http://www.cs.cornell.edu/courses/cs1110/2020sp

# Lecture 15: Recursion

(Sections 5.8-5.10)

CS 1110

Introduction to Computing Using Python

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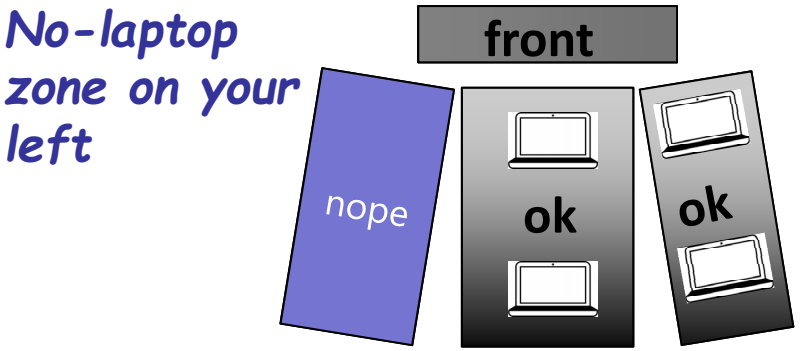
# Announcements




- Prelim 1 feedback expected by Sunday
- Read § 5.8-5.10 (if you haven't done so already)

# Thinking about upcoming changes

- We know everyone is stressed out ☹ As situation and planning evolve we'll keep you posted
- **Lecture**: recording is available. If you want to avoid lecture room even before break, it's ok. You can view recording instead.
- **Labs**: exercises online. We're trying to work out ways to provide interactive help somehow. Will need combination of technologies and platforms
- **Office/consulting hours**: ditto
- How will future **exams** work? This is difficult to deal with. In discussion inside and outside CS to come up with solution.
- Please use **Piazza**! Good way to get answers to clarification questions.

# Recursion

**Recursive Function**:

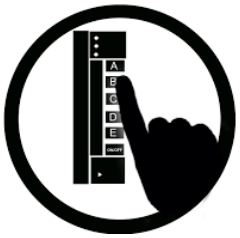
A function that calls itself

(see also Recursive Function)

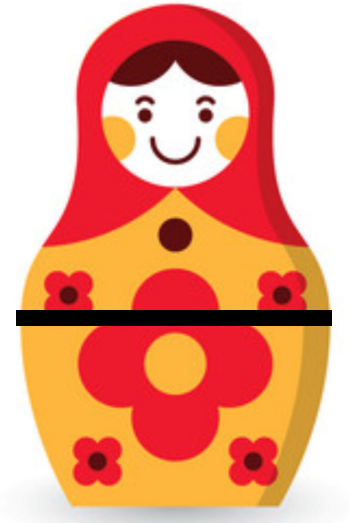
**Two parts to every recursive function:**

1. A simple case: can be solved easily
2. A complex case: can be made simpler (and simpler, and simpler… until it looks like the simple case)

# Russian Dolls!




# What is the simple case that can be solved easily?



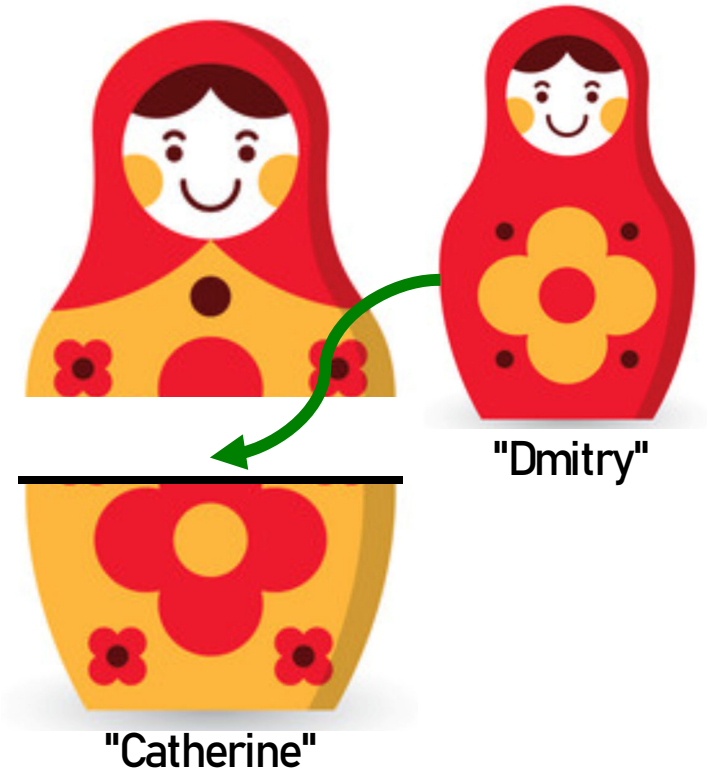
A: The case where the doll has a seam and another doll inside of it.

B: The case where the doll has no seam and no doll inside of it.

C: A & B are both simple

D: I do not know

# Russian Dolls!




Global Space

| | |
|---|---|
| d1 | id1 |
| d2 | id2 |

Heap Space

id1 — Doll

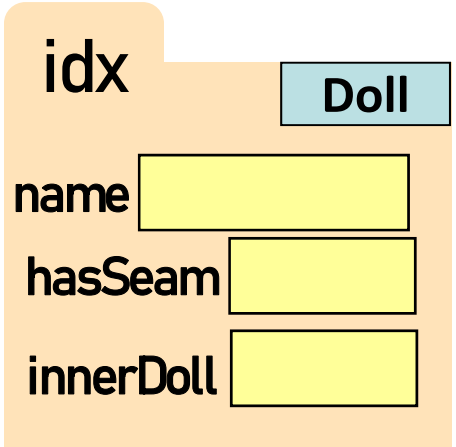
| | |
|---|---|
| name | "Dmitry" |
| hasSeam | False |
| innerDoll | None |

id2 — Doll

| | |
|---|---|
| name | "Catherine" |
| hasSeam | True |
| innerDoll | id1 |

```
import russian
d1 = russian.Doll("Dmitry", None)
d2 = russian.Doll("Catherine", d1)
```

```
def open_doll(d):
    """Input: a Russian Doll
     Opens the Russian Doll d """
     print("My name is "+ d.name)
     if d.hasSeam:
         inner = d.innerDoll
         open_doll(inner)
     else:
         print("That's it!")
```



# Examples

- Russian Dolls
- **Blast Off!**
- Factorial
- Deblank

# Blast Off!

```
blast_off(5) # must be a non-negative int
5
4
3
2
1
BLAST OFF!

blast_off(0)
BLAST OFF!
```

# Blast Off!

```
def blast_off(n):
    """Input: a non-negative int
     Counts down from n to Blast-Off!
     """
    if (n == 0):
        print("BLAST OFF!")
    else:
        print(n)
        blast_off(n-1)
```

# A Mathematical Example: Factorial

- Non-recursive definition:

  $n! = n \times n\text{-}1 \times \ldots \times 2 \times 1$

  $= n\,(n\text{-}1 \times \ldots \times 2 \times 1)$

- Recursive definition:

  $n! = n\,(n\text{-}1)!$ for $n > 0$ **Recursive case**

  $0! = 1$ **Base case**

What happens if there is no base case?

# Factorial as a Recursive Function

```
def factorial(n):
    """Returns: factorial of n.
    Pre: n ≥ 0 an int"""
    if n == 0:
        return 1

    return n*factorial(n-1)
```

- $n! = n\ (n\text{-}1)!$
- $0! = 1$

**Base case(s)**

**Recursive case**

What happens if there is no base case?

