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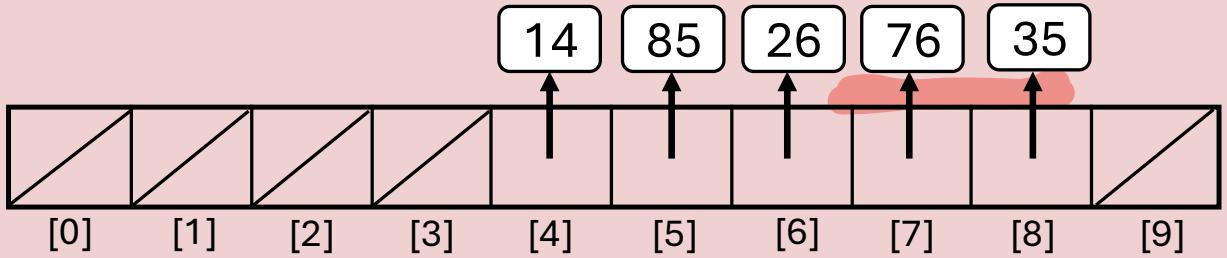
Suppose we add 14, 26, 35, 76, 85 (in some order) to a probing hash table (where their hash codes are their values). The state of the hash table is:



* Performance of probing hash tables depends on insertion order!

Which could *not* be the order these elements were added?

14, 85, 26, 76, 35	**(A)**	85, 26, 76, 35, 14	**(C)**
14, 26, 85, 35, 76	**(B)**	26, 14, 76, 85, 35	**(D)**

Lecture 21: Graphs

CS 2110

April 9, 2026

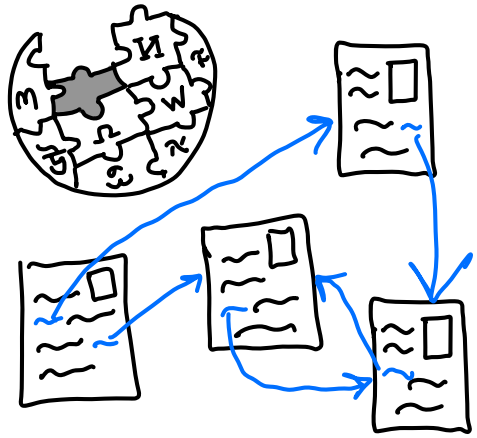
Today's Learning Outcomes

88. Translate between a formal description (list of vertices, edges) and an illustration of a (weighted) graph.

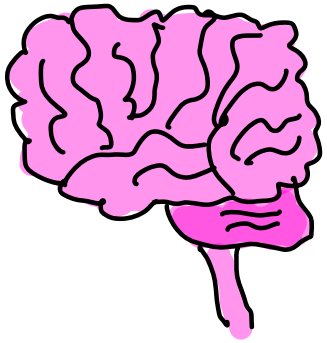
89. Identify substructures in graphs such as neighborhoods, paths, and cycles both visually and programmatically.

90. Describe adjacency list and matrix representations of a graph and compare the space/time complexities of operations on each of these representations.

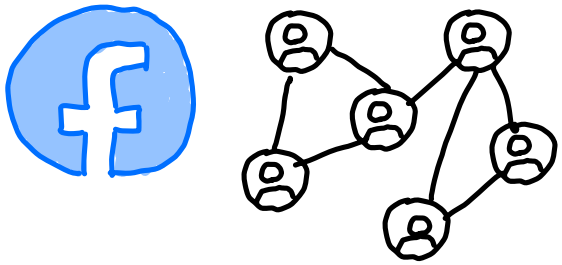
Modeling Structure in Real-World Settings



