later

2. Interface

```
interface SeqStructure {
   void put(Object o);
   Object get();
   boolean isEmpty();
   int size();
}
```

2.1 Operations

- `put:` insert items in any order…
- `get:` remove item with highest priority
  - might see altenate ops: `findMin` or `findMax`
  - return smallest/largest item means return highest/ lowest priority ("small" things usually go first)

2.2 Implementations

- linear: as array, as list
- hierarchical: as BST (better, splay tree), heap (special kind of tree)

3. PQ Elements

- PQ Element has data (**`Object`**) and priority (**`int`**)
- kind of like a key-value pair
- design elements to be compared for priorities
- code:

```
public class PQElement implements Comparable {
   private Object item;
   private int priority;
   public PQElement(Object o, int p) {
      item = o;
      priority = p;
   }
   public int getPriority() { return priority; }
   public void setPriority(int p) { priority = p; }
   public Object getItem() { return item; }
   public String toString() {
       return  "("+item+","+priority+")"; }

   // return pos means this.p > o.p
   // return nil means this.p = o.p
   // return neg means this.p < o.p
   public int compareTo(Object o) {
      if (o instanceof PQElement)
         return (priority - ((PQElement)o).priority);
      else {
         System.out.println("Crap!");
         return 0; // should really throw an Exception
      }
   }
} // Class PQElement
```

4. Array Implementation

4.1 Very similar to `SortedArray` for searching

- but, must allow for duplicates
- sort in ascending order
- item with highest priority from end of array
- so,
  - get: O(1)
  - put: O(n)
- see **`PQAsSortedArray.java`**

4.2 Get

```
// get the rightmost item,
// because sorted in ascending order
// (want highest priority item):
public Object get() {
   // check for empty PQ:
   if (size == 0) {
      System.out.println("Empty array error!");
      return null;
   }
   return a[--size];
}
```

4.3 Put

- need to set cursor (current item) with binary search
- allow for repeats

```
// insert into right place in sorted array
// (ascending order):
public void put(Object o) {
   if (size == MAXSIZE) {
      System.out.println("Overflow error!");
      return;
   }
   // set the cursor to point to insertion point:
   boolean found = search(o);

   // find leftmost object equal to o
   // (could have repeats):
   while (cursor > 0 &&
      ((PQElement)(a[cursor - 1])).compareTo(o) == 0)
      cursor--;

   // make room for new element by shifting
   // elements to left of cursor
   for (int i = size -1; i >= cursor;i--)
      a[i+1] = a[i];

   // insert new element:
      a[cursor] = (PQElement) o;
      size++;
}
```

5. List Implementation

5.1 Operations

- put: either store “randomly” (FIFO) or sort list:
  - FIFO makes **`put`** easy, but **`get`** hard
  - sorted list makes put work a lot to insert and set cursors (same kind of cursors as array PQ) but **`get`** is very easy
- **`get`**: see above

5.2 Implementations

- **`PQAsList`**: easy put, hard get
- **`PQAsListAlt`**: hard put, easy get
- either way, at least one operation is O(n)

5.3 Example from PQAsList

```
public class PQAsList implements SeqStructure{
   // fields

   public void put(Object o) {
      list.add(o); size++; }

   public Object get() {
      if (isEmpty()) {
         System.out.print("Empty!");
         return null;
      }

   // search list, starting from head:
   ListNode n = list.getHead();
   Object max = n.getItem();
   ListNode next = n.getNext();

   // find and update max:
   while (next != null) {
      Object current = next.getItem();
      int comp = ((Comparable)current).compareTo(max);
      if (comp > 0) max = current;
      next = next.getNext();
   }

   list.remove(max);
   size--;
   return max;

   // methods
}
```

6. Tree Implementation

6.1 Any other way to do this?

- array & list have O(*n*) at some point
- so maybe a search tree could help us?