# Recursion vs Iteration

- **Recursion** is ***provably equivalent*** to **iteration**
  - Iteration includes **for-loop** and **while-loop** (later)
  - Anything can do in one, can do in the other
- But some things are easier with recursion
  - And some things are easier with iteration
- Will **not** teach you when to choose recursion
  - That's for upper level courses
- We just want you to ***understand the technique***

# Recursion is great for Divide and Conquer

**Goal**: Solve problem P on a piece of data

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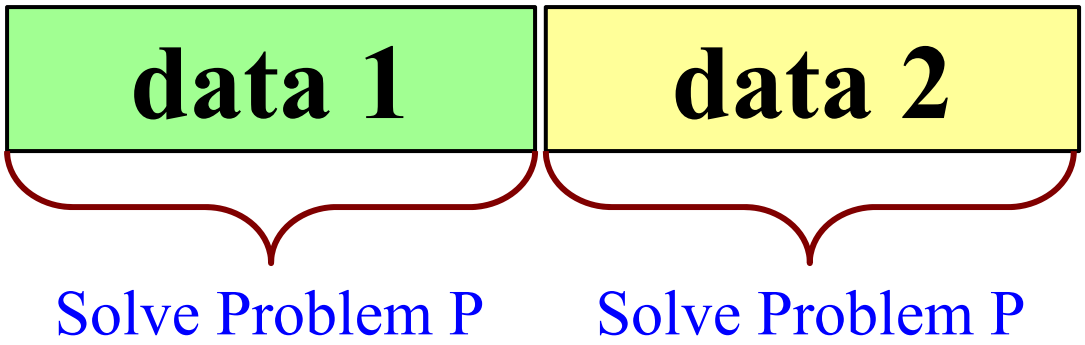
**Idea**: Split data into two parts and solve problem



# Recursion is great for Divide and Conquer

**Goal**: Solve problem P on a piece of data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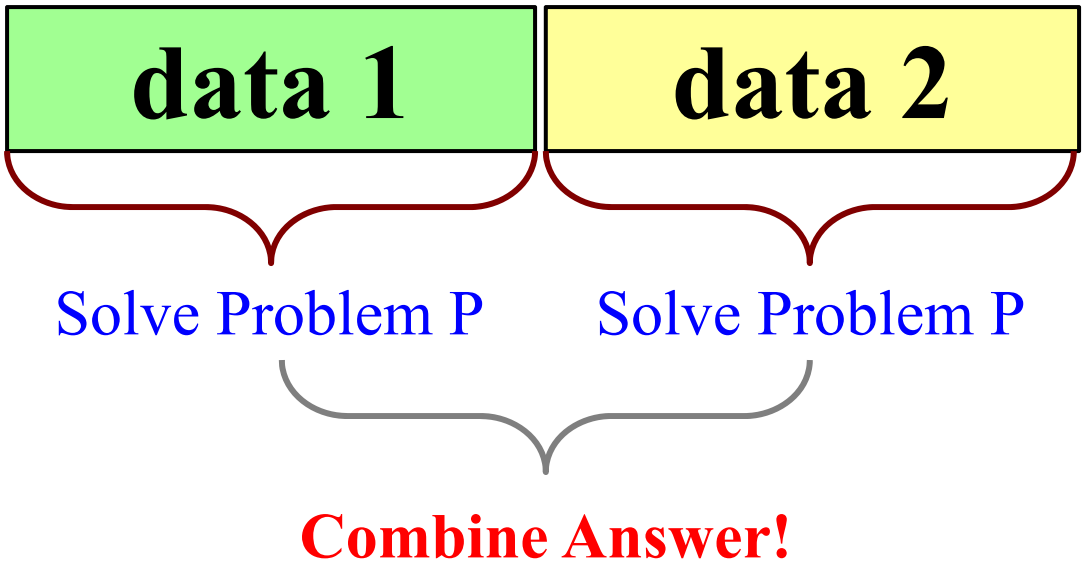
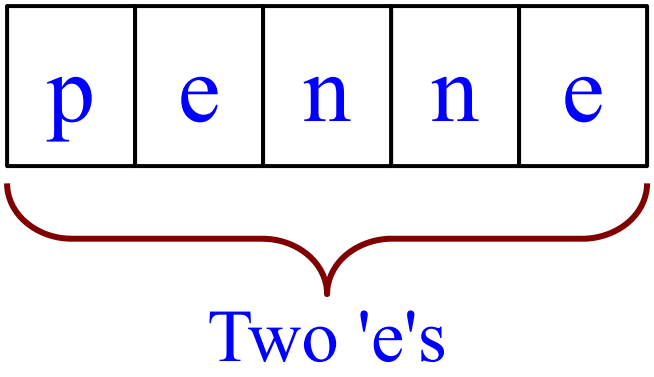
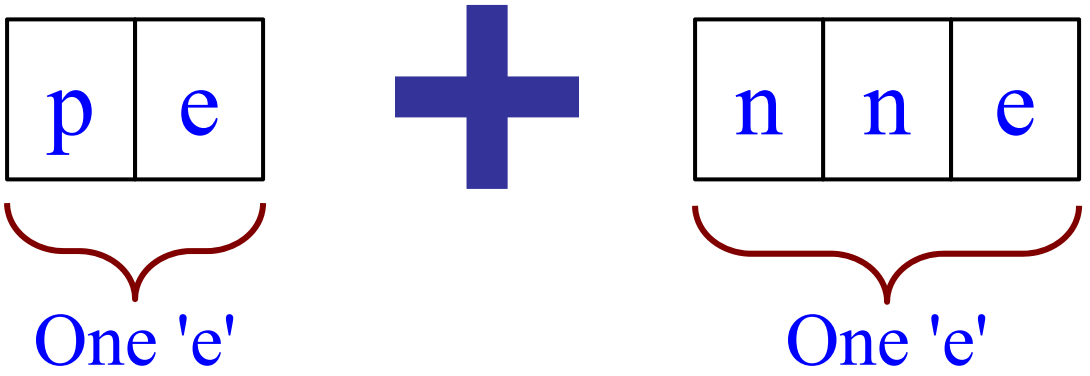


# Divide and Conquer Example

Count the number of 'e's in a string:



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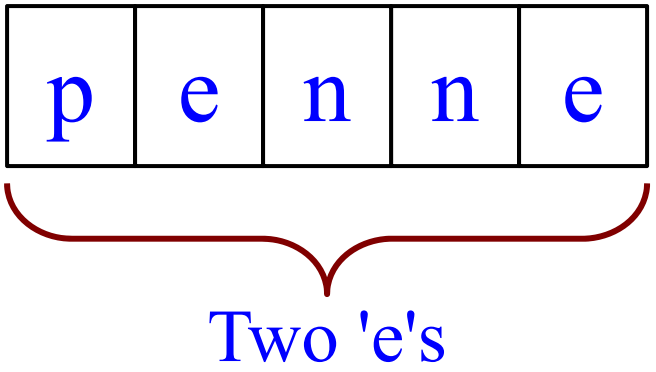