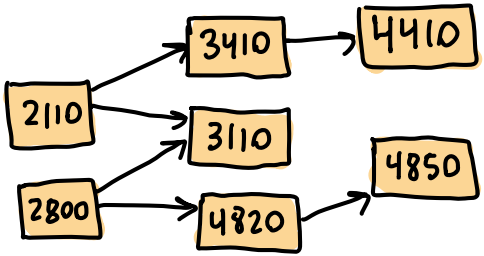
Wikipedia Links



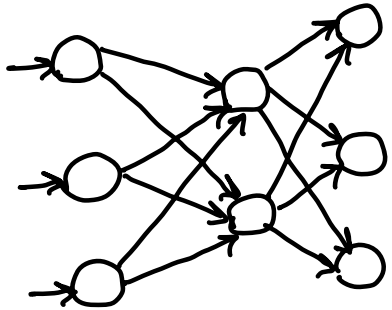
Neural Networks

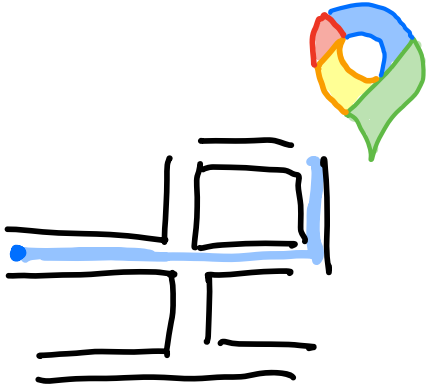


Social Networks



Course Planning





Route Planning

Directed Graphs

A directed graph is a structure described by two sets.

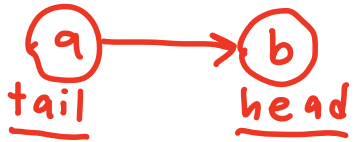
Vertices (V): units we're connecting

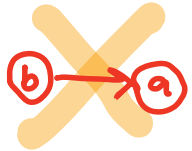
(people, addresses, cities, webpages, ...)

Edges (E): directed connections between pairs of vertices

(friendships, roads, flights, prereqs)

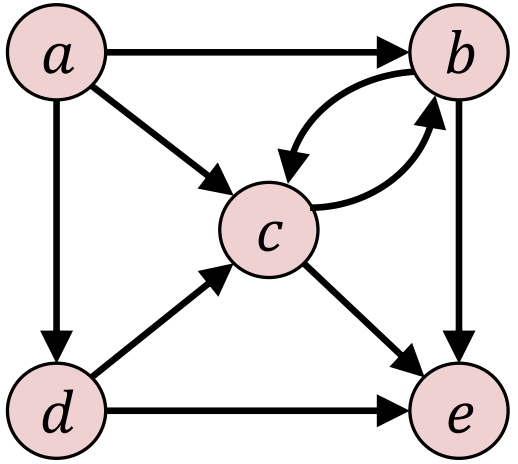
written as ordered pairs

(a,b) = 

not 

$V = \{a, b, c, d, e\}$

$E = \{(a,b), (a,c), (a,d), (b,c), (b,e), (c,b), (c,e), (d,c), (d,e)\}$



Other Graph Varieties

Undirected graphs: All edges connect in both directions

Self-loops: Edges that start/end at same vertex

Multigraphs: Have multiple parallel edges between the same pair of vertices (in same order)

Simple graphs don't have self-loops or parallel edges.

* We'll focus on simple directed graphs in CS2110.

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What is the maximum number of edges that a *simple* directed graph with 6 vertices can have?

6 choices for tail * 5 choices for head = 30 possible edges

More generally, a graph can have up to $|V| \cdot (|V|-1)$ $= O(|V|^2)$ edges

Connected graphs have

$|E| = O(|V|^2)$ ← upper bound

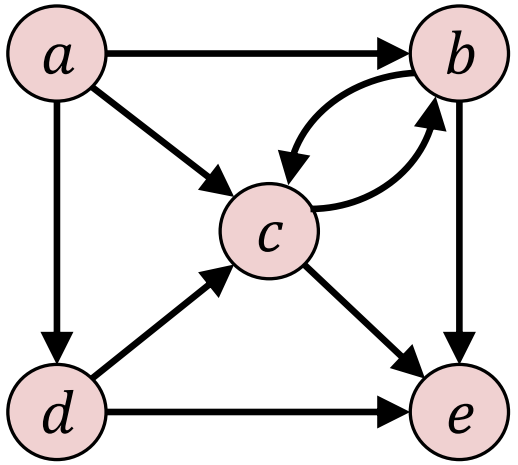
$|E| = \Omega(|V|)$ ← lower bound

Neighborhoods in Graphs

If $(u,v) \in E$, we say v is a neighbor (or an out-neighbor) of u.

