LECTURE 15: SORTING

ANNOUNCEMENTS:

READING:

DS&SD: pg. 92, 98-100; 5.3, 18.1-18.4, 18.5 is optional

DS&A: Chap. 15

OVERVIEW:

Motivation and applications for sorting

Brief mathematical theory (optional)

Example basic technique: select sort

More sophisticated: quick sort, merge sort

Choosing a sorting technique

Realistic choice: API's `sort`

Suggested exercises

1. Motivation

1.1 Help Searching

- see binary search

Study of Algorithms

1.2 Trend Analysis

- economic trends
- scientific trends
- grading trends

2. Mathematical Theory

2.1 See DS&A, pp. 491-492

2.2 The Gist:

- you need sortable values
- you need to collect some values together
- you need a comparison relationship

With these "standards" you can mathematically organize your information!

2.3 ***Stable sort***:

- why learn? API uses this term a lot!
- problem: equal elements pose some problems
- stable sort maintains equal element's relative positions

3. Sorting Classifications

Insertion

- insert sort

selection

- select sort
- heap sort

exchange

- bubble sort
- quick sort

merge

- merge sort

distribution

- bucket sort
- radix sort

4. Select Sort

4.1 Motivation

- ease in remembering
- ease in understanding

4.2 Algorithm

- pick an element from end of unsorted portion of array
- compare that element with the rest of the elements
- if another element is the largest/smallest, swap with the current element
- repeat

4.3 Code

See **`SelectSortUp`** and **`BasicSorting`**

```
// sort in ascending order:
public static void selectSortUp(Comparable[] a) {

      for (int i=0; i < a.length; i++) {

            int minPos=i;

            for (int rem=i+1; rem < a.length; rem++)
                  if (a[rem].compareTo(a[minPos]) < 0)
                        minPos=rem;

            Comparable tmp = a[i];
            a[i]=a[minPos];
            a[minPos]=tmp;

      }
}
```

4.4 Analysis

See DS&SD, pp. 98-100

- $T(n) = 2n^2 + 7n - 7$
- $\Rightarrow T(n) \in O(n^2)$

The Gist: count comparisons (**`compareTo`**s)

- assume n elements in array
- 1st time: n-1 comparisons for i=0 elem
- 2nd time: n-2 comparisons for i=1 elem
- ...
- last time: 1 comparison for i=n-1 elem

Derivation:

- Total: $(n-1) + (n-2) + \ldots + 1$
- Math: $1 + 2 + \ldots + n = \frac{(n)(n+1)}{2}$
- So, $TA(n) = \frac{(n-1)n}{2} \in O(n^2)$

5. Quick Sort

5.1 Motivation

- want to improve time
- want to keep sorting in place (not create tmp space)

5.2 Algorithm

Divide and conquer!

- Given array x and pivot value p
- Partition array into subarrays Right and Left
  R contains elems of x >= p
  L contains elems of x < p
- Concatenate elements into [L p R]
- Sort R and L separately (recursive!)

5.3 Diagram

```
60 20 30 10 40 50 70 90 80    pick p=40


20 10 30 40 60 50 70 90 80    pick p=30 and 50


20 10 30 40 50 60 70 90 80    pick p=20 and 80


10 20 30 40 50 60 70 80 90
```

5.5 Partitioning

```
B Y Y B Y Y B Y B B Y Y   blue and yellow
|                     |
L                     R   L and R “fingers”
```

We want to move blues and yellows to their own sides
(blue on left, yellow on right)

So, move the fingers on each side until hit wrong color

```
B Y Y B Y Y B Y B B Y Y
  |                 |
  L                 R
```

We can swap colors and keep advancing indices

```
B B Y B Y Y B Y B Y Y Y
    |           |
    L           R
B B B B Y Y B Y Y Y Y Y
        |   |
        L   R
```

Eventually, L and R fingers will cross or match:

```
B B B B B Y Y Y Y Y Y Y
          |
         L,R
```

We’re done!

5.4 Top-Level Design

Need recursive method for sorting:

```
//    sort x[a..b]:
      void quickSort(Comparable[] x, int a, int b)
```

QuickSort picks pivot, splits array, sorts each portion…

So, we need way to partition array:

- move items < pivot to one side (left)
- move items > pivot to the other side (right)

```
//    reorganize array into [left,pivot,right]:
      int partition(Comparable[] x, int low,
                        int high, Comparable pivot)
```

5.6 Code for Partition

```
public static int partition(Comparable[] x,
                int low, int high, Comparable pivot) {
      int i = low; int j = high;
      boolean flag = true;

      while (flag) {

      // advance left index to the right:
            while((x[i].compareTo(pivot)<0) &&
                        (i < x.length))  i++;
      // advance right index to the left:
            while((pivot.compareTo(x[j])<0) &&
                        (j >= 0)) j--;

      //    swap:
            if (i < j) {
                  Comparable temp = x[i];
                  x[i] = x[j];
                  x[j] = temp;
                  i++;
                  j--;
            }

      //    indices have "met" (i==j):
            else  flag=false;
      }
      return i;
}
```

5.7 Sorting Left and Right

Partitioning moves elements to left (<) or right (>=)

How to pick pivot?