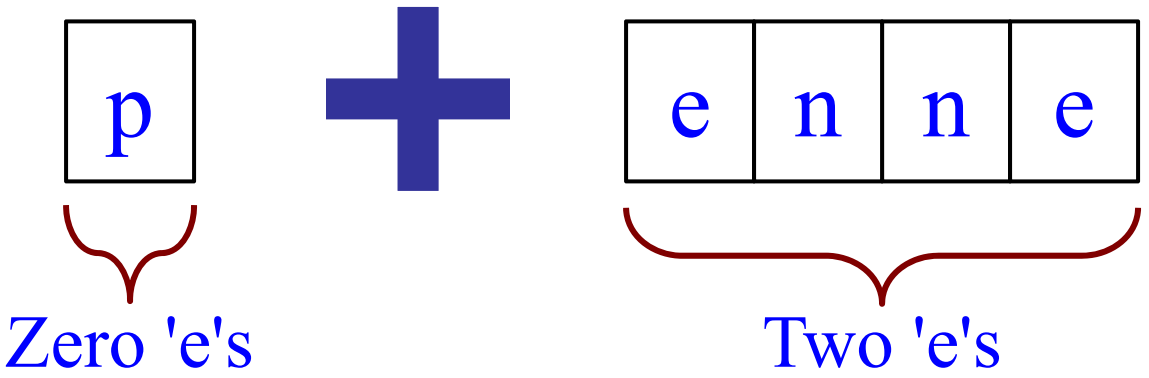

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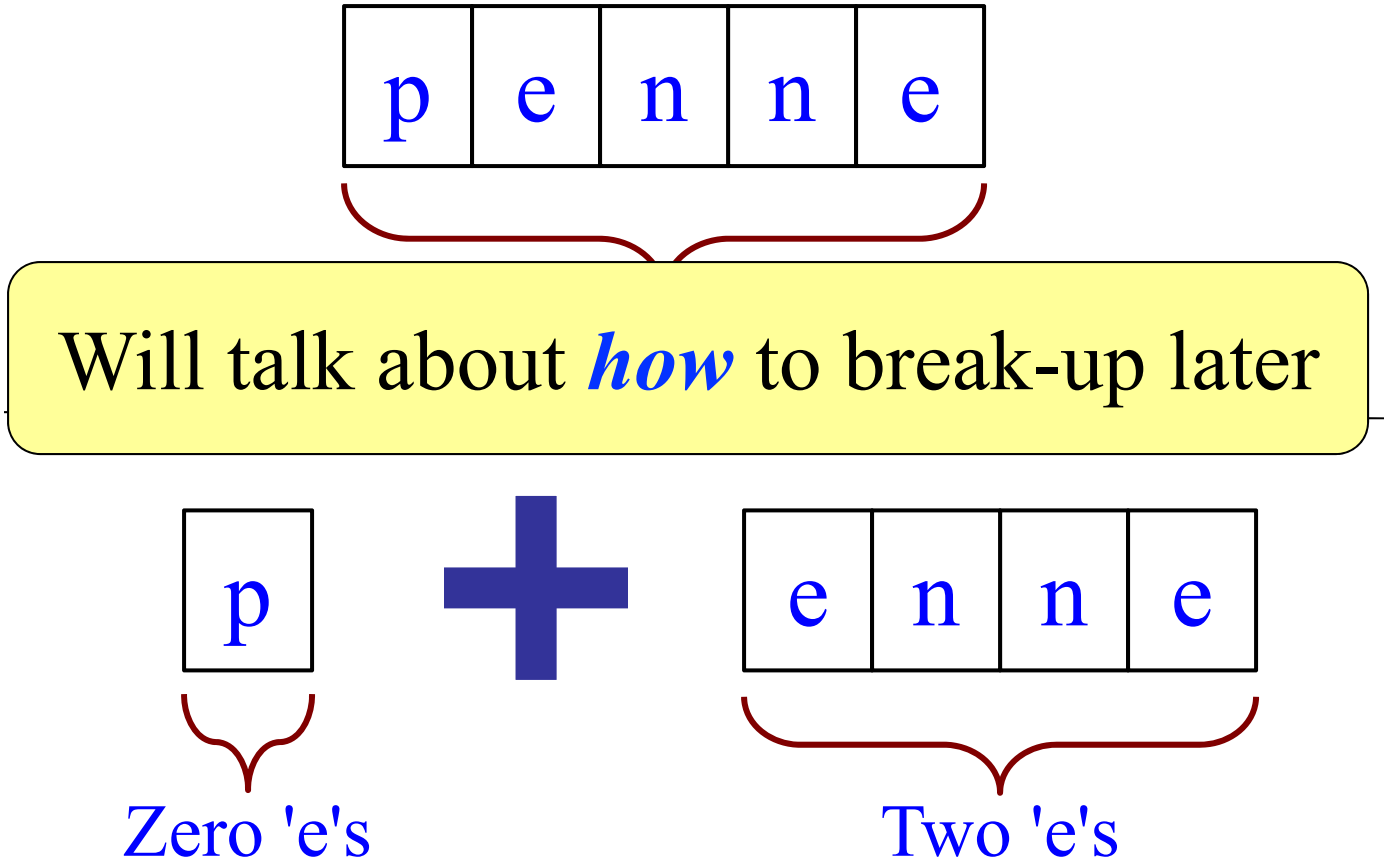

# Divide and Conquer

**Goal**: Solve really big problem P

**Idea**: Split into simpler problems, solve, combine

**3 Steps:**

1. Decide what to do for simple cases
2. Decide how to break up the task
3. Decide how to combine your work

# Three Steps for Divide and Conquer

1. Decide what to do on "small" data
   - Some data cannot be broken up
   - Have to compute this answer directly
2. Decide how to break up your data
   - Both "halves" should be smaller than whole
   - Often no wrong way to do this (next lecture)
3. Decide how to combine your answers
   - Assume the smaller answers are correct
   - Combining them should give bigger answer

# Divide and Conquer Example

```
def num_es(s):
    """Returns: # of 'e's in s"""
    # 1. Handle small data
    if s == '':
        return 0
    elif len(s) == 1:
        return 1 if s[0] == 'e' else 0

    # 2. Break into two parts
    left = num_es(s[0])
    right = num_es(s[1:])

    # 3. Combine the result
    return left+right
```

# Divide and Conquer Example

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def num_es(s):
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```

"Short-cut" for

```
if s[0] == 'e':
    return 1
else:
    return 0
```

# Divide and Conquer Example

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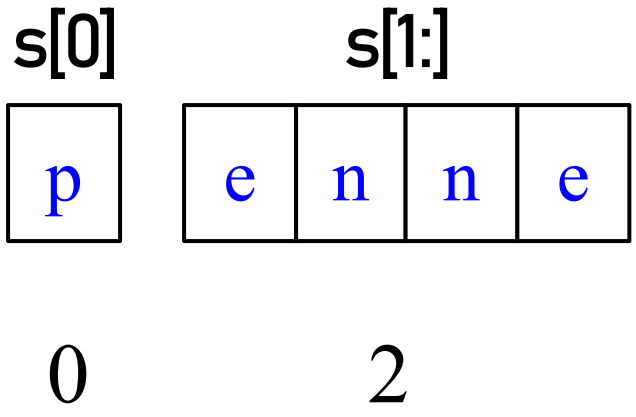

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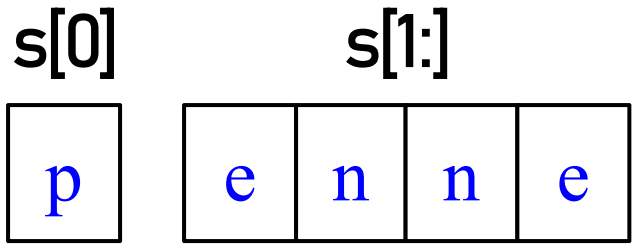


0 + 2

# Divide and Conquer Example

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def num_es(s):
    """Returns: # of 'e's in s"""
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    # 2. Break into two parts
    left = num_es(s[0])
    right = num_es(s[1:])

    # 3. Combine the result
    return left+right
```

Base Case

Recursive Case

# Exercise: Remove Blanks from a String

```
def deblank(s):
    """Returns: s but with its blanks removed"""
```

1. Decide what to do on "small" data

- If it is the **empty string**, nothing to do

```
if s == '':
    return s
```

- If it is a **single character**, delete it if a blank

```
if s == ' ':    # There is a space here
    return ''  # Empty string
else:
    return s
```

# Exercise: Remove Blanks from a String

```
def deblank(s):
    """Returns: s but with its blanks removed"""
```

2. Decide how to break it up

```
left = deblank(s[0])      # A string with no blanks
right = deblank(s[1:])    # A string with no blanks
```

3. Decide how to combine the answer

```
return left+right         # String concatenation
```

# Putting it All Together

```
def deblank(s):
    """Returns: s w/o blanks"""
    if s == '':
        return s
    elif len(s) == 1:
        return '' if s[0] == ' ' else s

    left = deblank(s[0])
    right = deblank(s[1:])

    return left+right
```

Handle small data

Break up the data

Combine answers

# Putting it All Together

