CS 412/413

Introduction to

Compilers and Translators

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Lecture 27: Dataflow analysis theory

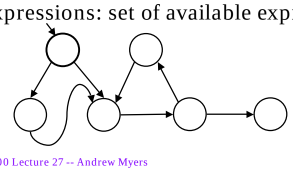
5 April 00

Administration

- Homework 4 due Monday
- Prelim review April 11, 7-9PM
- Prelim April 13, 7:30PM-9:30PM
  - static semantics, IR and assembly code generation, object-oriented languages, data-flow analysis, optimization

Dataflow analysis

- Abstractly: propagates dataflow values representing information about program through flowgraph. Space of values: *L*
- Solution: *in*[*n*], *out*[*n*]$\in L$ for every node *n*
- Live variable analysis: set of live variables
- Available expressions: set of available exprs



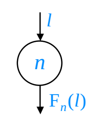
Dataflow analysis framework

Dataflow analysis characterized by:

1. Space of values L
2. Flow function F_n for every node *n*

$out[n] = F_n(in[n])$

$F_n : L \rightarrow L$



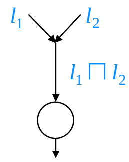
"If $l \in$ L is true before executing node *n*, $F_n(l)$ is true afterward"

Live vars: $F_n(l) = use[n] \cup (l - def[n])$

Combining operator

1. Space of values L
2. Flow function F_n for every node *n*
3. Combining operator $\sqcap$

"If we know either l_1 or l_2 holds on entry to *n*, we know at most $l_1 \sqcap l_2$"



$in[n] = \sqcap_{n' \in pred[n]} out[n']$

live vars: $\sqcap = \cup$ avail exprs: $\sqcap = \cap$

Iterative analysis

4. maximum information $\top \in$ L

```
for all n, in[n] = out[n] = ⊤
  repeat until no change
     for all n
          in[n] = ⊓n' ∈ pred[n] out[n']
          out[n] = Fn(in[n])
     end
  end
```

Questions

Will iterative analysis

- produce a solution when it terminates?
- produce the best solution possible?
- terminate?

- Depends on properties of L, F_n, $\sqcap$

L as partial order

- Best solution has as much information as possible – allows most optimization
  - Live variables: smallest possible set
  - Available expressions: largest possible set
- Some dataflow values contain more information: $l_1 \sqsubseteq l_2$ if l_2 has more information than l_1
- Live variables: $l_1 \sqsubseteq l_2 \Leftrightarrow l_1 \supseteq l_2$
- Available expressions: $l_1 \sqsubseteq l_2 \Leftrightarrow l_1 \subseteq l_2$

Partial orders

- L is a *partial order* defined by ordering operator $\sqsubseteq$
- Some elements are incomparable
- Properties of a partial order

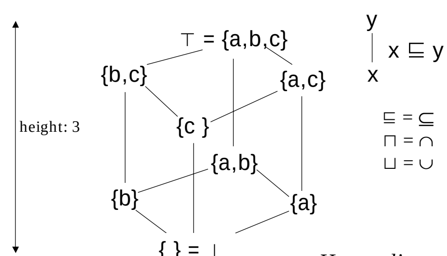
$$x \sqsubseteq x \qquad \textit{(reflexive)}$$

$$x \sqsubseteq y \;\&\; y \sqsubseteq z \Rightarrow x \sqsubseteq z \qquad \textit{(transitive)}$$

$$x \sqsubseteq y \;\&\; y \sqsubseteq x \Rightarrow x = y \qquad \textit{(anti-symmetry)}$$

- Examples: integers ordered by $\leq$, types ordered by <:, sets ordered by $\subseteq$ or $\supseteq$.

Example: subsets of {a,b,c}



Hasse diagram

Greatest lower bound

- Combining operator $l_1 \sqcap l_2$ gives element l such that $l \sqsubseteq l_1$, $l \sqsubseteq l_2$
- l is a *lower bound* for l_1, l_2
- Want *greatest* such element (most info): *greatest lower bound* (GLB)
- Partial order with GLB/meet ($\sqcap$) and LUB/join ($\sqcup$) is a *lattice*
- With only GLB, a *lower semi-lattice*

Meet-over-paths solution

- Consider a traversal of flowgraph visiting nodes $a, b, c, \ldots, n$
- Assume l_0 is initial information
- Information known is
  $F_n(\ldots(F_c(F_b(F_a(l_0)))))$
- Best possible solution is l such that
  $l \sqsubseteq F_n(\ldots(F_c(F_b(F_a(l_0)))))$
  for *all* paths $a, b, c, \ldots, n$
- MOP soln: $\bigsqcap_{\textit{all paths } p} F_{p1}(F_{p2}(F_{p3}(\ldots)))$

