# CS211, LECTURE 18: TREES

## ANNOUNCEMENTS:

- Partners?
- A4 and A6...

## READING:

- DS&SD: Chap 13.1-13.3.2; 13.3.4-13.4
- DS&A: Chap 9

## OVERVIEW:

- foundation data struuctures continued!
- motivation for trees
- node and tree classes
- binary trees
- general trees

# 1. Motivation

- Dynamic vs. Static
  - arrays
  - lists
- Linear vs. Hierarchical
  - lists
  - trees
  - no cycles (graphs are later)
- Examples of Trees
  - directories, file systems
  - hierarchy of people
  - more? see Lecture 7
- Want more generic tree classes for ADTs

# 2. Tree Definitions (reminders)

Summarized From Lecture 7:

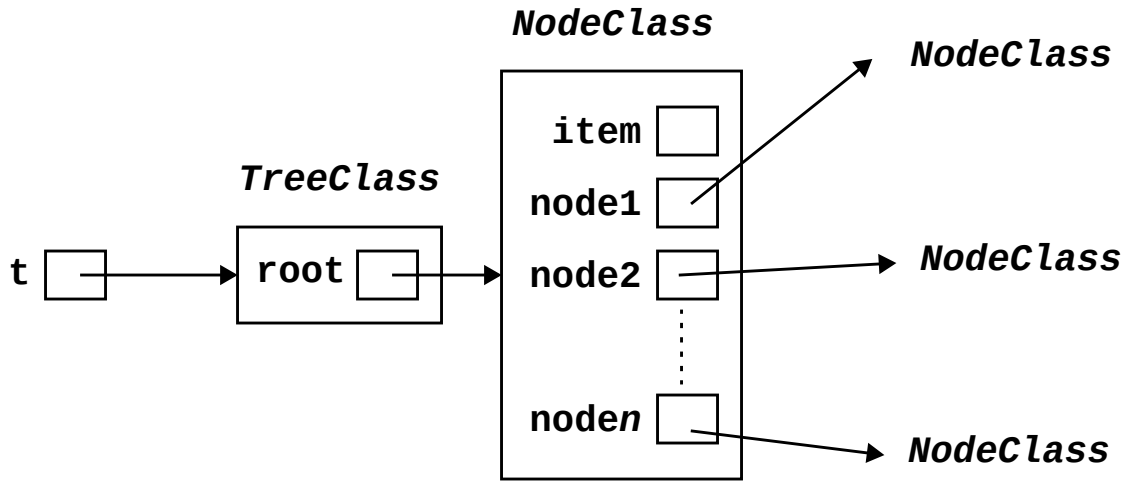
| | |
|---|---|
| ***Node***, ***Point***, ***Vertix*** | **a state or position** |
| ***Line***, ***Edge*** | **connection between nodes** |
| ***Graph*** | **set of vertices (also called *nodes*, *points*) and set of edges (also called *lines*, *arcs*) with each edge connecting a pair of vertices; nodes can connect to any other node** |
| ***Path*** | **sequence of edges in which each edge connects to another edge at each vertex** |
| ***Tree*** | **a graph in which every pair of vertices has a unique path** |
| ***Rooted Tree*** | **The "top" node has no "incoming" edges (no parent); all other nodes have a path to the root; a rooted tree is usually just called *tree*** |
| ***Root*** | **"topmost" node to which all other nodes have a path** |
| ***Parent*** | **node "above" another node, leading towards the root** |
| ***Child*** | **node "below" another node, leading away from root** |
| ***Internal Node*** | **node that has a parent and at least one child** |
| ***Leaf*** | **child node with no children** |
| ***Sibling*** | **node at the same level as another** |
| ***Level***, ***Depth*** | **length of the path from the root to a node; root starts at level 0 (DS&SD say 1)** |
| ***Height*** | ***height of tree*: longest path from root to leaf**<br>***height of node*: longest path length from the node to a leaf** |

# 3. Some Design Decisions

## 3.1 "singly-linked" nodes (links for children only)

- parent links?
- sibling links?

## 3.2 overall design



## 3.3 Java structure: the gist

```
class TreeClass {
   private TreeNode root;

   // constructors
   // getters, setters
   // analysis of ENTIRE Tree

}

class TreeNode { // <-- make inner class of TreeClass?
   private Object item;
   private TreeNode n1;
   private TreeNode n2;

   // constructors
   // getters, setters
   // analys of THIS subtree

}

public class Driver {
   public static void main(String[] args) {

      // build tree (see t in 3.2)
      // add/del nodes & leaves
      // search/sort tree

   }
}
```