u's neighborhood includes all of u's neighbors. $N_G(u)$

u's degree is the size of its neighborhood. $d_G(u)$



$N_G(c) = \{b, e\}$

$d_G(c) = 2$

Paths and Cycles

A path is a sequence of distinct contiguous edges in a graph.

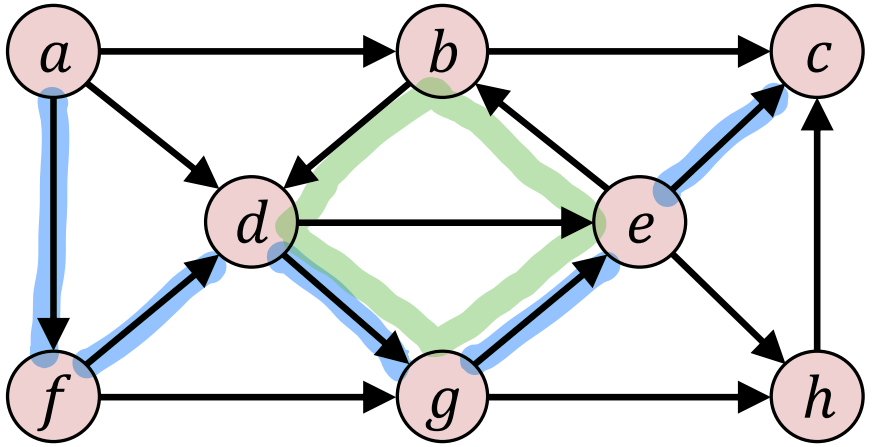
$P = ((v_0, v_1), (v_1, v_2), \ldots, (v_{\ell-1}, v_\ell))$ where $(v_0, v_1) \in E$, $(v_1, v_2) \in E$, $(v_{\ell-1}, v_\ell) \in E$

source: v_0; same vertex: v_1, v_2; destination: v_ℓ

length of path = # of edges (ℓ)

shorthand: $v_0 \to v_1 \to v_2 \to \ldots \to v_\ell$

a $v_0 \rightsquigarrow v_\ell$ path



Path $a \to f \to d \to g \to e \to c$ has length 5

Cycle $d \to g \to e \to b \to d$

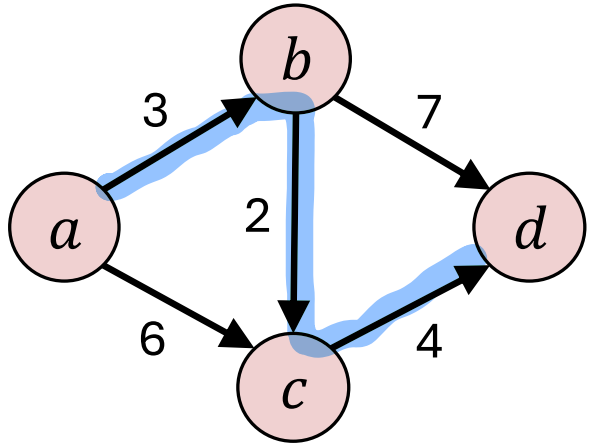
A cycle is a path with source = destination

Labeled Graphs

Sometimes, we label vertices/edges with extra info

Common: Label edges with number
- cost, weight, length

Length of paths overloaded term that refers to sum of edge labels along path



shortest a ⇝ d path

a → b → c → d

has length 9

Shortest path has minimum label sum, not necessarily fewest edges.