- median value (expensive)
- use first value of array (kind of random)
- can use middle element or average of 1st, last, middle

5.8 Code for `quickSort`

```
public static void quickSort(Comparable[] x,
                                               int a, int b) {
      if (a < b) {
      //   pick "random" pivot and partition x:
           int p = partition(x,a+1,b,x[a]);

      //   move pivot into its final resting place
      //   swap x[p-1] and x[a]:
           Comparable temp = x[p-1];// a..p-1,p,p+1..b
           x[p-1] = x[a];
           x[a] = temp;

      //   sort left and right partitions:
           quickSort(x,a,p-1);
           quickSort(x,p,b);
      }
}
```

5.9 Analysis

Worst case: wretched pivot (largest or smallest elem)

- $T(n) \in O(n^2)$ (see books for proof)

Best case: picked good pivot

- $T(n) \in O(n \log n)$ (see books for proof)

5.10 Improvements on Quick Sort

Pad left and right ends of array with sentinels:

- **`Integer.MIN_VALUE`**
- **`Integer.MAX_VALUE`**
- So, **`while((x[i].compareTo(pivot)<0) &&(i < x.length)`**) becomes **`while(x[i]<pivot)`**

Quick sort is bad on small arrays

- Use insert sort (see **`BasicSort`** example) for base case when array is 10 elements or less

Even more improvements! (see books)

6. Merge Sort

6.1 Motivation

- not worried about needing extra space for sorting
- want a dependable, “cheap” algorithm

6.2 Algorithm: Divide and conquer!

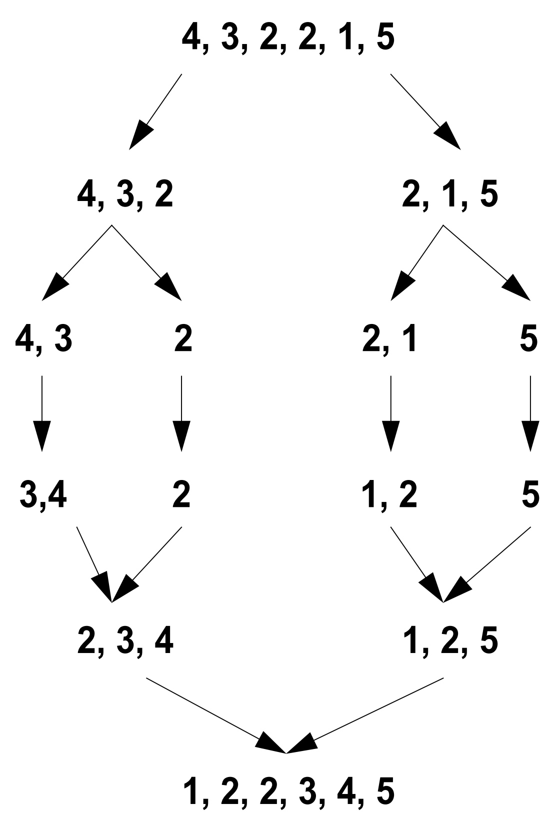
Divide array in halves (keep track of indices)

sort each part (call merge sort recursively)

merge sorted arrays a1 and a2:

- create array m: size(m) = size(a1)+size(a2)
- keep 3 indices: p1 for a1, p2 for a2, pm for m (p1, p2, m initialized to 0)
- compare a1[0] and a2[0] & move smaller (say a2) into 1st element of m, increment p2 and pm
- general: compare a1[p1] with a2[p2] and move smaller one into m[pm] and increment corresponding index
- if a1 or a2 depleted, copy remaining elems into m

6.3 Diagram



6.4 Code

```
public static Comparable[] mergeSort(
                 Comparable[] x, int low, int high) {

      // at least three elements:
      if (low < high - 1) {
            int mid = (low + high)/2;
            Comparable[] x1=mergeSort(x,low,mid);
            Comparable[] x2=mergeSort(x,mid+1,high);
            return merge(x1,x2);
      }

      // 0, 1, or 2 elements:
      else {
            int length = high - low +1;
            Comparable[] r = new Comparable[length];
            if (length == 1) r[0] = x[low];
            if (length == 2)
                  if(x[low].compareTo(x[high]) < 0) {
                        r[0] = x[low];
                        r[1] = x[high];
                  } else {
                  r[0] = x[high];
                  r[1] = x[low];
                  }
            return r;
      }
}
```

```
public static Comparable[] merge(
                 Comparable[] a1, Comparable[] a2) {

      Comparable[] m =
            new Comparable[a1.length + a2.length];

      int p1 = 0;
      int p2 = 0;
      int pm = 0;

      while ((p1 < a1.length) && (p2 < a2.length))
            if (a1[p1].compareTo(a2[p2]) <= 0)
                  m[pm++] = a1[p1++];
            else m[pm++] = a2[p2++];

      // either a1 or a2 will empty at this point:

            // merge any leftovers from a1
            for(; p1 < a1.length; p1++)
                  m[pm++] = a1[p1];

            // merge any leftovers from a2
            for(; p2 < a2.length; p2++)
                  m[pm++] = a2[p2];

      return m;

      }
}
```

6.5 Analysis of Merge Sort

Asymptotic complexity: $O(n \log n)$

refer to textbooks for proof

MS is asymptotically optimal algorithm for sorting

disadvantage: needs extra storage for merging

7. API ("Real World")

See http://java.sun.com/j2se/1.4/docs/api/java/util/
Arrays.html#sort(java.lang.Object[])

`public static void sort(Object[] a)`

- a must implement **`Comparable`**
- guaranteed to be stable
- modified merge sort!

See http://java.sun.com/j2se/1.4/docs/api/java/util/
Collections.html#sort(java.util.List)

`public static void sort(List list)`

- a must implement **`Comparable`**
- guaranteed to be stable
- modified merge sort!

8. Suggested Exercises

- determine how to do bubble and insert sorts; determine their asymptotic complexity
- write select sort from left to right and right to left; sort up and sort down
- write non-recursive merge sort (see DS&SD: 5.3)
- improve quick sort with "median of 3" pivot