# 4. Tree Class

- Name:
  - flexible
  - usually **`BinaryTree`**, **`UnaryTree`**, **`NaryTree`**, …
- Fields:
  - root: "top" of tree
- Methods:
  - building
  - printing
  - traversing
  - height
  - comparing
  - searching (next lecture: search structures)
- Interfaces?
  - see DS&SD **`LinkedSimpleTreeUOS`** and DS&A pg. 261 for examples
  - lots of disagreement in refs about what's crucial
  - see Methods above for generally accepted operations

# 5. Node Classes

## 5.1 General Tree

"random" (dynamic) number of nodes:

```
public class GeneralNode {
   private Object item;      // stored value
   private LinkedList nodes; // list of nodes
   private int degree;       // # of nodes
   // more stuff…
}
```

## 5.2 N-ary Tree

fixed number of nodes:

```
public class NaryNode {
   private Object item;      // stored value
   private NaryTree[] nodes; // array of nodes
   private int degree;       // # of nodes
   // more stuff…
}
```

## 5.3 Binary Tree

2 nodes:

```
public class BinaryNode {
   private Object item;      // stored value
   private BinaryNode left;  // left node
   private BinaryNode right; // right node
   // more stuff…
}
```

# 6. Binary Tree Focus

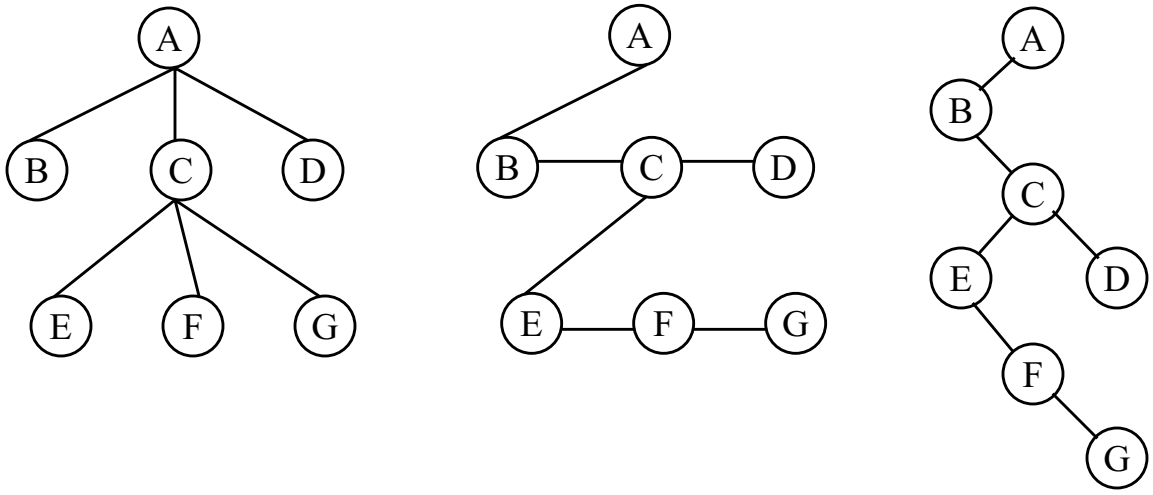
## 6.1 Why?

- clarity of concepts (design, traversal, …)
- applications:
  expression parsing (see A3)
  decision trees
- search techniques (search structures)

## 6.2 general trees can be made from binary trees

Algorithm:

- convert general tree to level-order order tree
- for a node, **`left`** ← **`child`**, **`right`** ← **`sibling`**



# 7. BinaryNode Class

## 7.1 Basics

```
public class BinaryNode {

   private Object item;
   private BinaryNode left;
   private BinaryNode right;

   public BinaryNode() {
      this(null,null,null);
   }
   public BinaryNode(Object item) {
      this(item,null,null);
   }
   public BinaryNode(Object item, BinaryNode left,
                                     BinaryNode right) {
      this.item=item;
      this.left=left;
      this.right=right;
   }

   public Object getItem() { return item; }
   public BinaryNode getLeft() { return left; }
   public BinaryNode getRight() { return right; }
   public void setItem(Object o) { item = o; }
   public void setLeft(BinaryNode n) { left = n; }
   public void setRight(BinaryNode n) { right = n; }

   // more methods

}
```

## 7.2 `toString`

- why: how should a binary tree appear?
- Algorithm:

  pre/post/in/level order?