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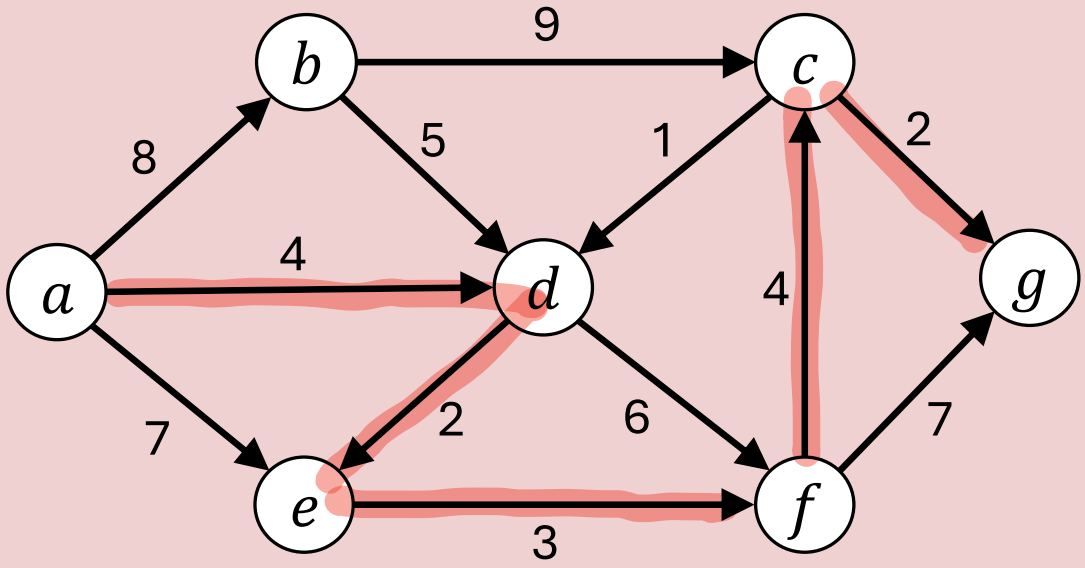
What is the length of the shortest $a \rightsquigarrow g$ path in this graph?



15	(A)
16	(B)
17	(C)
18	(D)

A Graph ADT

Brainstorm: What operations should a Graph support?

Query: Access info

- check for vertex
- check for edge
- get # vertices
- get # edges
- get edge weight
- get degree
- check for path/cycle

Mutate: Change

- add vertex
- add edge
- remove vertex
- remove edge
- update weight

Iterate

- over all vertices
- over all edges
- over neighbors of one vertex
- over edges in a path

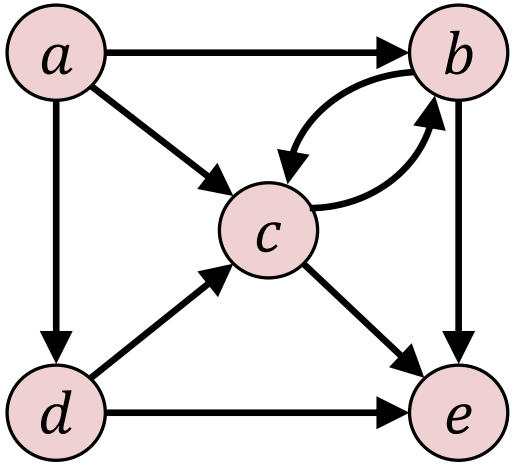
No built-in Graph ADT...

we'll need to build ourselves

Coding Demo: Graph ADTs

Adjacency Matrix Representation

Keep track of edges using 2D array/list structure



tail

head

	a	b	c	d	e
a		✓	✓	✓	
b			✓		✓
c		✓			✓
d			✓		✓
e					

can be booleans to indicate presence/absence or Edge variables with references/null