```
def deblank(s):
    """Returns: s w/o blanks"""
    if s == '':
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    left = deblank(s[0])
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    return left+right
```
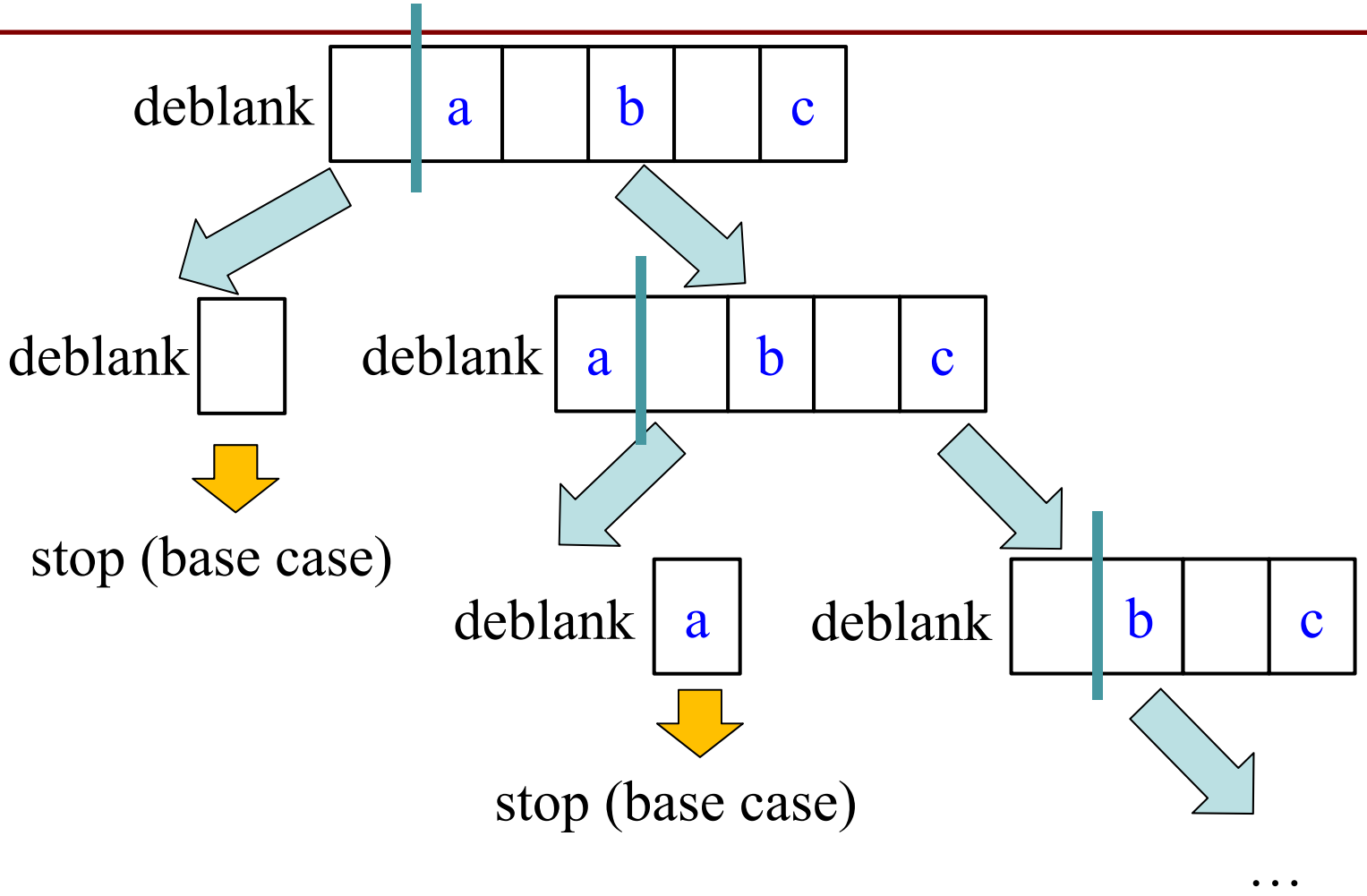

Base Case