  - stringify item, stringify children

  - stringification of children stringifies *their* children
- Code:

```
// using pre-order for clarity:
   public String toString() {
      return ""+_____+"("+_____+","+_______+")";
   }
```

## 7.3 `toTree`

- why? like **`toString`**, help with debugging
- Algorithm:

  **`tree`**←**`root`** of subtree (or leaf if no more tree)

  if **`left`** isn't empty,

  ```
  tree +=spacing + left.toString()
  ```

  if **`right`** isn't empty,

  ```
  tree += spacing + right.toString()
  ```

- Code:

```
// Start recursion:
   public String toTree() {
      return toTree("|   ","|___");
   }

// Build tree:
public String toTree(String blank,String spacing)
{
   String s = item.toString() + "\n";
   if (left != null)
      s +=  spacing +
            left.toTree(blank, blank+spacing);
   if (right != null)
      s +=  spacing +
            right.toTree(blank, blank+spacing);
   return s;
}
```

## 7.4 `getHeight`

- Why? help with analysis and search
- Algorithm (also recursive):

  height of leaf is 0

  height of children is

  1+ max( height(**`left`**) , height(**`right`**) )
- Code:

```
public int getHeight() {
   return getHeight(this);
}

private int getHeight(BinaryNode node) {
   // bottom of tree:
   if (node == null)
       return 0;

   // node to children adds 1 to height of node:
   else
       return 1 + Math.max(getHeight(node.left),
                           getHeight(node.right));
}
```

## 7.5 Saving For later/another time

- merging
- comparison
- **`equals`**
- traversing (using iterators for trees: needs stacks, queues)
- API: **`TreeMap`**, **`TreeSet`**
- parent links in tree

## 7.6 Demo

```
public class TestBinaryTree {
   public static void main( String [ ] args ) {
      // I built this tree in Lecture 7:

       /*
             0
          1     2
         3 4   5 6

       */

      System.out.println(t0);
      System.out.println(t0.toTree());

    }
}

/* Output:

0(1(3(null,null),4(null,null)),2(5(null,null),
6(null,null)))
0
|___1
|   |___3
|   |___4
|___2
|   |___5
|   |___6

*/
```

# 8. General Tree

- General (dynamic), N-ary (kind of static)
- showing general here

```
import java.util.*;
public class GeneralNode {
   private Object item;
   private LinkedList nodeList;

   public GeneralNode() {
      nodeList = new LinkedList();
   }
   public GeneralNode(Object item) {
      this.item=item;
      if (nodeList==null)
         nodeList = new LinkedList();
   }
   public void addNode(GeneralNode node) {
      nodeList.add(node);
   }

   public String toString() {
      String s = ""+item;
      Iterator it = nodeList.iterator();
      if (it.hasNext()) s += "(";
      while(it.hasNext()) {
         s += it.next();
         if (it.hasNext()) s += ",";
         if (!it.hasNext()) s += ")";
      }
      return s;
   }
```

```
    public String toTree() {
       return toTree("|   ","|___");
    }

    public String toTree(String blank, String spacing) {
       String t = item.toString() + "\n";
       for ( Iterator it = nodeList.iterator();
                                        it.hasNext(); ) {
          if (it.hasNext()) {
             GeneralNode next = (GeneralNode) it.next();
             t += spacing +
                (next.toTree(blank, blank+spacing));
          }
       }
    return t;
    }

} // Class GeneralNode

public class GeneralTree {
    private GeneralNode root;
    public GeneralTree(GeneralNode node) {
       root = node;
    }

    public String toString() {
       return root.toString();

    }
    public String toTree() {
       return root.toTree();
    }
}
```

```
public class TestGeneralTree {
   public static void main( String [ ] args ) {
      GeneralNode b0 = new GeneralNode("0");
      GeneralTree t0 = new GeneralTree(b0);
      GeneralNode b1 = new GeneralNode("1");
      GeneralNode b2 = new GeneralNode("2");
      GeneralNode b3 = new GeneralNode("3");
      GeneralNode b4 = new GeneralNode("4");
      GeneralNode b5 = new GeneralNode("5");
      GeneralNode b6 = new GeneralNode("6");
      GeneralNode b7 = new GeneralNode("7");
      GeneralNode b8 = new GeneralNode("8");
      GeneralNode b9 = new GeneralNode("9");

      b0.addNode(b1);
      b0.addNode(b2);
      b0.addNode(b3);
      b1.addNode(b4);
      b4.addNode(b5);
      b5.addNode(b6);
      b5.addNode(b7);
      b5.addNode(b8);
      b5.addNode(b9);

      System.out.println(t0);
      System.out.println(t0.toTree());
   }
}
```

# 9. Suggested Exercises

Write method(s) to get the depth of any node.

Why is there a need for the **`merge`** method in **`BinaryTree`**? That is, why shouldn't we just provide the constructor? See 13.3.1 in DS&SD:

```
public BinaryTree(Object item, BinaryTree t1,
                  BinaryTree t2) {
   root = new BinaryNode(item);
   root.setLeft(t1.root);
   root.setRight(t2.root);
}
```

Write method(s) to get the number of nodes for any subtree in a binary tree.

Write a program (or collection of methods) that converts a general tree to a binary tree.

Add **`getHeight`** and setters/getters to the general tree classes.