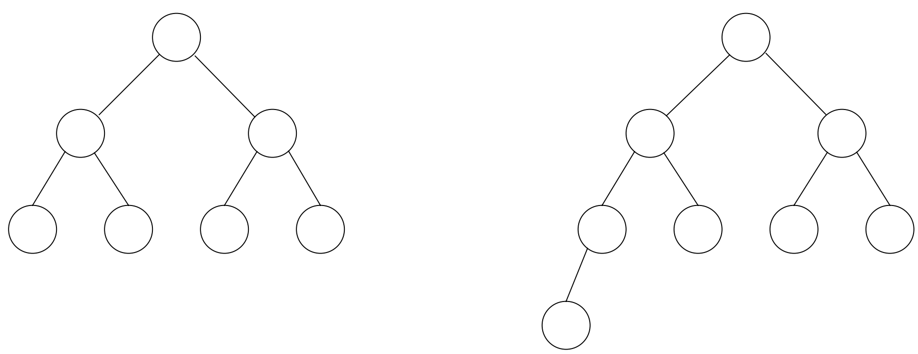
6.2 Search Trees

- designed for searching (BST gives O(log *n*) on avg
  - search is proportional to height (search path from root to leaf)
  - nodes = 2^height-1, so height = log[2](nodes+1)
  - if BST is "bushy", gives O(log *n*) time
  - if BST is "skinny", resembles list: O(*n*) time
  - actually, there's a lot more math involved here…
- problems:
  - input not randomized (queues used for simulation)
  - support more operations than really needed
  - need to have "bushiness" of tree to get O(log n)
- something called a ***splay tree*** helps sometimes
- ***heap*** to the rescue!

7. Binary Trees Revisited

7.1 Special types of binary tree to define heap

- we need some tree definitions
- ***full binary tree***: every node has two children
- ***complete binary tree***: full binary tree, except…
  - next-to-last level may be partially filled
  - must fill last level from left to right
- full, complete help give "bushiness" to trees



full complete

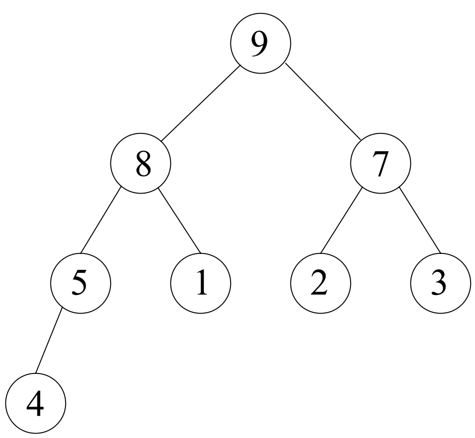
8. Heap Types

8.1 ***heap***: complete binary tree with ordering

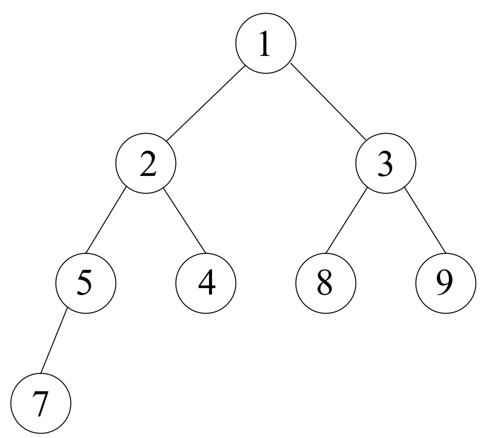
- different kinds of ordering (min, max, min-max, …)
- not related to “the heap”
- use **`Comparable`** to achieve ordering

8.2 Specific heap types

- ***maxheap***: object in a node ≥ all children
- ***minheap***: object in a node ≤ all children
- ***min-max heap*** (and more): http://www.diku.dk/forskning/performance-engineering/Jesper/heaplab/heapsurvey_html/node1.html



maxheap



minheap

8.3 Real Life Examples

- ages of people in family tree: parent is always older than children (max heap), but you may have an uncle/aunt younger than you
- people's salaries: bosses make more than subordinates ("pions"), but a 2nd-level manager may make more money than a 1st-level manager in a different sub-division

8.4 Min or Max for PQ?

- we'll pick maxheap (DS&SD 18.6)
- why? want to find max priority item
- minheap has analogous operations (see DS&A, Chap 11)

8.5 Methods to implement

- **`put`**: add something to the heap, but must preserve the heap property (min, max, min-max, …)
- **`get`**: remove largest item from the heap