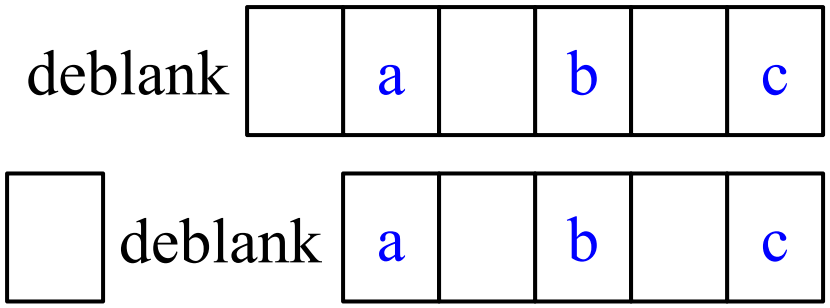
Recursive Case

# Following the Recursion

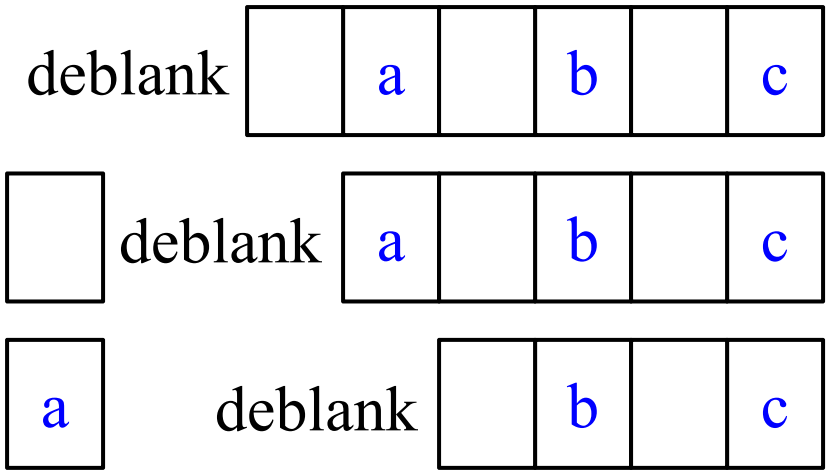



You really, really, really want to **visualize a call of `deblank` using Python Tutor**. Pay attention to the recursive calls (call frames opening up), the completion of