

**Topics:** 2-d example, simulation, course wrap-up

**Reading:** review MATLAB Essentials (previous handouts)

## Simulation of systems

Simulation is the application of mathematical and computer models that imitate the behavior of a system. Simulation is a useful tool for design, training, and games!

## Simple dice game

Simulate the rolling of a *fair* die. The function below allows the user to specify the number of rolls. Be careful about using the random number generator for generating integers *with equal probability*.

```
function freq = rollDice(rolls)
% Simulate rolling of fair 6-sided die
% Usage:  freq = rollDice(rolls)
%   ROLLS is the number of times to roll die
%   FREQ is vector of frequencies of possible outcomes

SIDES= 6;                                % number of sides on die

freq=                                     % bins for storing frequencies

% Roll FAIR die
allRolls=

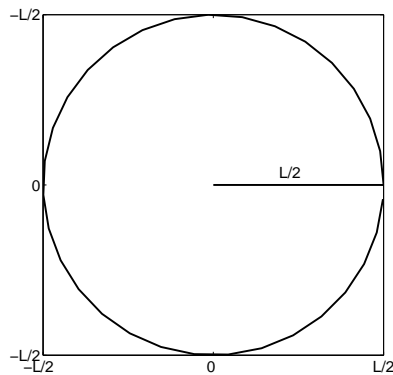
% Count outcomes


% Show histogram of outcome
% YOU ARE NOT RESPONSIBLE FOR LEARNING hist
hist(allRolls,1:SIDES);
title(['Outcomes from ' num2str(rolls) ' rolls of fair die']);
xlabel('Outcome'); ylabel('Frequency');
```

## Estimate Pi

The mathematical “constant”  $\pi$  can be approximated in many ways. One method is to use Monte Carlo simulations of dart throwing!

Let  $N$  be the number of darts thrown randomly over a square domain of area  $L \times L$ . The largest circle that can fit inside this domain has a diameter of  $L$  and an area of  $\pi L^2/4$ .



Let the number of darts  $N$  be the area of the square domain:

$$N = L \times L. \quad (1)$$

Then the number of darts that fall inside the circle,  $N_{in}$ , is the area of the circle:

$$N_{in} = \frac{\pi L^2}{4}. \quad (2)$$

Substitute equation (1) into (2) to get  $\pi$ :

$$\pi = \frac{4N_{in}}{N} \quad (3)$$

The following function performs Monte Carlo simulations of dart throwing. The function argument is the number of darts to be thrown.

```
function myPi = approxPi(nDarts)
% Approximate Pi using Monte Carlo simulations
% Usage: myPi = approxPi(nDarts)
% NDARTS is number of "darts" thrown
% myPi is Monte Carlo approximation of Pi, one trial only

L= 10; % length of square

% Throw darts in L-by-L area, centered at 0,0
throws=

x= throws(:,1); % x-coordinates of darts
y= throws(:,2); % y-coordinates of darts

% Location of darts relative to center
dist= % distance from center

nIn= % #darts inside circle

myPi= 4*nIn/nDarts;

% Plot darts in domain
% YOU ARE NOT RESPONSIBLE FOR LEARNING AXIS FORMATS
% Circle data
theta= 0:0.2:2*pi;
xcircle= cos(theta)*L/2;
ycircle= sin(theta)*L/2;
plot(xcircle,ycircle,'r',x,y,'*', 'linewidth',2)
axis([-L/2 L/2 -L/2 L/2]); axis('square');
title(['Pi = ' num2str(myPi)]);
```

## Simple dice game

```
function freq = rollDice(rolls)
% Simulate rolling of fair 6-sided die
% Usage:  freq = rollDice(rolls)
%  ROLLS is the number of times to roll die
%  FREQ is vector of frequencies of possible outcomes

SIDES= 6;                % number of sides on die
freq= zeros(1,SIDES); % bins for storing frequencies

% Roll FAIR die
allRolls= ceil(rand(1,rolls)*SIDES);

% Count outcomes
for i= 1:rolls
    freq(allRolls(i)) = freq(allRolls(i)) + 1;
end

% Show histogram of outcome
% YOU ARE NOT RESPONSIBLE FOR LEARNING hist
hist(allRolls,1:SIDES);
title(['Outcomes from ' num2str(rolls) ' rolls of fair die']);
xlabel('Outcome'); ylabel('Frequency');
```

## Estimate Pi

```
function myPi = approxPi(nDarts)
% Approximate Pi using Monte Carlo simulations
% Usage:  myPi = approxPi(nDarts)
%  NDARTS = number of "darts" thrown
%  myPi = Monte Carlo approximation of Pi

L= 10; % length of square

% Throw darts in L-by-L area, centered at 0,0
throws= L*rand(nDarts,2) - L/2;
x= throws(:,1); % x-coordinates of darts
y= throws(:,2); % y-coordinates of darts

% Location of darts relative to center
dist= sqrt(x.^2+y.^2); % distance from center
nIn= sum(dist <= L/2); % #darts inside circle

myPi= 4*nIn/nDarts;

% Plot darts in domain
% YOU ARE NOT RESPONSIBLE FOR LEARNING AXIS FORMATS
% Circle data
theta= 0:0.2:2*pi;
xcircle= cos(theta)*L/2;
ycircle= sin(theta)*L/2;
plot(xcircle,ycircle,'r',x,y,'*', 'linewidth',2)
axis([-L/2 L/2 -L/2 L/2]); axis('square');
title(['Pi = ' num2str(myPi)]);
```