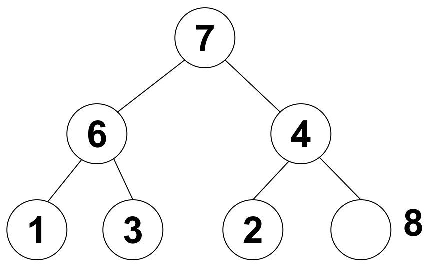
8.6 Heapsort

- If we're always pulling out the largest (or smallest) item, then technically, we could use a heap for sorting!
- DS&SD go into ***<u>heapsort</u>*** (section 18.6)

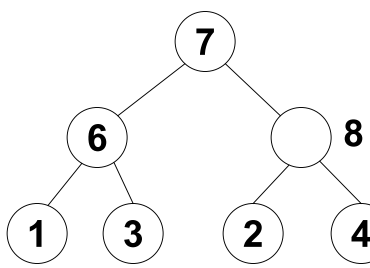
9. Heap Operations

9.1 put

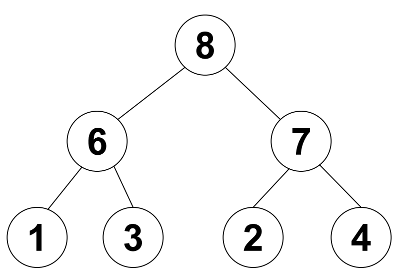
- method will insert item at first free node (left-bottom)
- must maintain heapness, so will have to reheapify



Get heap
Does it have enough space?
Find/create 1st empty location
See if item (8) goes there



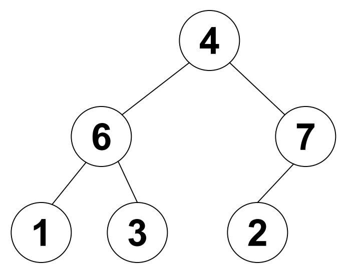
Parent (4) is < item (8)
so, move parent to that empty
Now, compare item with parent



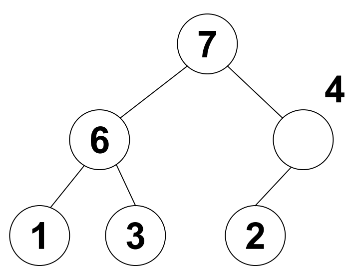
Parent (7) is < item (8)
so, move parent to that empty
Now, compare item with root
Nothing left to compare
Done.

9.2 get

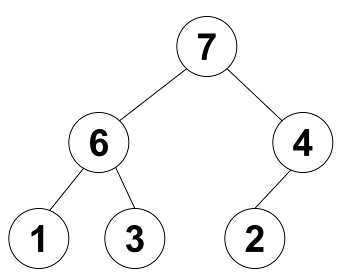
- want to remove max, which means root (uh oh!)
- tree could becomes forest! crap!
- but, we could “fix” (reheap) the heap



Get max (root, which was 8)
Replace with 1st leaf (4)
Delete (or nullify) that leaf
Want happy heap? Reheap!