a call (sending the result to the call frame "above"), and the resulting accumulation of the answer.

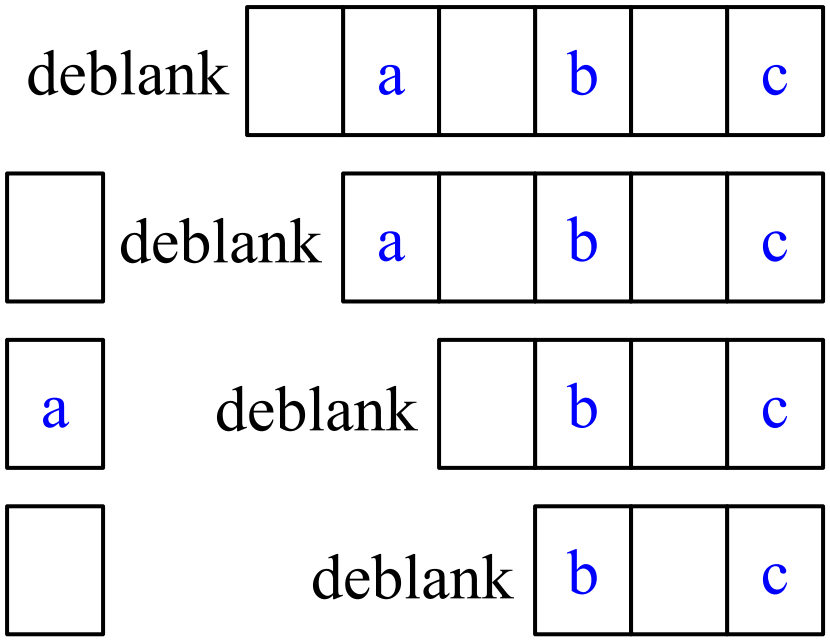
# Breaking it up (1)



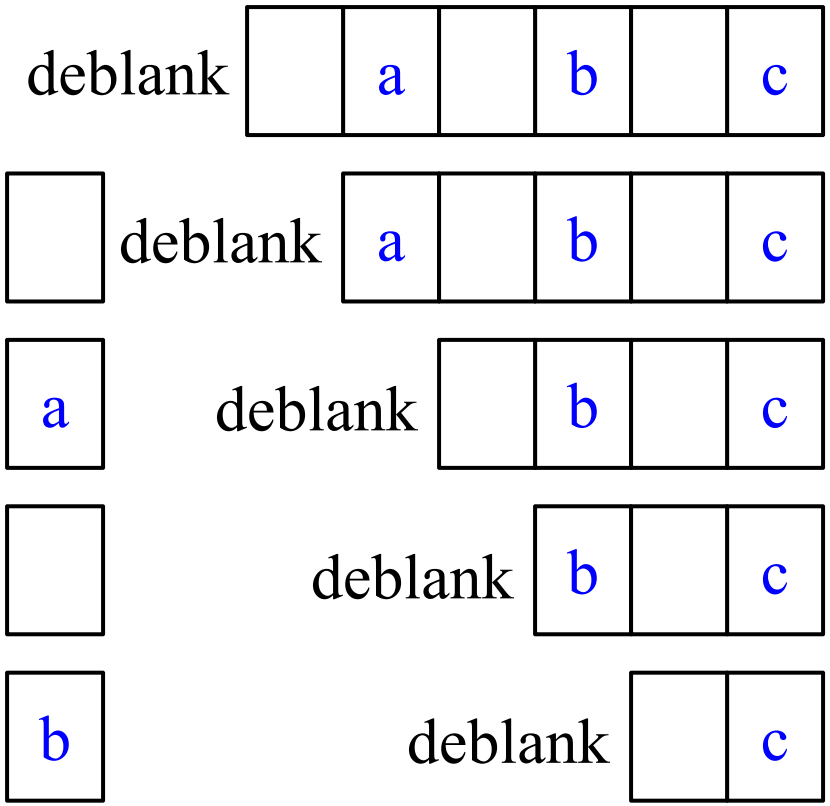
# Breaking it up (2)



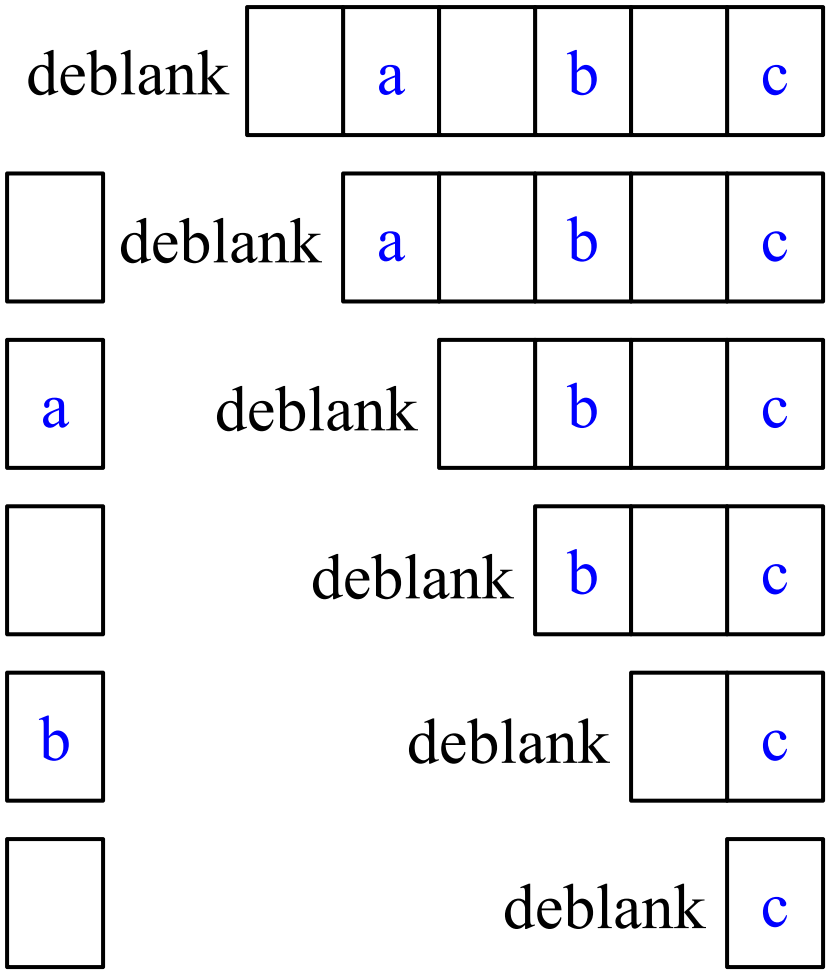