Data-flow equations

- Algorithm repeatedly recomputes each *out*[*n*] as

$$F_n(\sqcap_{n' \in pred[n]} out[n'])$$

- Let $x_1 \ldots x_n$ be *out*[1]...*out*[*n*]. Algorithm:

$$x_i = F_i(\sqcap_{j \in pred[i]} x_j)$$

- Solution is point in L^n : $\mathbf{X} = (x_1,\ldots x_n)$
- Total set of equations is $\mathbf{X} = F(\mathbf{X})$ where $F(x_1,\ldots x_n) = (F_1(\sqcap_{j \in pred[1]} x_j), F_2(\ldots),\ldots)$

Fixed points

- Iterative analysis: initialize all x_i with top of lattice ($\mathbf{X}_0 = (\top, \top, \top,\ldots)$), apply $F(\mathbf{X})$ until fixed point is reached: $F^k(\mathbf{X}_0) = F^{k+1}(\mathbf{X}_0)$
- $F^k(X_0)$ is a fixed point of F: a value that F maps to itself
- Wanted: maximal fixed point (we know that minimum-information solution $\perp$ works)

Monotonicity

- Flow functions map lattice values to other lattice values; must be *monotonic*
- Monotonicity:

$$l_1 \sqsubseteq l_2 \Rightarrow F(l_1) \sqsubseteq F(l_2)$$

"If you have more information entering a node, you have at least as much leaving"

- Example: *reaching definitions*. Lattice is all sets of defining nodes ordered by subset relation:

$$F_n(x) = gen[n] \sqcup (x - kill[n])$$

Termination

- First step either lowers some x_i or terminates
- Second step sees same x_i as first step ($\top$), or possibly lower: $F(\mathbf{X}_0) \sqsubseteq \mathbf{X}_0$
- Monotonicity $l_1 \sqsubseteq l_2 \Rightarrow F(l_1) \sqsubseteq F(l_2)$

$\Rightarrow$ output of second step $F^2(\mathbf{X}_0)$ is lower than first step (or it terminates): $F^2(\mathbf{X}_0) \sqsubseteq F^1(\mathbf{X}_0)$

- Induction: each iteration moves at least one node lower in lattice: $F^{i+1}(\mathbf{X}_0) \sqsubseteq F^i(\mathbf{X}_0)$
- # algorithm steps to fixed point is at most *height* of lattice *H* times number of nodes n: k = $O(nH)$

Solution quality

- MOP is best possible solution:

$$\sqcap_{all\ paths\ p} F_{p1}(F_{p2}(F_{p3}(\ldots)))$$

- Does iterative analysis

$$x_i = F_i(\sqcap_{j \in pred[i]} x_j)$$

produce the MOP solution?

- Flow functions must *distribute* over the meet operator:

$$\sqcap_i F(x_i) = F(\sqcap_i x_i)$$

Reaching definitions

- L is all sets of defining nodes in call flow graph. Maximum information means *smallest possible* lists of reaching definitions, so:
- Top ($\top$) is the empty set { }, meet ($\sqcap$) is set union ($\cup$)

$$x_n = out[n]$$

$$F_n(x) = gen[n] \cup (x - kill[n])$$

$$x_i = F_i(\sqcap_{j \in pred[i]} x_j) \Rightarrow \begin{array}{l} in[n] = \bigcup_{n' \in prev[n]} out[n'] \\ out[n] = gen[n] \cup (in[n] - kill[n]) \end{array}$$

Monotonic?

$F_n(x) = gen[n] \cup (x - kill[n])$

$x_1 \sqcap x_2 = x_1 \cup x_2$

- Is $F_n(x)$ monotonic?

If $x \sqsubseteq y$, $x \supseteq y$

$F_n(x) = gen[n] \cup (x - kill[n]) =$

$gen[n] \cup ((x \cup y) - kill[n]) =$

$gen[n] \cup (x - kill[n]) \cup (y - kill[n]) \supseteq F_n(y)$

MOP?

$F_n(x) = gen[n] \cup (x - kill[n])$

$x_1 \sqcap x_2 = x_1 \cup x_2$

- Does $F_n(x)$ distribute over $\sqcap$?

$F_n(x \sqcap y) = F_n(x \cup y)$
$= gen[n] \cup ((x \cup y) - kill[n])$
$= (gen[n] \cup (x - kill[n]))$
$\cup (gen[n] \cup (y - kill[n]))$
$= F_n(x) \cup F_n(y) = F_n(x) \sqcap F_n(y)$

∴ Iterative analysis always terminates, finds the best possible (meet-over-paths) solution to reaching defintions

Other analyses

- Live variables

$F_n(l) = use[n] \cup (l - def[n])$

$\sqcap = \cup$

- Available expressions

$F_n(l) = gen[n] \cup (l - kill[n])$

$\sqcap = \cap$

- Computes MOP solutions?

Summary

- Standard optimizations require data-flow analyses that fit into data-flow analysis framework
- Iterative analysis finds solution if flow function monotonic in $\sqsubseteq$, combining function $\sqcap$ defines semi-lattice
- Solution is MOP if distribution condition $\sqcap_i F(x_i) = F(\sqcap_i x_i)$ holds