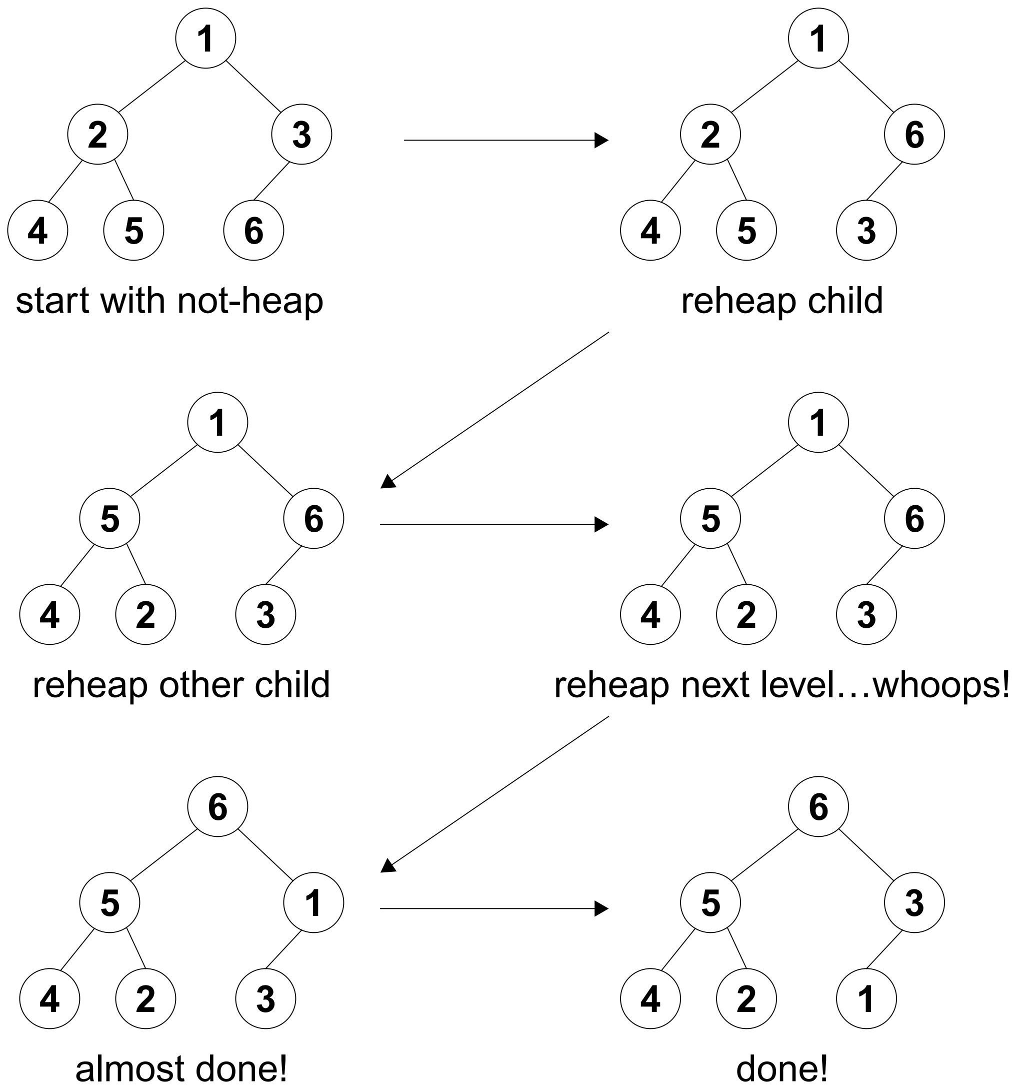
Item (4) is > children
Put item in current parent



Done!
Our heap is ready for more action.

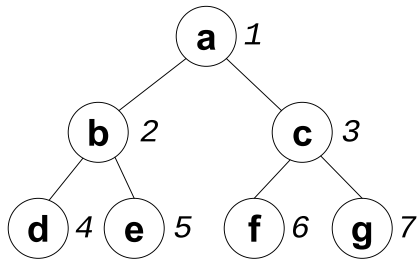
9.3 create heap

- could use **`put`**, which gives O(n log n)
- better: reheap each internal node until reaching root:



10. Heaps and Arrays!

10.1 Number Nodes in Heap



10.2 Now, put items in array with indicies

	a	b	c	d	e	f	g
0	1	2	3	4	5	6	7

10.3 Now, think BINARY TREE

- binary tree should somehow use powers of 2
- ex) size = 2^(h+1)-1 = 2^(2+1)-1 = 8-1 =7

10.4 Older files to check out:

- **`MaxHeap.java`**
- **`HeapDecoder.java`**
- **`TestMaxHeap.java`**

10.5 Interesting Features

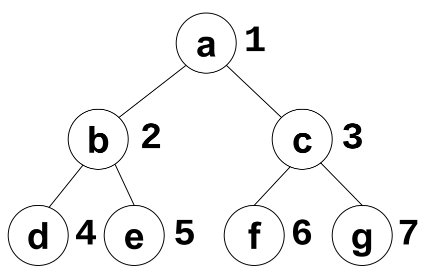
- children of node i: $2i$ and $2i+1$
- parent of node i (not root): $i/2$
- root: $i/2=0$, so $i = 1$

10.6 Binary Path

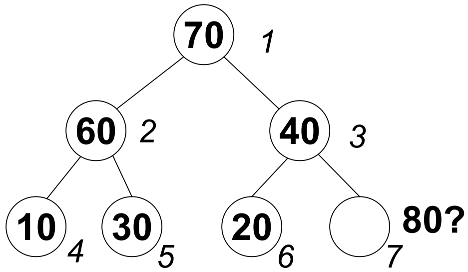
- find binary representation of a node number:
  - formula:

    $$d = b_n \times 2^n + b_{n-1} \times 2^{n-1} + \ldots + b_1 \times 2^1 + b_0 \times 2^0$$

  - take number d, divide by 2
  - remainders are $b_0, b_1, \ldots, b_{n-1}, b_n$
  - ex) b0=6%2=0; b1=3%2=1; b2=1%2=0; so, 6→110
- drop leading 1 (if exists):
  - pattern: 1 = R, 0 = L
  - ex) 6 → 110, so to find node 6, go RK (10):



10.7 Example (put)



Want to put 80 into heap

80 > 40?