Coding Demo: AdjMatrixGraph

Adjacency Matrix Operation Complexities

```
class AdjMatrixGraph implements Graph<...> {
   record AdjMatrixEdge(...) implements Edge<...> { }

   class AdjMatrixVertex implements Vertex<...> {
      private String label;
      private int index;
   }

   private HashMap<String, AdjMatrixVertex> vertices;
   private ArrayList<ArrayList<AdjMatrixEdge>> edges;

   // methods
}
```

Iterate over neighbors: $O(|V|)$

Count vertices: $O(1)$ - `vertices.size()`

Count edges: $O(|V|^2)$

Check for an edge: $O(1)$ using vertices and index

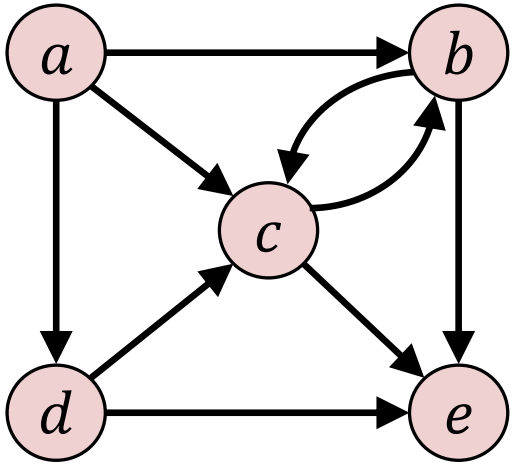
Add vertex: $O(|V|)$ amortized

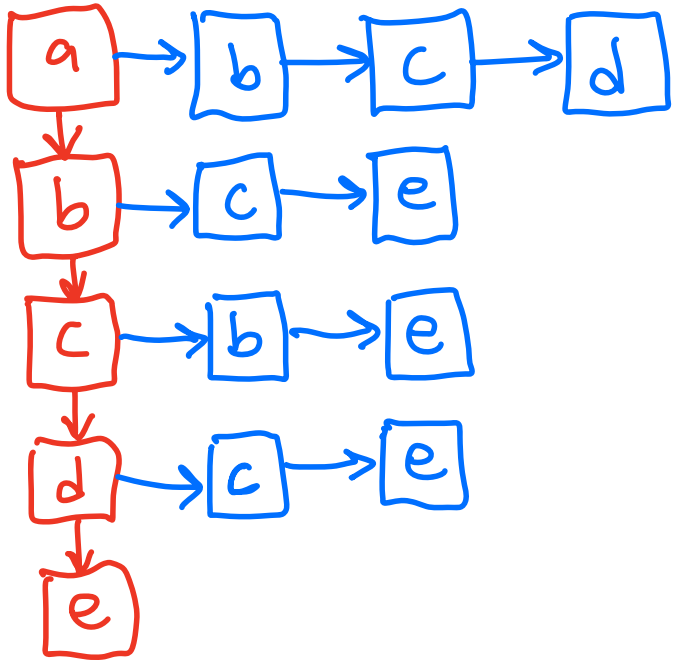
Add edge: $O(1)$

Iterate over all edges: $O(|V|^2)$

Adjacency List Representation (Classical)

Maintain a list of all vertices, and have each vertex maintain a list of all its neighbors