# Breaking it up (3)



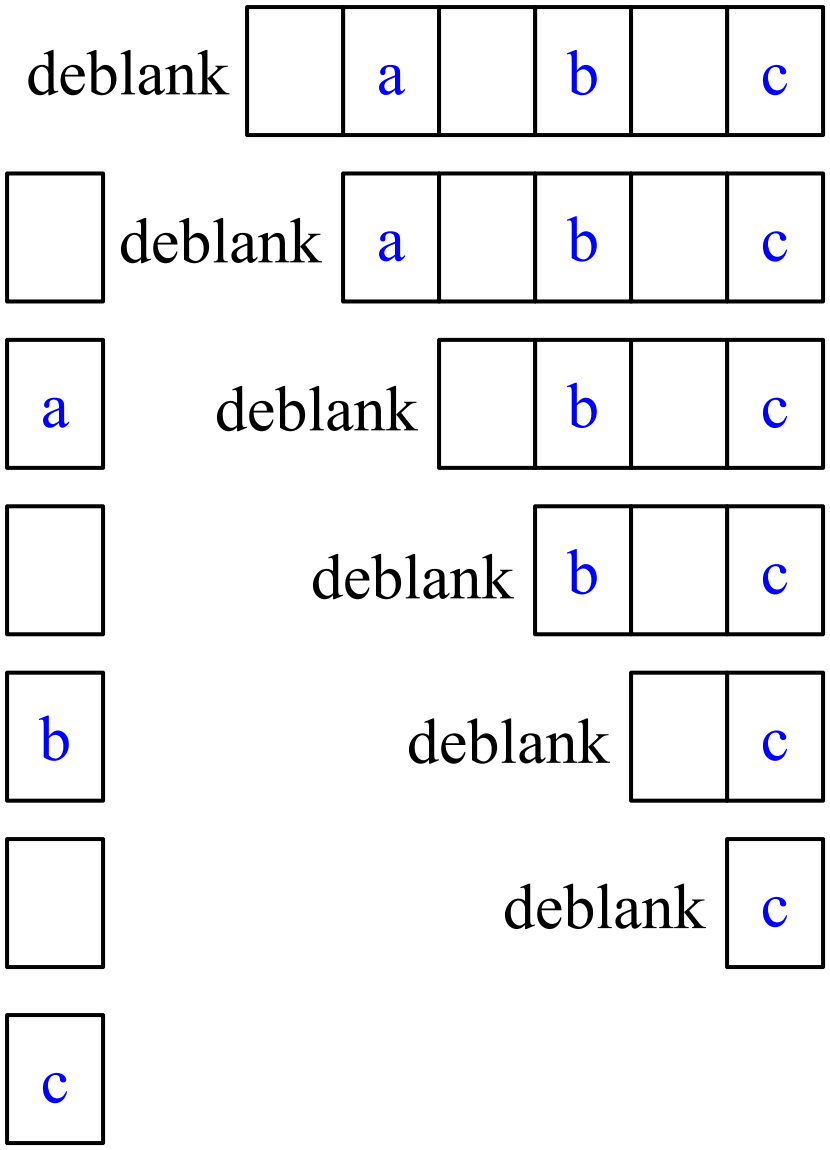
# Breaking it up (4)



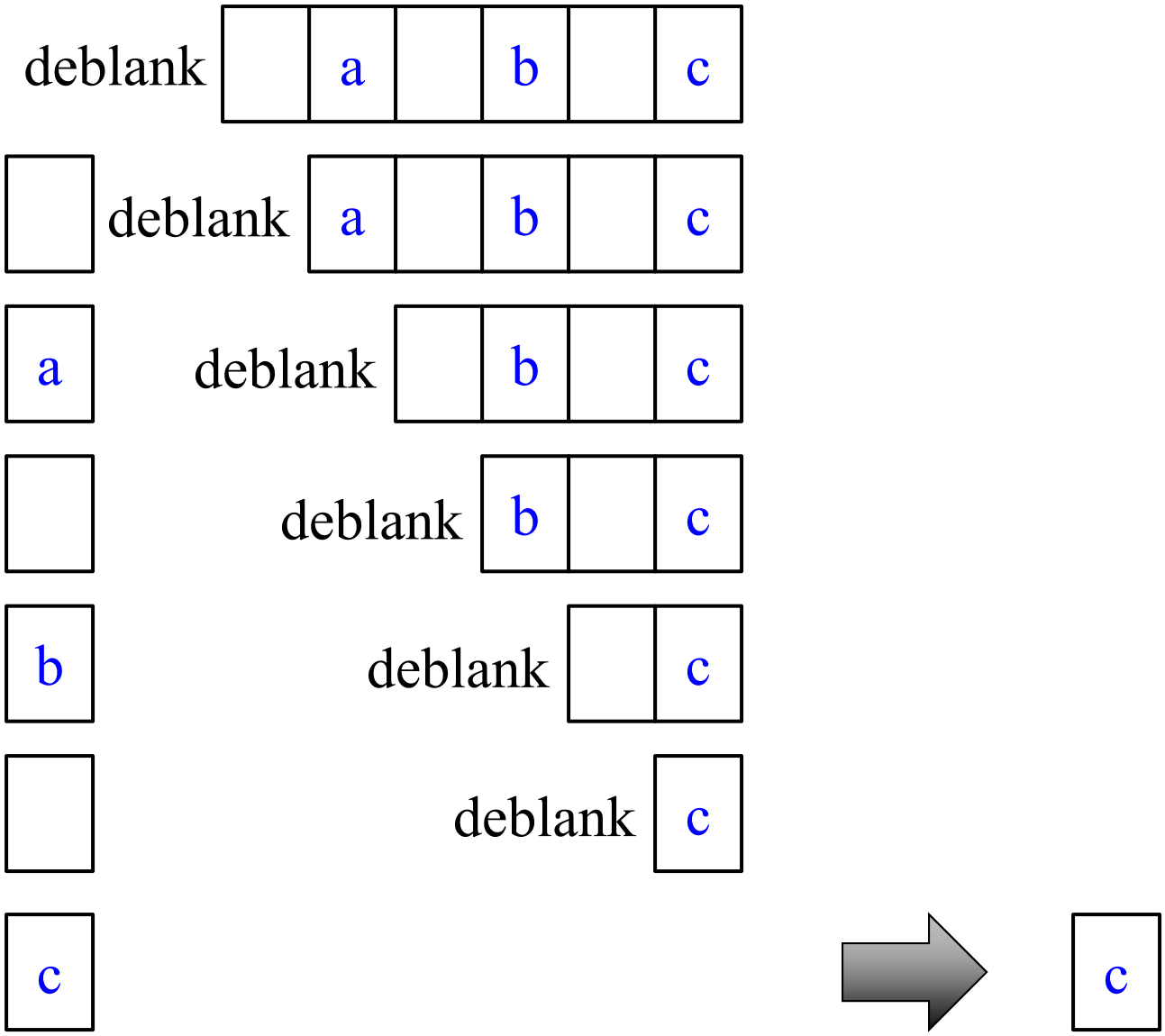
# Breaking it up (5)



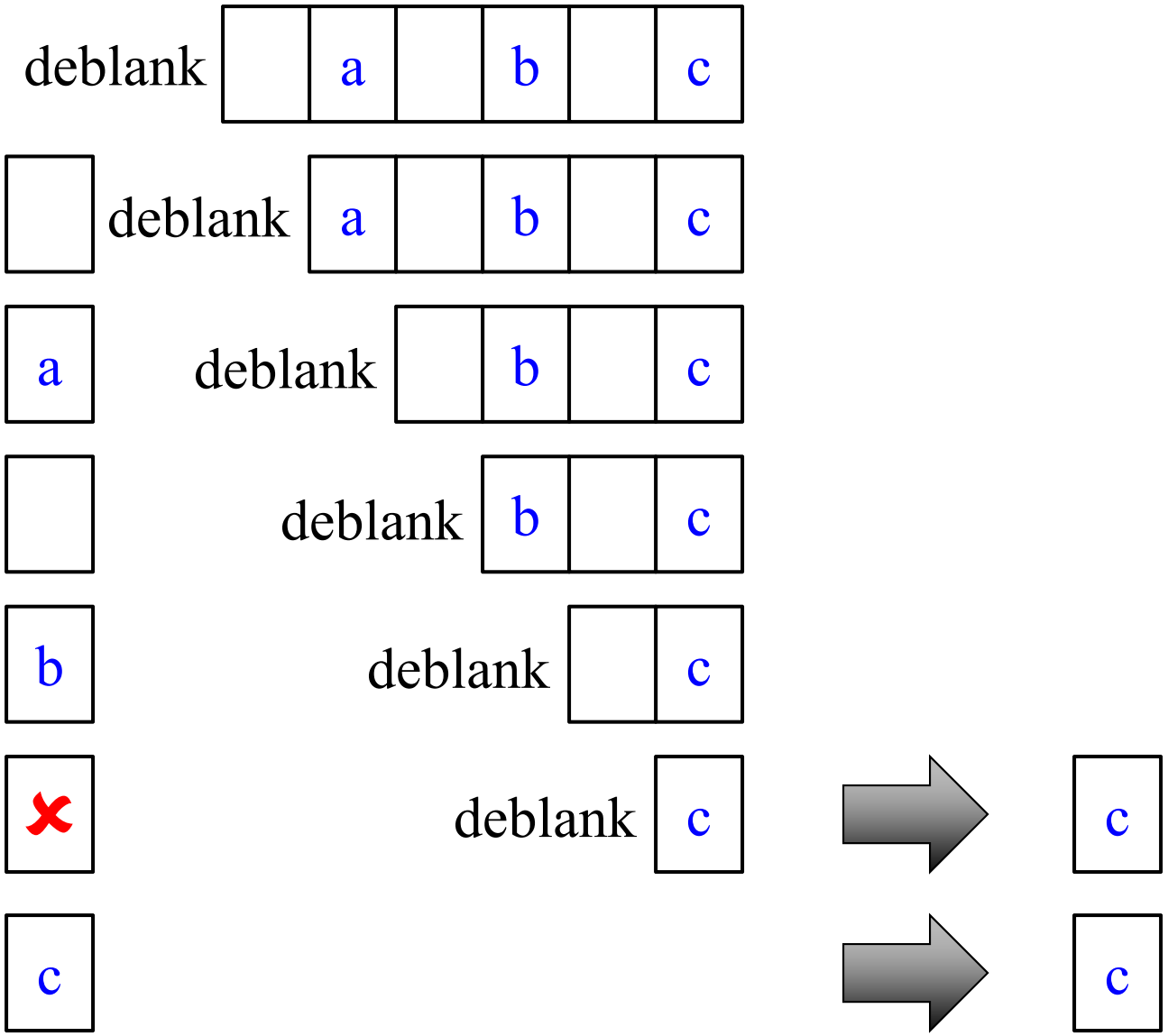
# Breaking it up (6)



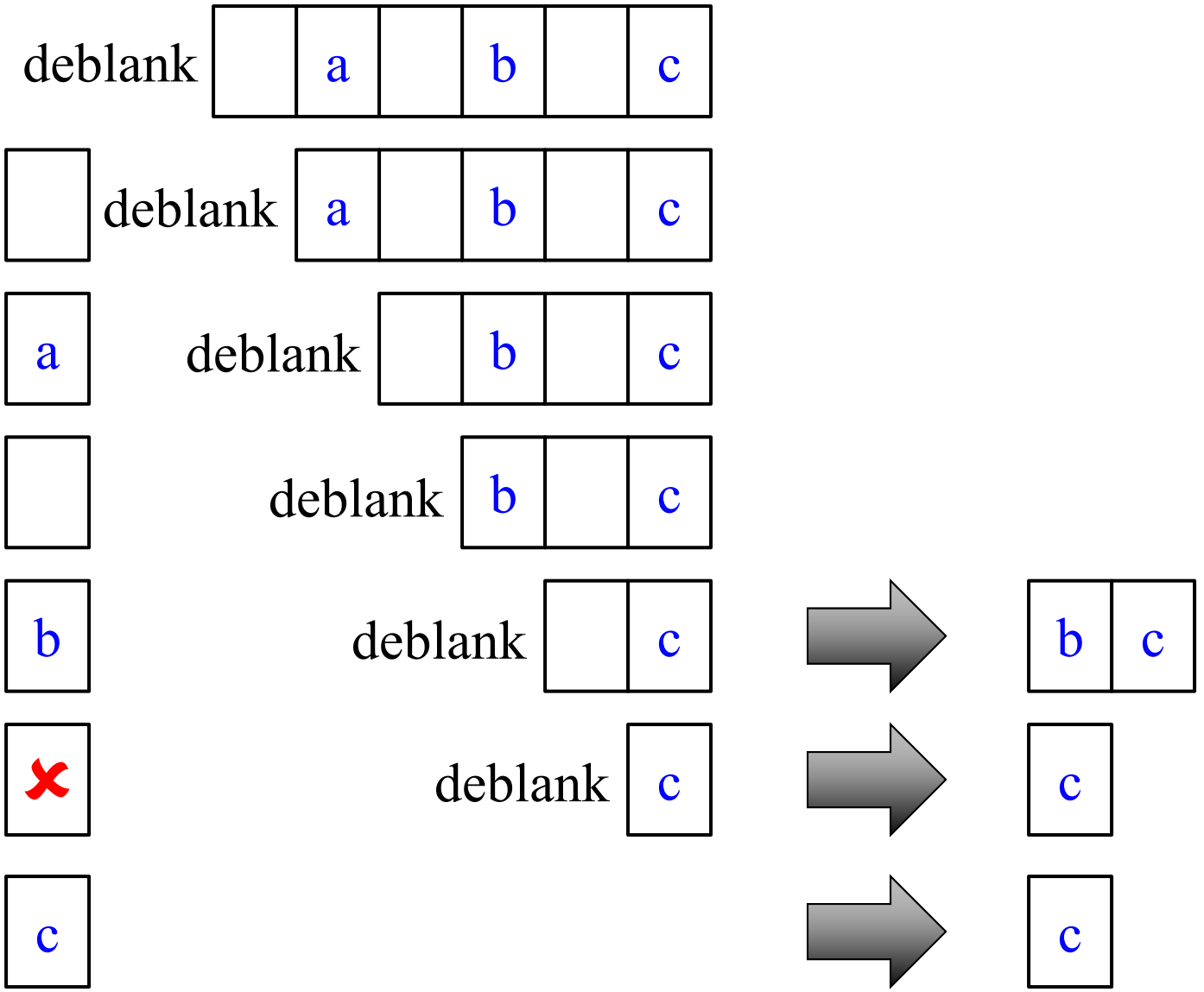
# Combining Left+Right (1)



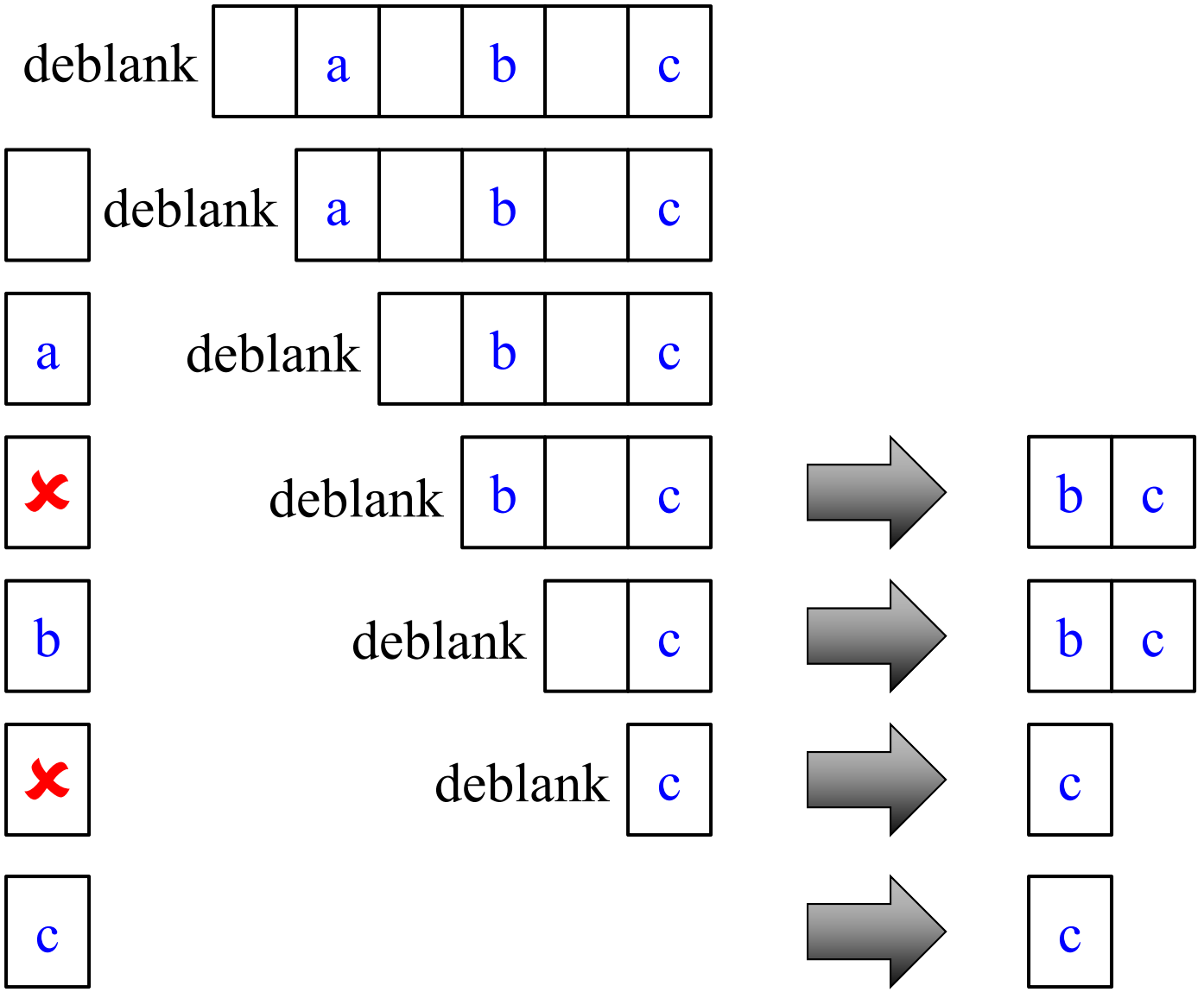