	70	**60**	**40**	**10**	**30**	**20**	
0	*1*	*2*	*3*	*4*	*5*	*6*	*7*

Want to insert item (80)
Locate free space (index 7)
Find parent 7/2=3→40

80 > 70?

	70	**60**	**80**	**10**	**30**	**20**	**40**
0	*1*	*2*	*3*	*4*	*5*	*6*	*7*

80>40, so move 40 to index 7
Find parent: 3/2=1→70

	80	**60**	**70**	**10**	**30**	**20**	**40**
0	*1*	*2*	*3*	*4*	*5*	*6*	*7*

80>70, so move 80 to index 1
Find parent: 1/2=0
So, we reached root…done!

11. Maxheap Code

From **MaxheapAsArray.java**
See also **PQAsMaxheapAsArray.java**

11.1 Fields, Constructors

```
private Object[] heap;
private int MAXSIZE;
private int size; // gives pointer to last index in heap

public MaxheapAsArray(int size) {
   MAXSIZE = size;
   heap = new Object[MAXSIZE];
}

public MaxheapAsArray(Object[] stuff) {
   // set # of elems and allotted space:
   MAXSIZE = size = stuff.length+1;

   // need to have empty 0th pos:
   heap = new Object[MAXSIZE];

   // copy stuff into unheaped heap:
   System.arraycopy(stuff,0,heap,1,MAXSIZE);

   // start at root and work "down"
   for (int i = heap.length/2; i > 0; i--) reheap(i);
}
```

11.2 Put

```
public void put(Object o) {

   size++; // increment size more this new item

   // increase array if out of space
   if (size > MAXSIZE) increaseHeapSize();

   int index = size; // index of current free location
   int parent = index/2; // parent of free location

   // Until reaching root, move the parents down
   // while item o > parents:
   while (index > 1 &&
      ((Comparable) o).compareTo(heap[parent]) > 0) {

      heap[index]=heap[parent]; // parent ->child
      index = parent;           // update index
      parent = index/2;         // update parent

   }

   // done finding appropriate location
   // to maintain heapness:
   heap[index] = o;
}
```

11.3 Get

```
public Object get() {
   Object root = null;
   if (!isEmpty()) {
      root = heap[1];     // max item to return
      heap[1]=heap[size]; // root<-recent item added
      size--;             // reduce size
      reheap(1);          // reheap entire tree
   }
   return root;
}
```

`reheap`? See `MaxheapAsArray.java`

11.4 PQ AS Maxheap (AsArray)

```
public class PQAsMaxheapAsArray {
   public static void main(String[] args) {
      SeqStructure pq = new MaxheapAsArray(10);
      pq.put(new PQElement("Bill",3));
      pq.put(new PQElement("Monica",1));
      pq.put(new PQElement("Hillary",4));
      pq.get());
      pq.put(new PQElement("Gennifer",3));
   }
}
```

12. Time analysis

12.1 Nodes and height

- $n = 2^h - 1$
- $2^h = n + 1$
- so, $h = \log(n + 1)$

12.2 put

- height is $\log(n + 1)$
- adding does 1 level at a time, so O(log n)

12.3 get

- similar analysis as height
- O(log n)

13. Exercises

- Implement a PQ with a circular array.
- Implement a PQ with a sorted list. Allow for duplicate items.
- Write **`toTree`** for **`HeapAsArray`** that produces a text-based tree, as we did for you binary trees.
- Rewrite **`HeapAsArray`**'s **`put`** such that the unused array position (index 0) stores the item. Doing so helps to move the **`index > 1`** test in the **`while`** loop.
- Write a **`heapsort`** method inside **`HeapAsArray`**.