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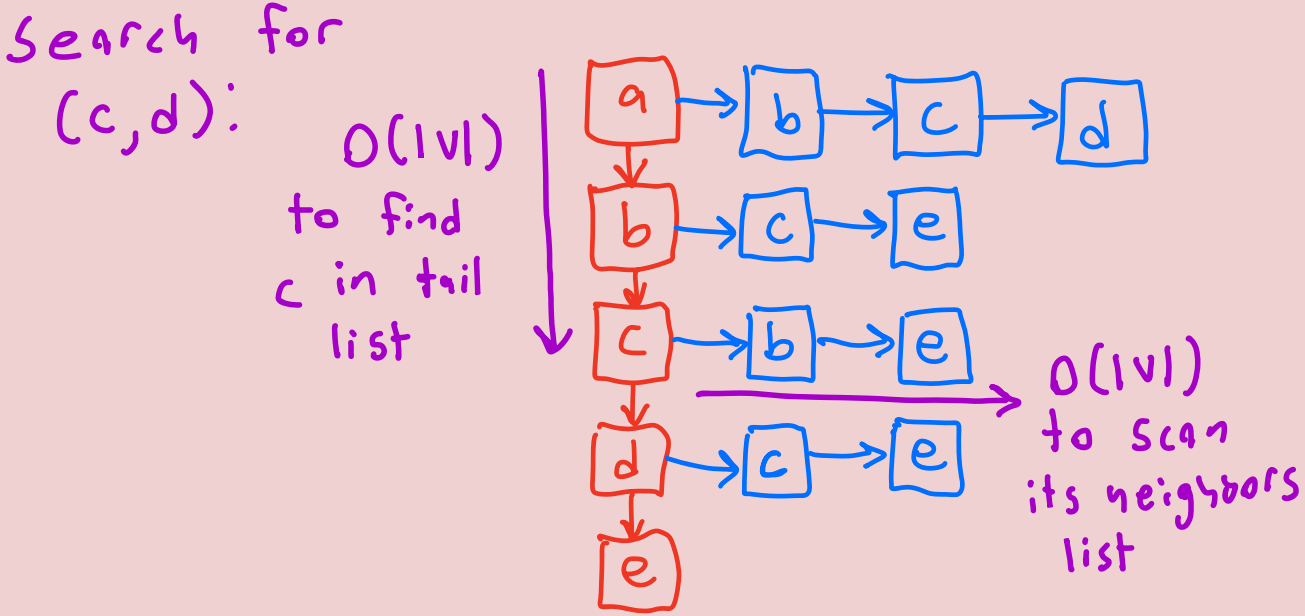
Given an adjacency list representation of a graph backed by nested `LinkedLists`, what is the tightest runtime complexity we can obtain for `hasEdge()`?



$O(\|V\|)$	**(A)**
$O(\|E\|)$	**(B)**
$O(\|V\| + \|E\|)$	**(C)**
$O(\|V\|^2)$	**(D)**

Adjacency List Representation (Improved)

Iterating over nested list structure is slow

- linear scan over vertices to find tail's neighbors list
- linear scan over neighbor list to search for edge

To improve efficiency: Replace both layers of lists with hash maps keyed on vertex labels

$O(|V|)$ scan → expected $O(1)$ lookup

Coding Demo: AdjListGraph

(We'll pick up from here in the next lecture.)

Adjacency List Operation Complexities

```
class AdjListGraph implements Graph<...> {
   record AdjListEdge(...) implements Edge<...> { }

   static class AdjListVertex implements Vertex<...> {
      String label;
      HashMap<String, AdjListEdge> outEdges;
   }

   private HashMap<String, AdjListVertex> vertices;

   // methods
}
```

Iterate over neighbors: $O(\text{degree}) = O(|V|)$

Count vertices: $O(1)$ using `vertices.size()`

Count edges: $O(|V|)$ using `outEdges.size()`

Check for an edge: $O(1)$ using map operations

Add vertex: $O(1)$

Add edge: $O(1)$

Iterate over all edges: $O(|V| + |E|)$

Comparing Representations

Representation	Memory Usage	Time to Check for an Edge	Time to Iterate over Neighborhood	Time to Iterate over all Edges
Adj Matrix	$O(\|V\|^2)$	$O(1)$	$O(\|V\|)$	$O(\|V\|^2)$
Adj List (Linked)	$O(\|V\| + \|E\|)$	$O(\|V\|)$	$O(\|V\|)$	$O(\|V\| + \|E\|)$
Adj List (Map)	$O(\|V\| + \|E\|)$	$O(1)$	$O(\|V\|)$	$O(\|V\| + \|E\|)$

dense graphs have $|E| \approx O(|V|^2)$: AM + AL have similar performance

memory layout likely makes AM ops faster

sparse graphs have $|E| \approx O(|V|)$: AL has much better performance

↖ most real-world graphs