# Combining Left+Right (2)



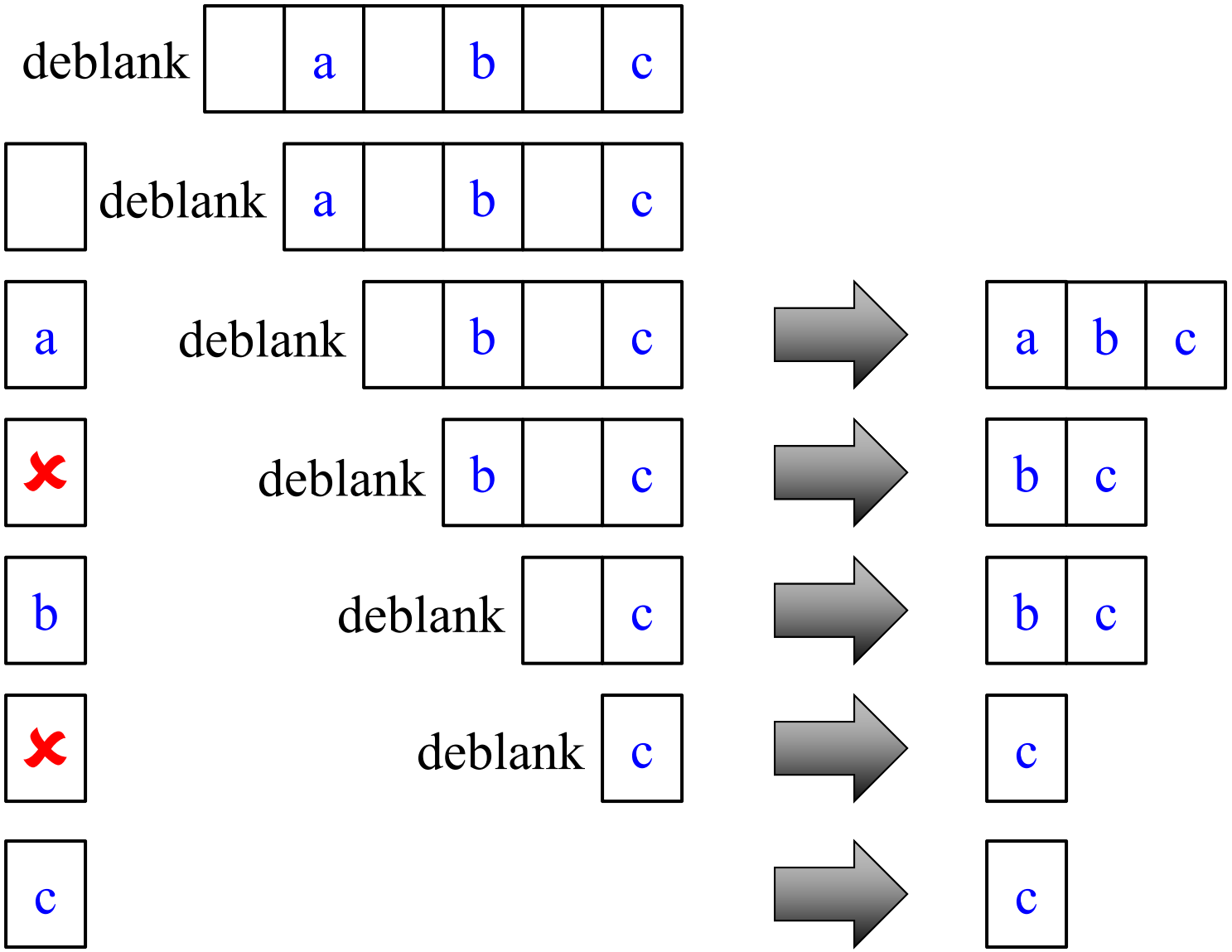
# Combining Left+Right (3)



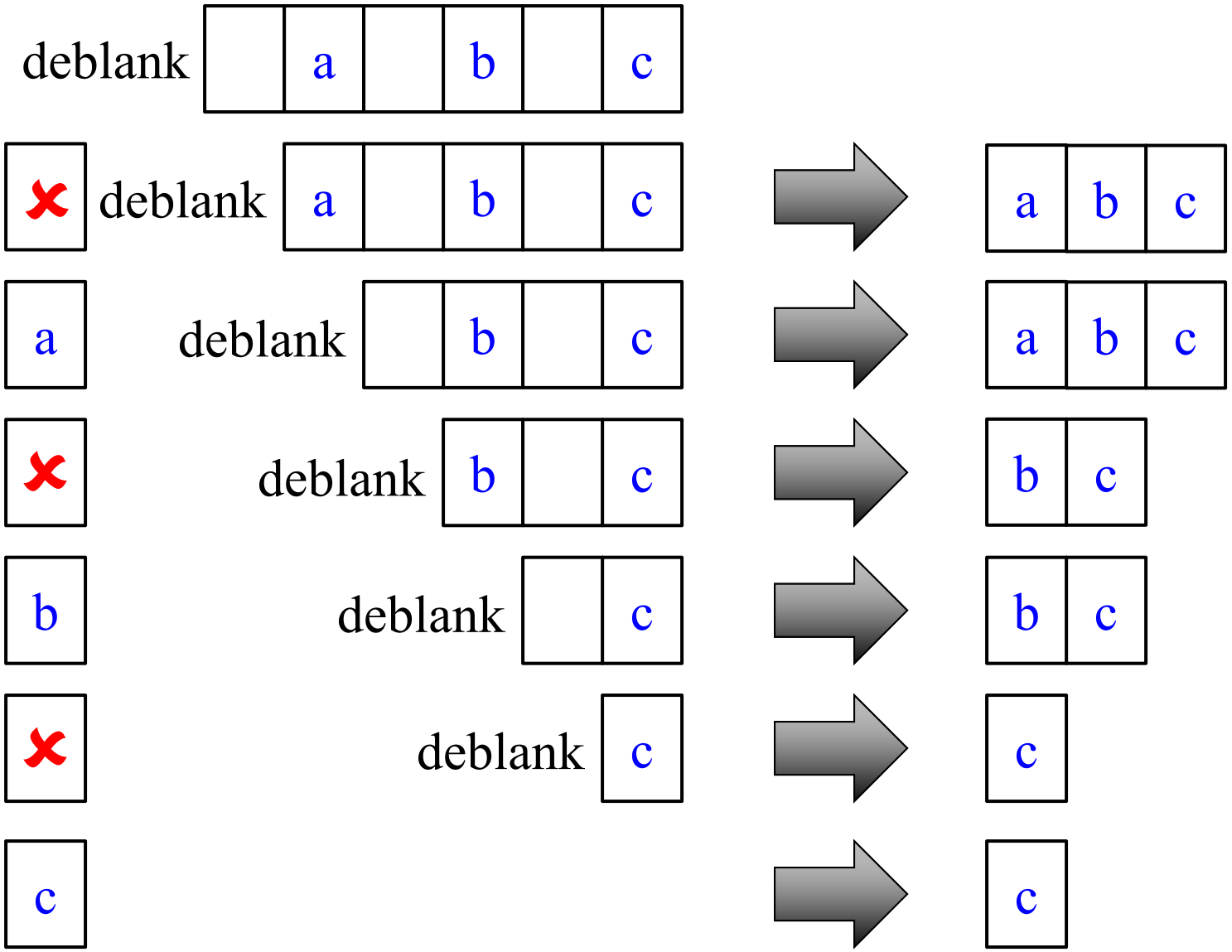
# Combining Left+Right (4)



# Combining Left+Right (5)



# Combining Left+Right (6)



# Combining Left+Right (7)